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ON THE INFRARED BEHAVIOR OF THE PRESSURE IN THERMAL FIELD THEORIES

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On the Infrared Behavior of the Pressure in Thermal Field Theories

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We study non-perturbatively, via the Schwinger-Dyson equations, the leading infrared behavior of the pressure in the ladder approximation. This problem is discussed firstly in the context of a thermal scalar field theory, and the analysis is then extended to the Yang-Mills theory at high temperatures. Using the Feynman gauge, we find a system of two coupled integral equations for the gluon and ghost self-energies, which is solved analytically. The solutions of these equations show that the contributions to the pressure, when calculated in the ladder approximation, are finite in the infrared domain.

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I. INTRODUCTION

Relativistic field theories at finite temperature have been actively studied in the past years, because of their relevance to the theory of the early universe and to the quark-gluon plasma which may be created in heavy ion collisions[1]. The study of such physical problems requires the computation of the thermodynamic potential, from which other thermodynamic properties, like the pressure, may be determined. It was pointed out by Linde[2] that the thermodynamic potential of the Yang-Mills theory cannot be calculated beyond the fifth order of the coupling constant. This is related with the infrared singularities of the non-abelian gauge theories at finite temperatures, which arise from the fact that the magnetic mass vanishes at least up to second order in perturbation theory. The infrared problem in these theories at finite temperature is qualitatively different from that at $T = 0$, manifesting itself by the presence of infrared power divergences in higher orders of the perturbative expansion.

One may hope that the use of non-perturbative methods can throw some light on this problem. Such methods generally start with a discussion of the relevant Schwinger-Dyson equations[3]. It is well known that this set of integral equations is not closed since an n-point function is generally related to the $n+1$ and $n+2$ functions. This is what makes the solutions of such problems exceedingly difficult. For this reason one must resort in practice to some approximations, which result from truncating in some way the set of the Schwinger-Dyson equations. This non-perturbative approach has been used by Jackiw and Templeton[4], to show how super-renormalizable interactions might cure their infrared divergences. It has also been employed by Mandelstam[5] to analyze the infrared behavior of the gluon propagator,using the ladder approximation at $T = 0$. He showed that the ensuing set of the Schwinger-Dyson equations does provide confinement in the strong coupling regime.

The purpose of this work is to study, in the ladder approximation, the leading infrared behavior of the pressure in the Yang-Mills theory at high temperatures. Our approach, via the Schwinger-Dyson equations, is similar in spirit with that advocated by Kajantie and Kapusta[6]. Namely, we regard the simplified set of these equations just as a convenient procedure for summing particular classes of an infinite number of diagrams in the weak coupling regime. In order to regularize the theory in the event that no magnetic masses are generated, we shall put an infrared cut-off λ on momenta integrations which arises when calculating the contributions to the pressure. Our aim is to study the possibility of cancellation of the infrared singularities in the limit $\lambda \to 0$, when summing the whole class of diagrams which contribute in the ladder approximation.

As we shall see, even this more modest task is non-trivial and we begin by considering in Section II the scalar $g\phi^3$ theory in six dimensions. This model has some similarities with the Yang-Mills theory, such as a dimensionless coupling constant and asymptotic freedom. Strictly speaking, in this case one could employ the resummation method developed by Braaten and Pisarski[7], because a scalar thermal mass is generated in lowest order. Indeed, such a procedure leads to an infrared finite expression for the pressure[8]. However, we will use here instead the method described above, in order to illustrate in a simple way some relevant features which will appear in the Yang-Mills theory. We consider the truncated set of Schwinger-Dyson equations which yield consistently the scalar self-energy in the ladder approximation. The ensuing integral equation is equivalent, under certain conditions to be discussed later, to a second order differential equation which can be solved analytically in terms of modified Bessel functions. Using this solution, we show that the corresponding contribution to the pressure obtained in this approximation, is finite in the limit $\lambda \to 0$.

In Section III we consider the leading infrared behavior of the $SU(N)$ Yang-Mills

theory at high temperatures , in the ladder approximation. We work in the Feynman gauge and obtain a system of two coupled Schwinger-Dyson integral equations, linking the gluon polarization tensor and the ghost self-energy function. Using the same reasoning as in the previous section, we show that this system is equivalent to a fourth order differential equation which describes in this approximation the leading infrared behavior. The solution of this equation is expressed in terms of modified Bessel functions with complex argument and the corresponding complex conjugate functions. We find that the relevant contributions to the pressure, as calculated in the ladder approximation, are finite in the infrared limit $\lambda \to 0$. Some of the mathematical details which arise during these calculations are given in the Appendices A and B. Finally, in Appendix C we consider the possibility of the cancellation of the infrared divergences in the pressure, beyond the ladder approximation.

II. THE SIX-DIMENSIONAL SCALAR THEORY

We consider here the scalar theory with a $g\phi^3$ interaction at the temperature T. In this theory the formula giving the pressure in the ladder approximation is rather simple and can be expressed directly in terms of the self-energy function $\tilde{\Pi}$. The Schwinger-Dyson equation for the scalar self-energy in the ladder approximation is shown in Fig. (1). Note that the diagrams contributing to $\tilde{\Pi}$ have the same combinatorial factor $\frac{1}{2}$ to all orders in perturbation theory. The corresponding contributions to the pressure are represented in Fig. (2.a).

