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Contribution of the Proton-Proton Reaction $p + p \rightarrow D + e^{+} + \nu_e$ to the Solar Neutrino Production

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Abstract. We calculate the "electron neutrinos" υ_e flux due to the proton-proton reactions $p + p \rightarrow D + e^{+} + \upsilon_e$ that occurs in the solar core described by the Solar Standard Model. These electron neutrinos are also named "pp neutrinos". Since this paper was written to graduate and postgraduate students of physics the calculations will be performed in a simple and didactical way, but, as rigorously as possible. In addition, only a few references will be cited and commented.
Key words: pp reaction; solar neutrinos flux.

1) Introduction. The proton-proton cross section.

As well known,[1-4] the reactions $p + p \rightarrow D + e^{+} + \upsilon_e$ play a fundamental role in the "proton-proton chain" in the stellar evolution process. According to the Solar Standard Model[3,4] they constitute 99.77% of the nuclear reactions responsible for the solar thermonuclear power in the proton-proton chain. Our objective is to calculate the total number of the neutrinos υ_e, named "pp neutrinos" or "electron neutrinos" (see Section 3), generated by the Sun. In spite of considerable progress of the experimental techniques in nuclear and elementary particle physics the cross section or the rate for this primary reaction have not been measured in laboratory. It is too slow to be measured at relevant energies since the transformation proceeds via the weak interaction.[1] The cross section and the rate for this reaction was accurately calculated by Bethe and Critchfield in 1938,[1] using the measured deuteron characteristics and the theory of low-energy weak interactions. Up to now their predictions are taken as exact and no modification of their results has been proposed in the literature.[3] To calculate[1,2] the cross section two steps are taken into account: (1) The probability of finding the two protons within the confines of a deuteron. It involves the collision of two protons with the penetrability of their mutual potential barrier. (2) The probability of emission of the positron and the neutrino during the collision predicted by the Fermi β-decay theory. Thus, according to (1) and (2) the probability to the deuteron formation, P_D, per second, is given by

$$P_D = \left| \int \psi_{pp} \, \psi_D \, dV \right|^2 \beta \qquad (1.1),$$

where the first term express the probability of finding the two protons within the confines of the deuteron; ψ_{pp} is the normalized wave function of the relative motion of two protons in a volume V, ψ_D is the wave function of the deuteron. The integration goes over the space of the relative position **r** of the nucleons. The first term of (1.1) is the square of the matrix element of the transition pp →D that was exactly calculated by Bethe and Critchfield.[1] The second term $\beta \approx 6.79\ 10^{-5}\ s^{-1}$ expresses the probability (per second) of the β-emission according to Fermi´s theory.[1,2] Now, instead of following the exact calculation of Bethe and Critchfield,[1] we will adopt an approximate method proposed by Nauenberg and Weisskopf.[2] In this way ψ_{pp} is taken as

$$\psi_{pp} = G_a(\mathbf{r}) \exp(i\mathbf{p}\cdot\mathbf{r})/\sqrt{V} \qquad (1.2),$$

where V is the volume of the Sun, **p** and **r** are the relative momentum and distance of the protons, $G_a(\mathbf{r})$ is the Gamow factor representing the effect of Coulomb repulsion between them, which reduces the wave function at small **r.** At $\mathbf{r} = 0$ this function is given by $G_a(\mathbf{0}) = (v^*/v)^{1/2}\exp(-v^*/2v)$, $v^* = e^2/h = 1.38\ 10^9$cm/s. Here v is the relative speed of the colliding protons. The deuteron wavefunction ψ_D is written in the simple form

$$\psi_D = \exp(-r/\delta)/(\pi\delta^3)^{1/2} \qquad (1.3),$$

where $\delta = 4.3\ 10^{-13}$ cm is the "size" of the deuteron. Since is small enough we get

$$\int \psi_{pp} \, \psi_D \, dV = G_a(\mathbf{0}) \int \psi_D \, dV = 8\pi^{1/2} (v^*/v)^{1/2}\exp(-v^*/2v)(\delta^3/V)^{1/2} \qquad (1.4).$$

It is important to note that the proposed matrix element (1.2) used by Nauenberg and Weisskopf[2] can be obtained using the WKB method taking into account, for instance, Fermi[5] calculations. As will be shown in Section 2 the elaborate calculation of Bethe and Critchfield[1] and the approximate one used by Nauenberg and Weisskopf[2] to get (1.4) are in excellent agreement.