In order to derive the expression giving the pressure, we use the relation between its functional derivative and the self-energy function[1]:

$$\left(\frac{\delta P}{\delta \mathcal{D}_0}\right)_{1\mathrm{PI}} = -\frac{T}{2}\tilde{\Pi} \qquad (1$$

where $\mathcal{D}_0$ is the free particle propagator and 1PI indicates that only one particle

irreducible diagrams contribute to $\tilde{\Pi}$. With the help of the graphical representation given in Fig. (2.a), it is easy to see that (1) implies the relation:

$$P = P_0 - \frac{T}{2}\frac{1}{4}\sum_{p_0}\int \frac{d^5p}{(2\pi)^5}\frac{1}{p^2+p_0^2}\left[\tilde{\Pi}(p,p_0) + \frac{1}{3}\tilde{\Pi}^{(1)}(p,p_0)\right] \tag{2}$$

The factor $\frac{1}{4}$ arises because in general there are only four ways we can cut the diagram (2.a) in order to get $\tilde{\Pi}$ in the ladder approximation. The only exception occurs in the lowest order, where the factor is $\frac{1}{3}$ instead. The graphical representation of this formula is shown in Fig. (2.b).

The sum over p_0 should be taken over even frequencies $p_0 = 2\pi nT$. However, the dominant infrared contributions arise only from the terms with zero frequency ($n = 0$). When calculating the contributions from eq. (2) corresponding to this mode, we can put an ultraviolet cut-off of order T on the momentum integration. This cut-off arise naturally when summing over all modes n. As mentioned in the Introduction, we shall also put a cut-off λ in order to regularize the perturbative infrared divergences. In this way we find that the leading infrared contributions to the pressure are given by:

$$P = P_0 - \frac{T}{8}\int_\lambda^T \frac{d^5p}{(2\pi)^5}\frac{1}{p^2}\tilde{\Pi}(p,p_0=0) + \dots \tag{3}$$

where dots denote additional subleading and infrared convergent contributions.

Then, using the Schwinger-Dyson equation for the scalar self-energy function we arrive at the following integral equation for $\tilde{\Pi}(p,p_0=0)$:

$$\tilde{\Pi}(p) \equiv \Pi(p) + p^2 = -\frac{g^2T}{(2\pi)^5}\int_\lambda^T \frac{d^5k}{(\mathbf{p}+\mathbf{k})^2}\left[\frac{1}{2k^2} + \frac{\Pi(k)}{k^4}\right] \tag{4}$$

In order to solve this integral equation analytically, we will try to convert it into a Volterra type. This is possible provided we adopt the following maximization procedure. Let us define:

$$(\mathbf{p}+\mathbf{k})^2_{\max} \equiv \begin{cases} p^2 & \text{for } p > k \\ k^2 & \text{for } k > p \end{cases} \tag{5}$$

Using this approach, we then obtain from eq. (4):

$$\Pi_{\max}(p) = -p^2 + \frac{g^2T}{12\pi^3}\left[\frac{p}{3} - \frac{T}{2} + \frac{1}{6}\frac{\lambda^3}{p^2} - \frac{1}{p^2}\int_\lambda^p dk\Pi_{\max}(k) - \int_p^T \frac{dk}{k^2}\Pi_{\max}(k)\right] \tag{6}$$

Comparing this with the original equation (4), we note that the first iteration of (6) with $\lambda \to 0$ yields:

$$\Pi^{(1)}_{\max} = -p^2 + \frac{g^2T^2}{24\pi^3}\left[1 - \frac{2}{3}\frac{p}{T}\right] \tag{7}$$

a result which completely agrees in the high temperature limit with the leading contribution which follows from (4). To next order, our procedure yields in this regime results which are in satisfactory agreement with the ones obtained from equation (4), which becomes increasingly cumbersome to handle . Since we wish to study only the main features of the leading infrared behavior, it will be sufficient for our purpose to restrict our attention to the Volterra equation (6). For simplicity of notation we shall drop in what follows the suffix max appearing in (6). After successive iterations, we encounter the presence of infrared divergent terms in the perturbative series of Π:

$$\Pi(p) = -p^2 + \frac{\alpha}{2}T^2 - \alpha^2T^2\left(\frac{T}{p}\right) + \alpha^3T^2\left(\frac{T}{p}\right)^2 ln\left(\frac{p}{\lambda}\right) - \frac{7}{6}\alpha^4T^2\left(\frac{T}{p}\right)^3\frac{p}{\lambda} + \\ + \frac{19}{36}\alpha^5T^2\left(\frac{T}{p}\right)^4\left(\frac{p}{\lambda}\right)^2 - \frac{149}{540}\alpha^6T^2\left(\frac{T}{p}\right)^5\left(\frac{p}{\lambda}\right)^3 + \dots \tag{8}$$

where $\alpha \equiv \frac{g^2}{12\pi^3}$.

With the help of this result and using equation (3), we find that the perturbative expansion of the pressure exhibits power infrared divergences given by:

$$P_{\rm div} = -\frac{T^6}{96\pi^3}\left[\alpha^3 ln\frac{T}{\lambda} - \frac{7}{6}\alpha^4\left(\frac{T}{\lambda}\right) + \frac{19}{36}\alpha^5\left(\frac{T}{\lambda}\right)^2 - \frac{149}{540}\alpha^6\left(\frac{T}{\lambda}\right)^3 + \ldots\right] \qquad (9)$$

Despite the fact that equation (6) cannot be solved perturbatively because the iterations yield infrared divergent terms, we will show that nevertheless, a non-perturbative solution does exist. To this end, it is convenient to define:

$$x \equiv \frac{\alpha T}{p} \qquad ; \qquad f(x) \equiv \frac{\Pi}{p^2} \qquad (10)$$

Then (6) reduces, dropping the inhomogeneous λ^3 term, to the following integral equation:

$$f(x) = -1 + \frac{x}{3} - \frac{x^2}{2\alpha} - x^2\int_\alpha^x \frac{dy}{y^2}f(y) - x^4\int_x^{\frac{\alpha T}{\lambda}} \frac{dy}{y^4}f(y) \qquad (11)$$

Perturbation theory now corresponds to solving (11) by a power series in x, a procedure which, as we have seen, yields power infrared divergences.

But the differential equation which follows from (11):

$$x^2 f'' - 5xf' + (8-2x)f = x - 8 \qquad (12)$$

has a well behaved solution. The complementary one involves modified Bessel functions[9] and two constants:

$$f_c(x) = Ax^3 I_2(\sqrt{8x}) + Bx^3 K_2(\sqrt{8x}) \qquad (13)$$

Using standard methods we can then find a particular solution in terms of these modified Bessel functions. The general solution, consisting of the sum of $f_c(x)$ and the particular one has the form:

$$f(x) = \left[A + 2\int_x^{\frac{\alpha T}{\lambda}} dt\left(\frac{8}{t^4} - \frac{1}{t^3}\right)K_2(\sqrt{8t})\right]x^3 I_2(\sqrt{8x}) + \\ + \left[B + 2\int_\alpha^x dt\left(\frac{8}{t^4} - \frac{1}{t^3}\right)I_2(\sqrt{8t})\right]x^3 K_2(\sqrt{8x}) \qquad (14)$$

Substituting this into the integral equation (11), fixes the constants A and B in terms of the parameters $\frac{\alpha T}{\lambda}$ and α. Since I_2 grows exponentially at large x, it would produce a divergence in the integral equation as $\lambda \to 0$, unless $A = 0$ in this limit. Indeed, the consistency conditions on $f(x)$ at $x = \frac{\alpha T}{\lambda}$ and $x = \alpha$, demand A to vanish when $\lambda \to 0$, while B becomes:

$$B = \frac{1}{K_3(\sqrt{8\alpha})}\left[2I_3(\sqrt{8\alpha})\int_\alpha^\infty dt\left(\frac{8}{t^4} - \frac{1}{t^3}\right)K_2(\sqrt{8t}) - \sqrt{\frac{8}{\alpha^7}}\right] \qquad (15)$$

Then, since K_2 decreases exponentially at large x, it is not difficult to show that $f(x)$ is a decreasing function of x in this domain. Because large values of x correspond to small values of the momenta [see eq.(10)], this behavior leads to a convergent integral in (3) as $\lambda \to 0$. Thus, the pressure calculated in the ladder approximation remains finite in the infrared domain. This also happens in the case of the Yang-Mills theory to which we now turn.

III. THE YANG-MILLS THEORY

We now consider the leading infrared behavior of the pressure in thermal Yang-Mills theory. Since the $\tilde{\Pi}_{00}(p \to 0, p_0 = 0)$ component of the polarization tensor is non-zero, the longitudinal gluons are screened at large distances. As a result, the infrared problem is related in this case only to the behavior of the transverse part of the gluon propagator. Using the fact that at finite temperatures, the leading infrared contributions arise from terms with zero frequencies, we can reduce our problem to a study in 3-dimensional Euclidian Yang-Mills theory.

Although the analysis is now more complicated due to the presence of the 4-gluon couplings and ghosts, it proceeds in parallel with that described previously in the Section II. In particular, in order to derive the expression for the pressure in the ladder approximation, we start from a basic relation which is the analogue of eq. (1)

By a reasoning similar to that used in deriving eq. (3), we find that the expression giving the leading infrared contributions to the pressure has the form:

$$P = P_0 - T\int_\lambda^T \frac{d^3p}{(2\pi)^3}\frac{1}{p^2}\left\{\frac{1}{8}\tilde{\Pi}^{aa}_{ii}(p) - \frac{1}{4}\tilde{\Sigma}^{aa}(p) + \frac{1}{16}\frac{T^2}{2\pi^2 p^2}V^{abcc}_{ijkk}\tilde{\Pi}^{ab}_{ij}(p) + \ldots\right\} \tag{16}$$

This relation is represented graphically in Fig. (3). Here $\tilde{\Pi}$ and $\tilde{\Sigma}$ denote respectively the gluon and ghost self-energy functions, which must be calculated consistently in the ladder approximation. The vertex V stands for the bare gluon four-point function and dots indicate infrared convergent contributions associated with two-loop diagrams. Note that expression (16) is much simpler than the exact formula giving the pressure in the Yang-Mills theory[10].