In this way, taking into account (1.1) the probability $P_D(v)$ for the formation deuteron process pp→D is written as:

$$P_D(v) = 64\pi (v^*/v) \exp(-v^*/v)(\delta^3/V)\beta \qquad (1.5).$$

2) The Maxwell-Boltzmann Velocity Distribution of the Solar Protons.

To take into account the average probability $< P_D >$ for the deuteron formation inside the solar core we must perform the average of the (1.5) over Maxwell-Boltzmann velocity distribution of the protons $F_T(v)$. This average value is given by the integral

$$< P_D > = \int P_D(v)\, F_T(v)\, dv \qquad (2.1),$$

where $F_T(v) = (4v_T^3/\sqrt{\pi})\, v^2 \exp(-v^2/v_T^2)$, $v_T = (4kT/m)^{1/2}$, k the Boltzmann constant and m the proton mass. The integral (2.1) which integrand has a minimum at $v = v_o = (v^* v_T^2/2)^{1/3}$ is given by[2]

$$< P_D > = (512\pi/9\sqrt{3})\, S_o^2 \exp(-S_o)\, (\delta^3/V)\, \beta \qquad (2.2),$$

where $S_o = 3(v^*/2v_T)^{2/3}$.

If N is the number of solar protons the total number of electron neutrinos N_υ produced by the pp →D reaction, per second, would be given by $N_\upsilon = (N^2/4) < P_D >$, where $N^2/2$ is the number of protons pairs and the factor ½ comes from the average over the protons spins. So, using (2.2) the total number of neutrinos created, per second, by unit of volume n_υ is written as

$$n_\upsilon = N_\upsilon/V = (N/V)^2 (512\pi/36\sqrt{3})\, \delta^3\beta\, S_o^2 \exp(-S_o) \qquad (2.3)$$

Let us indicate by M the mass, R the radius, V the volume and $\rho = M/V$ the density of the Sun. If ρ_H is the proton density $\rho_H = Nm/V$ its concentration C_H is given by $C_H = \rho_H/\rho$. In terms of these parameters (2.3) becomes

$$n_\upsilon = C_H^2 \rho^2 (512\pi/36\, m^2\sqrt{3})\, \delta^3\beta\, S_o^2 \exp(-S_o) \qquad (2.4)$$

At this point it important to note that according to Bethe and Critchfield calculation[1] the number of neutrinos $n_\upsilon(BC)$ is given by

$$n_\upsilon(BC) = n_\upsilon(r,T)\, \rho^2 (16\pi\Lambda^2/m^2 3^{5/2})\, \delta^3\beta\, S_o^2 \exp(-S_o) \qquad (2.5),$$

where $\Lambda \approx 2.84$. So, the numerical multiplicative factor of (2.4) is 8.21 and of (2.5) is 8.28.That is, there is only very small difference (less than1%) between the two approaches.

So, according to (2.4) the local neutrino density production $n_\upsilon(r,T)$ at a distance r from the center of the Sun which has a temperature $T = T(r)$, a concentration $C_H(r)$ and density $\rho(r,T)$ is expressed by

$$n_\upsilon(r,T) = A\, C_H(r)^2\, \rho(r,T)^2\, S_o(T)^2 \exp[-S_o(T)] \qquad (2.6),$$

where $A = (512\pi/36m^2\sqrt{3})\delta^3\beta$, $S_o(T) = 3(v^*/2v_T)^{2/3} = 33.8/T_6(r)^{1/3}$ with $T_6(r)$ measured in millions of Kelvin degrees.