We now proceed to investigate, in the Feynman gauge, the infrared behavior of the gluon polarization tensor and the ghost self-energy function. The corresponding Schwinger-Dyson (S-D) equations in the ladder approximation are shown in Fig. (4).

It is well known that any approximation of the S-D equations may impose severe constraints in the case of a gauge theory. The reason is that the Ward identities, which reflect its underlying gauge invariance, are satisfied only when we take into account all the relevant contributions, order by order in perturbation theory. In particular, the transversality property of the polarization tensor is guaranteed only by the full set of S-D equations. Since it is impossible to implement this program in practice, we follow the procedure adopted by Mandelstam[5], neglecting the longitudinal terms which might arise in connection with the approximate set of S-D equations. This is obviously correct to lowest order in perturbation theory, where our polarization tensor is manifestly transverse. In higher orders, some of the contributions associated with the exact 3-point and 4-point gluon vertices will cancel these longitudinal terms. Since in the ladder approximation the vertex corrections are also neglected, the above

procedure is justified and, as we shall see, a consistent solution of the approximate set of S-D equations does exist.

Thus, using the transversality property of the gluon polarization tensor $\tilde{\Pi}^{ab}_{ij} = \delta^{ab}\tilde{\Pi}_{ij}$,we can write it in the form:

$$\tilde{\Pi}_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right)(\Pi_T + p^2) \tag{17}$$

In this way, we arrive at the following set of coupled integral equations which relate the gluon polarization tensor to the ghost self-energy $\tilde{\Sigma}^{ab} = \delta^{ab}(\Sigma + p^2)$:

$$\begin{aligned}\Pi_T = -p^2 - \frac{11g^2NT}{48\pi^3}\int\frac{d^3k}{k^2} - \frac{g^2NT}{16\pi^3}\int\frac{d^3k}{k^4}\frac{2(2k^2+3p^2)(\mathbf{p}\cdot\mathbf{k})^2 - p^2k^2(6p^2+7k^2)}{p^2(\mathbf{p}+\mathbf{k})^2} - \\ -\frac{g^2NT}{24\pi^3}\int\frac{d^3k}{k^4}\Pi_T(k) + \frac{g^2NT}{8\pi^3}\int\frac{d^3k}{k^6}\frac{5p^2k^2 - 3(\mathbf{p}\cdot\mathbf{k})^2}{p^2(\mathbf{p}+\mathbf{k})^2}(p^2+k^2)\Pi_T(k) + \\ +\frac{g^2NT}{8\pi^3}\int\frac{d^3k}{k^4}\frac{p^2k^2 - (\mathbf{p}\cdot\mathbf{k})^2}{p^2(\mathbf{p}+\mathbf{k})^2}\Sigma(k)\end{aligned} \tag{18}$$

$$\begin{aligned}\Sigma(p) = -p^2 - \frac{g^2NT}{8\pi^3}\int\frac{d^3k}{k^4}\frac{(\mathbf{k}\cdot\mathbf{p})^2}{(\mathbf{p}+\mathbf{k})^2} + \frac{g^2NT}{8\pi^3}\int\frac{d^3k}{k^4}\frac{(\mathbf{p}\cdot\mathbf{k})}{(\mathbf{p}+\mathbf{k})^2}\Sigma(k) - \\ -\frac{g^2NT}{8\pi^3}\int\frac{d^3k}{k^6}\frac{p^2k^2 - (\mathbf{p}\cdot\mathbf{k})^2}{(\mathbf{p}+\mathbf{k})^2}\Pi_T(k)\end{aligned} \tag{19}$$

The first iteration of this system yields, as required, the lowest order perturbative contributions to the gluon and ghost self-energy functions. In higher orders the perturbative series of these self-energy functions lead, via equation (16), to the presence of power infrared divergences in the perturbative expansion of the pressure. These features are rather similar to the ones exhibited in Section II by the thermal scalar field [see eq. (8) and (9)].

Analogously to the scalar case, we will now show that a well behaved non-perturbative solution for the above set of integral equations does exist. In order to be able to solve it analytically, we will use consistently in the numerators and denominators of equations (18) and (19), the maximization procedure described in

the previous case. As we have seen, in the high temperature domain this procedure simplifies the equations, without modifying the qualitative features of their solutions. Then, it is convenient to define the dimensionless quantities:

$$\alpha \equiv \frac{g^2 N}{3\pi^2} \quad ; \quad x \equiv \frac{\alpha T}{p} \tag{20}$$

$$F(x) \equiv \frac{\Pi_T}{p^2} \quad ; \quad G(x) \equiv \frac{\Sigma}{p^2} \tag{21}$$

In this way, after performing the angular integrations, we find the following set of Volterra integral equations:

$$F(x) = -1 - \frac{13}{12}x - Hx^2 + x^2 \int_\alpha^x \frac{dy}{y^2} G(y) + x^4 \int_x^{\frac{\alpha T}{\lambda}} \frac{dy}{y^4} G(y) \tag{22}$$

$$G(x) = -\left(1 + \frac{\alpha}{2}\right) + x - \int_\alpha^x dy F(y) - x^2 \int_x^{\frac{\alpha T}{\lambda}} \frac{dy}{y^2} F(y) \tag{23}$$

where

$$H = \frac{1}{2\alpha} + \frac{11}{2} \int_\alpha^{\frac{\alpha T}{\lambda}} \frac{dy}{y^2} F(y) \tag{24}$$

Perturbation theory corresponds to solving (22) and (23) by a power series in x. On the other hand, the relevant momenta in the infrared domain are such that $p < \alpha T$, which correspond to large values of x.

We now consider the differential equations which follow from (22) and (23):

$$x^2 F''(x) - 5xF'(x) + 8F(x) + 2xG(x) = -8 - \frac{39}{12}x \tag{25}$$

$$xG''(x) - G'(x) - 2F(x) = -1 \tag{26}$$

To obtain the solution of this set of coupled equations, we use (25) to express $G(x)$ in function of $F(x)$ and its derivatives. Substituting this into (26), we get the following fourth order differential equation:

$$F''''(x) - \frac{4}{x}F'''(x) + \frac{12}{x^2}F''(x) - \frac{24}{x^3}F'(x) + \left(\frac{24}{x^4} + \frac{4}{x^2}\right)F(x) = \frac{2}{x^2} - \frac{24}{x^4} \tag{27}$$

which describes the infrared behavior of our system.

The complementary solution of the homogeneous part of equation (27) can be expressed in terms of the modified Bessel functions with complex argument. Using the functional relations satisfied by these functions[9] it is straightforward to show that:

$$F_c(x) = ix^2[C_1 I_2(\sqrt{8ix}) + C_2 K_2(\sqrt{8ix})] + \text{complex conjugate} \tag{28}$$

where C_1 and C_2 are complex constants to be determined.

In order to obtain a particular solution of (27), we employ the method of variation of the parameters, which is described in Appendix A. The general solution, consisting of the sum of $F_c(x)$ and $\tilde{F}(x)$ [equation (A7)] is given by:

$$F(x) = ix^2 \left\{ \left[C_1 + \int_x^{\frac{\alpha T}{\lambda}} dt \left(\frac{1}{t^2} - \frac{12}{t^4}\right) K_2(\sqrt{8it}) \right] I_2(\sqrt{8ix}) \right.$$
$$\left. + \left[C_2 + \int_\alpha^x dt \left(\frac{1}{t^2} - \frac{12}{t^4}\right) I_2(\sqrt{8it}) \right] K_2(\sqrt{8ix}) \right\} + \text{c.c.} \tag{29}$$

Since I_2 grows exponentially for large values of its argument, this result would produce a divergence as $\lambda \to 0$ in the integral equations (22) and (23), unless C vanishes. In Appendix B, we discuss the consistency conditions which follow from these equations in the limit $\lambda \to 0$. We find that, indeed, these conditions require the vanishing of the constant C_1. Futhermore, they fix the constant C_2 in terms of α, but the explicit expression has a rather complicated form involving the modified Bessel functions [see eq. (B2)].

From the behavior of $F(x)$ and $G(x)$ for large values of x, given by (B5) and (B6), and using eqs. (20,21), we see that $\Pi_T(p)$ and $\Sigma(p)$ will be proportional to p for small values of the momenta:

$$\Pi_T(p) \cong \frac{1}{2}p^2 \quad ; \quad \Sigma(p) \cong -\frac{39}{24}p^2 \qquad [p << \alpha T] \tag{30}$$

This behavior will ensure the convergence of the integral in eq. (16) as $\lambda \to 0$. Hence we find that the leading contributions to the pressure, when calculated in the ladder approximation of the $SU(N)$ Yang-Mills theory, are finite in the infrared domain.

In conclusion, we stress that our analysis was restricted to the ladder approximation, in which case the set of Schwinger-Dyson equations remains linear, allowing for an analytic solution. However, there are many other important contributions which must be taken into account when studying the infrared behavior of the pressure. These include the set of diagrams with crossed ghost and gluon lines, which can be reduced in the large N-limit to the class of planar diagrams . Furthermore, one must also consider additional self-energy and vertex corrections, which are relevant for the appearance of the effective coupling constant $g(T)$. It is conceivable that the inclusion of these contributions, may indicate the presence of new collective phenomena associated with the infrared behavior of the pressure. Nevertheless, in view of the above results, we believe that the cancellation of the infrared divergences is a possibility which deserves further investigation.