Consequently, the total electron solar neutrino production per unit of time N_υ would be given by

$$N_\upsilon = 4\pi \int_0^R n_\upsilon(r,T) r^2\, dr \qquad (2.7),$$

that, using (2.6), becomes

$$N_\upsilon = 4\pi A \int_0^R C_H(r)^2 \rho(r,T)^2\, S_o(T)^2 \exp[-S_o(T)]\, r^2\, dr \qquad (2.8).$$

Defining $x = r/R$ and $g[T(x)] = g(x) = T_6(x)^{-2/3} \exp[-33.8\, T_6(x)^{-1/3}]$ we see that (2.8) can be written as

$$N_\upsilon = 4\pi A (33.8)^2 R^3 \int_0^R C_H(x)^2 \rho(x)^2\, g(x)\, x^2\, dx \qquad (2.9).$$

Adopting the solar physical and chemical characteristics according to the "standard solar model"[3,6] (SSM) we have

$$C_H(x) \approx 0.65 - 0.309 * 10^{-7.45x}$$

$$\rho(x) \approx 148.0 * 10^{-2.146x} \quad (g/cm^3) \qquad (2.10).$$

$$T_6(x) \approx 15.6 * 10^{-0.778x} \quad (10^6\ K)$$

If the distance between Sun and Earth is *D* the predicted solar electron neutrino flux Φ_{th} on the Earth surface is given by $\Phi_{th} = N_\upsilon/4\pi D^2$. So, taking into account the numerical values of the parameters R, m, δ, β, D and using the SSM functions $C_H(x)$, $\rho(x)$ and $T(x)$ defined by (2.10) we get from (2.9):

$$\Phi_{th} \approx 6.0\ 10^{10}\ cm^{-2}\ s^{-1} \qquad (2.11),$$

in agreement with exaustive calculations found in the literature.[3]

According to recent experimental results[7] the measured flux Φ_{exp} of these pp neutrinos is

$$\Phi_{exp} \approx 2.4\ 10^{10}\ cm^{-2}\ s^{-1} \qquad (2.12),$$

showing that, the predicted flux Φ_{th} is much larger the measured Φ_{exp}, that is, $\Phi_{th} \approx 2.4\, \Phi_{exp.}$ Up to now this disagreement between theory and experiment was not satisfactorily explained. This problem of "missing υ_e neutrinos" is also known as the "solar neutrino puzzle". See comments in next Section.

3) Neutrinos and Conclusions.

It is not our intention to analyze the history and the physics of neutrinos since there is an extensive literature about them. To have an idea about neutrinos we suggest to reading, for instance, the site "What´s a Neutrino?"[8] the Bahcall[3] book and the papers of Altmann et al.[7] and of Valdiviesso and Guzzo.[9]

There are three kinds ("flavors") of neutrinos: *electron neutrino* υ_e, *muon neutrino* υ_μ and *tau neutrino* υ_τ. They appear, for instance, in the decays of the neutron (n), pion (π) and tao (τ), respectively:

$$n \rightarrow p + e^- + \upsilon_e,$$
$$\pi^\pm \rightarrow \mu^\pm + \upsilon_\mu$$
$$\tau^\pm \rightarrow \mu^\pm + \upsilon_\mu{}^* + \upsilon_\tau$$

One tentative to solve the problem of the missing υ_e neutrinos was adopt a theoretical approach[7,9] named "neutrino flavor oscillations". According to this approach the difference of the neutrino masses would induce a temporal mixing of neutrino flavors and, consequently, a temporal oscillation between the flavors. Let us suppose that υ_e is created at a given point in the solar interior. During the time taken to arrive at the Earth detector it would oscillate among the different flavors. So, it can be detected as υ_e,υ_μ or υ_τ. Experimental results[7,9] reveals that this model cannot satisfactorily explain the solar missing υ_e problem.

Note that up to now no proposed theoretical model was able to explain satisfactorily the "solar neutrino puzzle".[7,9]

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