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APPENDIX A

Here we derive a particular solution of eq. (27), using the method of variation of the parameters. The complementary solution was given in eq. (28), so we assume a particular one of the form:

$$\tilde{F}(x) = Z_1(x)x^2 I_2(\sqrt{8ix}) + Z_2(x)x^2 K_2(\sqrt{8ix}) + \text{c.c.} \tag{A1}$$

where $Z_1(x)$ and $Z_2(x)$ are two complex functions to be determined consistently. We have to our disposal four real functions, but only one condition, so that we are free to impose three more conditions. We choose these additional conditions as follows:

$$Z_1'(x)x^2 I_2(\sqrt{8ix}) + Z_2'(x)x^2 K_2(\sqrt{8ix}) + \text{c.c.} = 0 \tag{A2}$$

$$Z_1'(x)\left[x^2 I_2(\sqrt{8ix})\right]' + Z_2'(x)\left[x^2 K_2(\sqrt{8ix})\right]' + \text{c.c.} = 0 \tag{A3}$$

$$Z_1'(x)\left[x^2 I_2(\sqrt{8ix})\right]'' + Z_2'(x)\left[x^2 K_2(\sqrt{8ix})\right]'' + \text{c.c.} = 0 \tag{A4}$$

Then, the expressions giving $\tilde{F}'(x)$, $\tilde{F}''(x)$, $\tilde{F}'''(x)$ and $\tilde{F}''''(x)$ simplify considerably. We now impose the basic condition that (A1) be a solution of (27). Thus we substitute these expressions into (27) and obtain an identity. Since $x^2 I_2(\sqrt{8ix})$, $x^2 K_2(\sqrt{8ix})$ and their complex conjugates are solutions of the homogeneous equation, this identity reduces to:

$$Z_1'(x)\left[x^2 I_2(\sqrt{8ix})\right]''' + Z_2'(x)\left[x^2 K_2(\sqrt{8ix})\right]''' + \text{c.c.} = \frac{2x^2 - 24}{x^4} \tag{A5}$$

Together with the auxiliary conditions (A2), (A3) and (A4), we have a system of equations which determinate the complex functions $Z_1'(x)$ and $Z_2'(x)$. To this end we use the functional relations satisfied by $K_2(z)$ and $I_2(z)$, together with the fact that their Wronskian is $\frac{1}{z}$. Then, after a straightforward calculation we obtain:

$$Z_1'(x) = -i\left(\frac{1}{x^2} - \frac{12}{x^4}\right) K_2(\sqrt{8ix})$$
$$Z_2'(x) = i\left(\frac{1}{x^2} - \frac{12}{x^4}\right) I_2(\sqrt{8ix}) \qquad \text{(A6)}$$

Integrating these relations and substituting the result into (A1), we find that the particular solution $\tilde{F}(x)$ can be expressed in the form:

$$\tilde{F}(x) = ix^2 \left\{ I_2(\sqrt{8ix}) \int_x^{\frac{\alpha T}{\lambda}} dt \left(\frac{1}{t^2} - \frac{12}{t^4}\right) K_2(\sqrt{8it}) + \right.$$
$$\left. + K_2(\sqrt{8ix}) \int_\alpha^x dt \left(\frac{1}{t^2} - \frac{12}{t^4}\right) I_2(\sqrt{8it}) \right\} + \text{c.c.} \qquad \text{(A7)}$$

With the help of this relation, the expression (29) for the general solution $F(x)$ can now be easily deduced. Using the series representation of the modified Bessel functions[9], it can be verified $F(x)$ satisfies the perturbative boundary condition $F(0) = -1$.

APPENDIX B

We discuss here the determination of the constants C_1, C_2 in the infrared limit $\lambda \to 0$, and obtain the asymptotic forms of $F(x)$ and $G(x)$ for large values of x.

To this end we substitute (25) and (26) back into the original integral equations (22) and (23). When the expression (29) for $F(x)$ is used in these equations, we obtain two relations which must be satisfied identically for all values of x. The constants C_1 and C_2 are fixed by the consistency conditions which arise from these relations at the points $x = \frac{\alpha T}{\lambda}$ and $x = \alpha$, respectively. Using the properties of the modified Bessel functions[9], it is straightforward to show in the limit $\lambda \to 0$, that these conditions demand C_1 to vanish. Then C_2 is determined by the system:

$$\frac{x^3}{2}\frac{d}{dx}\left(\frac{G(x)}{x^2}\right)\Bigg|_{x=\alpha} = 1$$
$$\frac{x^3}{2}\frac{d}{dx}\left(\frac{F(x)}{x^4}\right)\Bigg|_{x=\alpha} = H + \frac{13}{8\alpha} + \frac{2}{\alpha^2} \qquad \text{(B1)}$$

This set of equations fixes the complex constant C_2 uniquely, since the discrimi nant D of the system formed by C_2 and C_2^* is non-vanishing. To evaluate it, we us the expression (29) which gives the general solution $F(x)$ in terms of the modifie Bessel functions, together with the fact that $C_1 = 0$. Futhermore, we express $G(x$ [eq.(25)] in terms of $F(x)$ and its derivatives, as well as H [eq. (24)] in terms of a integral over $F(x)$. These factors make the explicit expression of the discriminan have a rather complicated form involving the modified Bessel functions. Using th relations satisfied by these functions, we find after a straightforward calculation that

$$D = -i\alpha\left(2\alpha + \frac{11}{4}\right) K_2(\beta)K_2(\beta^*) - i\alpha^2(\alpha - 11) K_2'(\beta)K_2'(\beta^*) +$$
$$+ i\left[\frac{\alpha}{\beta}\left(\frac{33}{4} - 11i\alpha + \frac{3}{2}\alpha + 4i\alpha^2 - 11\alpha^2\right) K_2'(\beta)K_2(\beta^*) + \text{c.c.}\right] \qquad \text{(B2}$$

where $\beta \equiv \sqrt{8i\alpha}$ and $K_2'(z) \equiv \frac{d}{dz}K_2(z)$.

Since C_1 vanishes, we see from equation (28) that the complementary solutio decreases exponentially for large values of x. Then, the asymptotic form of $F(x)$ i determined by $\tilde{F}(x)$, which in the infrared limit $\lambda \to 0$, becomes:

$$F(x) \cong ix^2 \left\{ K_2(\sqrt{8ix}) \int_\alpha^x \frac{dt}{t^2} I_2(\sqrt{8it}) + I_2(\sqrt{8ix}) \int_x^\infty \frac{dt}{t^2} K_2(\sqrt{8it}) \right\} + \text{c.c.}$$
(B3

In order to evaluate $F(x)$ for large values of x, we use the asymptotic expansion of the modified Bessel functions[9]:

$$I_2(z) \cong \frac{1}{\sqrt{2\pi z}} e^z$$
$$K_2(z) \cong \sqrt{\frac{\pi}{2z}} e^{-z} \qquad [|z| >> 1] \qquad \text{(B4}$$

Then, the leading contributions arising from the integrations in (B3) can be ca culated explicitly. In this way, we find that the asymptotic behavior of $F(x)$, for larg values of x, is given by:

$$F(x) \cong \frac{1}{2} \qquad [x >> 1] \tag{B5}$$

With the help of this result, we obtain from (25) the asymptotic form of $G(x)$ for large values of x:

$$G(x) \cong -\frac{39}{24} \qquad [x >> 1] \tag{B6}$$

Since $x = \frac{\alpha T}{p}$, these expressions describe equivalently the behavior of our solutions for small values of the momenta.

APPENDIX C

Here we present a very simple model which illustrates a possible mechanism for the cancellation of the infrared divergences, outside the ladder approximation. The model has some similarities with the magnetic sector of the thermal Yang-Mills theory. We consider the following expression for the pressure:

$$P = P_0 + cT \sum_{p_0} \int \frac{d^3p}{p^2 + p_0^2} \tilde{\Pi}(p, p_0) \tag{C1}$$

The perturbative series of $\tilde{\Pi}(p \to 0, p_0 = 0)$ is given by:

$$\tilde{\Pi}(p) = -ag^2pT + b\left(g^2T\right)^2 + \ldots \tag{C2}$$

where a, b and c are positive constants. We see that a thermal mass of order g^2T is generated, a possibility which might occur also in the case of the magnetic mass. Our model assumes, for definiteness, that the contributions to $\tilde{\Pi}(p)$ exponentiate. Then, (C2) can be written as follows:

$$\tilde{\Pi}(p) = \frac{a^2}{2b} p^2 \left(e^{-\frac{\alpha T}{p}} - 1\right) \tag{C3}$$

where $\alpha \equiv \frac{2b}{a} g^2$.

Proceeding in parallel with previous cases, we obtain that the expression descril ing the dominant infrared behavior of the pressure has the form:

$$P \cong 2\pi \frac{a^2c}{b} T \int_\lambda^T p^2 e^{-\frac{\alpha T}{p}} dp \tag{C4}$$

Using this form, we find that the perturbative expansion of the pressure contain power infrared divergences given by:

$$P_{\text{div}} \cong 2\pi \frac{a^2c}{b} T^4 \left[-\frac{\alpha^3}{3!} ln\left(\frac{T}{\lambda}\right) + \frac{\alpha^4}{4!}\frac{T}{\lambda} - \frac{\alpha^5}{5!2}\left(\frac{T}{\lambda}\right)^2 + \ldots \right] \tag{C5}$$

Note that this behavior is similar to the one shown by the perturbative therme Yang-Mills theory.

However, the full expression (C4) is well behaved in the infrared limit $\lambda \to 0$ Indeed, introducing the variable $x = \frac{\alpha T}{p}$, we find in this limit that P becomes:

$$P \cong 2\pi \frac{a^2c}{b} T^4 \alpha^3 \int_\alpha^\infty \frac{dx}{x^4} e^{-x} \tag{C6}$$

This can be evaluated in terms of the exponential-integral function[9]:

$$P \cong \frac{\pi}{3} \frac{a^2c}{b} T^4 \left[\alpha^3 \text{Ei}(-\alpha) + (2 - \alpha + \alpha^2)e^{-\alpha}\right] \tag{C7}$$

Therefore, in our model the complete expression giving the pressure in the infrared domain is a well behaved function of α.

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FIGURES

FIG. 1. The Schwinger-Dyson equation for the scalar self-energy function in the ladder approximation.

FIG. 2. (a) A general contribution of the thermal scalar field to the pressure in the ladder approximation and (b) its graphical representation as given by equation (2).

FIG. 3. Graphical representation of equation (16). Wavy lines denote bare gluons and dashed lines represent bare ghost particles. Combinatorial factors are shown explicitly in the diagrams.

FIG. 4. (a) The Schwinger-Dyson equation for the gluon polarization tensor in the ladder approximation and (b) the corresponding equation for the ghost two-point function.

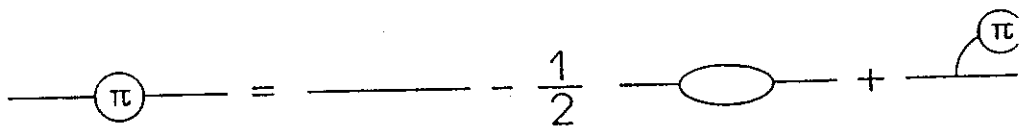
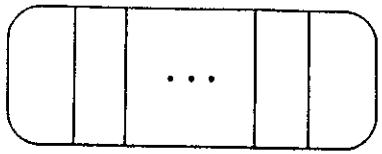
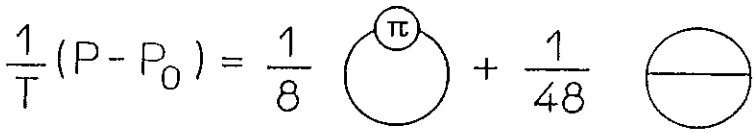


FIG. 1



(a)



(b)

FIG. 2

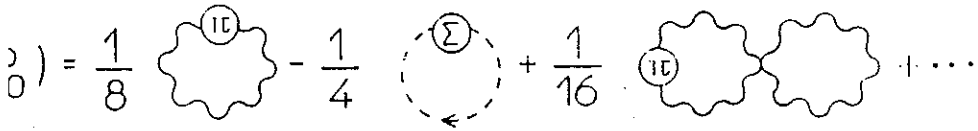
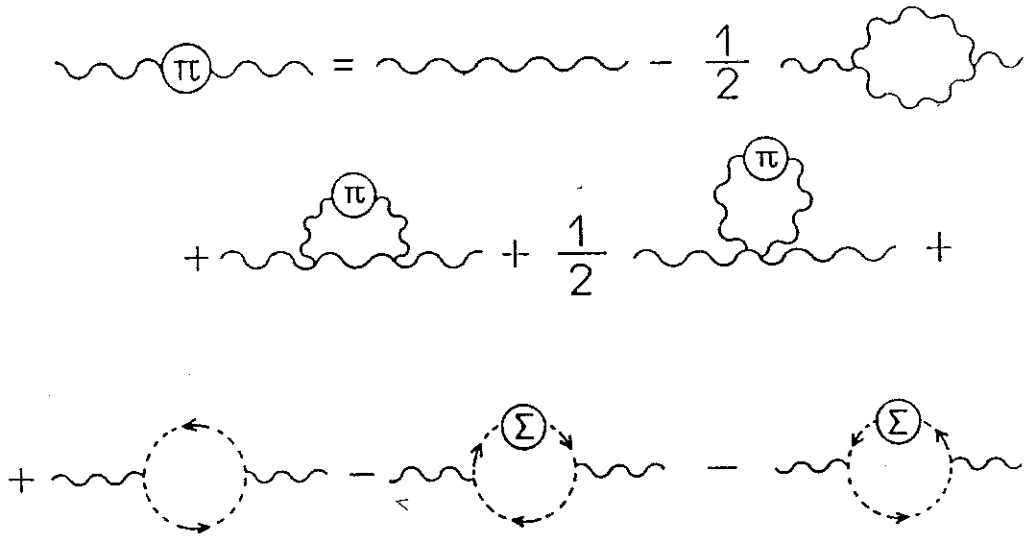
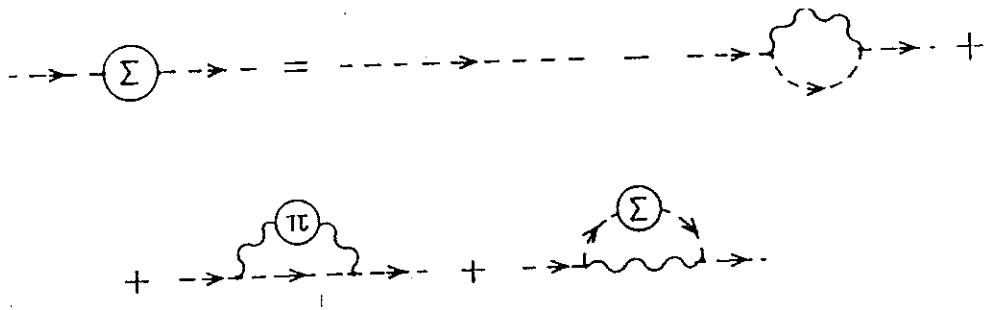


FIG. 3



(a)



(b)

FIG. 4