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**QUANTIZATION OF SPINNING PARTICLE WITH
ANOMALOUS MAGNETIC MOMENTUM**

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Abstract

A generalization of the pseudoclassical action of a spinning particle in the presence of an anomalous magnetic moment is given. The leading considerations, to write the action, are gotten from the path integral representation for the causal Green's function of the generalized (by Pauli) Dirac equation for the particle with anomalous magnetic momentum in an external electromagnetic field. The action can be written in reparametrization and supergauge invariant form. Both operator (Dirac) and path-integral (BFV) quantization are discussed. The first one leads to the Dirac-Pauli equation, whereas the second one gives the corresponding propagator. One of the nontrivial points in this case is that both quantizations schemes demand for consistency to take into account an operators ordering problem.

1 Introduction

In recent years numerous classical models of relativistic particles and superparticles were discussed intensively. First, the interest to such models was initiated by the close relation with the string theory, but now it is also clear that the problem itself has an important meaning for the deeper understanding of the structure of quantum theory.

One of the basic, in the above mentioned set of classical models, is the pseudoclassical model of Fermi particle with spin 1/2, proposed first in the works [1,2], investigated and quantized in many works, see for example [1-9]. The model can be formulated in gauge invariant (reparametrization and supersymmetric) form. The action of the model in an external electromagnetic field has the form [9]:

$$S = \int_0^1 \left[-\frac{\dot{x}^2}{2e} - e\frac{m^2}{2} - g\dot{x}^\alpha A_\alpha + ig\epsilon F_{\alpha\beta}\psi^\alpha\psi^\beta + i\left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - m\psi^5\right)\chi - i\psi_n\dot{\psi}^n \right] d\tau, \quad (1)$$

where x^α , e are even and ψ^n , χ are odd variables dependent on τ , the latter plays the role of the time in this theory, $A_\alpha(x)$ is an external electromagnetic field potential, $F_{\alpha\beta}(x)$ is the Maxwell strength tensor, and g the electrical charge. Greek indices run over $\overline{0,3}$ and Latin indices n, m run over $\overline{0,3,5}$. The metric tensors: $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1)$ and $\eta_{mn} = \text{diag}(1, -1, -1, -1, -1)$. There are two gauge transformations in the theory with the action (1), reparametrizations,

$$\delta x = \dot{x}\xi, \quad \delta e = \frac{d}{dt}(e\xi), \quad \delta\psi^n = \dot{\psi}^n\xi, \quad \delta\chi = \frac{d}{dt}(\chi\xi), \quad (2)$$

and supertransformations,

$$\delta x^\alpha = i\psi^\alpha\epsilon, \quad \delta e = i\chi\epsilon, \quad \delta\chi = \dot{\epsilon}, \quad \delta\psi^\alpha = \frac{1}{2e}(\dot{x}^\alpha + i\chi\psi^\alpha)\epsilon, \\ \delta\psi^5 = \left[\frac{m}{2} - \frac{i}{me}\psi^5\left(\dot{\psi}^5 - \frac{m}{2}\chi\right) \right] \epsilon, \quad (3)$$

where ξ are even and ϵ odd τ -dependent parameters. The spinning degrees of freedom in such a model are described by Grassmannian variables, that's

why the model is called pseudoclassical. The quantization of the model in different ways leads to the quantum mechanics of the Dirac particle, is very instructive and creates many useful analogies with problems of quantization of gauge field theories.

In this work a generalization of the model, when an anomalous magnetic momentum of the particle is present, is discussed. The relativistic quantum theory of a spinning particle, which has both the "normal" magnetic momentum $g/2m$ and an "anomalous" magnetic momentum μ , was formulated by Pauli [10]. In this case he generalized the Dirac equation to the following form:

$$\left(\hat{P}_\nu \gamma^\nu - m - \frac{\mu}{2} \sigma^{\alpha\beta} F_{\alpha\beta} \right) \Psi(x) = 0, \quad (4)$$

where $\hat{P}_\nu = i\partial_\nu - gA_\nu(x)$, $\sigma^{\alpha\beta} = \frac{i}{2}[\gamma^\alpha, \gamma^\beta]_-$, $[\gamma^\alpha, \gamma^\beta]_+ = 2\eta^{\alpha\beta}$, and the notations $[A, B]_\pm = AB \pm BA$ are used.

We present a generalization of the action (1), whose quantization gives the Dirac-Pauli theory. The work is organized as follow. First we construct a path-integral representation for the causal Green's function of the Dirac-Pauli equation (4). The form of this representation allows one to identify some terms of the effective action with a classical gauge invariant action of a spinning particle with anomalous magnetic momentum. We find gauge and supergauge transformations of the action, analyze Lagrangian and Hamiltonian structure of the theory. Both operator (Dirac) and path-integral (BFV) quantizations are discussed. The first one leads to the Dirac-Pauli equation, whereas the second one gives the corresponding propagator. One of the nontrivial points in this case is that both quantizations schemes demand for consistency to take into account an operators ordering problem.

2 Path Integral Representation for Causal Green's Function of Dirac-Pauli Equation

In this section we are going to write the path integral representation for the causal Green's function $S^c(x, y)$ of the equation (4). To get the result in supersymmetric form one needs to work with the transformed by $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$, $(\gamma^5)^2 = -1$, function $\tilde{S}^c(x, y) = S^c(x, y)\gamma^5$, which obeys

the equation

$$\left(\hat{P}_\nu \tilde{\gamma}^\nu - m\gamma^5 - \frac{\mu}{2} \tilde{\gamma}^5 \sigma^{\alpha\beta} F_{\alpha\beta} \right) \tilde{S}^c(x, y) = \delta^4(x - y), \quad (5)$$

where $\tilde{\gamma}^\mu = \gamma^5 \gamma^\mu$, $\tilde{\sigma}^{\mu\nu} = \frac{i}{2}[\tilde{\gamma}^\mu, \tilde{\gamma}^\nu]_-$.

The matrices $\tilde{\gamma}^\nu$, obey the same commutation relations as initial ones γ^ν , $[\tilde{\gamma}^\mu, \tilde{\gamma}^\nu]_+ = 2\eta^{\mu\nu}$, so that the tilde sign will be omitted hereafter. For all the γ -matrices we have $[\gamma^m, \gamma^n]_+ = 2\eta^{mn}$.

Similar to Schwinger [11] we present $\tilde{S}_{\alpha\beta}^c(x, y)$ as a matrix element of an operator $\tilde{S}_{\alpha\beta}^c$, but, in the coordinate space only,

$$\tilde{S}_{\alpha\beta}^c(x, y) = \langle x | \tilde{S}_{\alpha\beta}^c | y \rangle, \quad (6)$$

where spinor indices are written explicitly for clarity only once and will be omitted hereafter; $|x\rangle$ are eigenvectors for some hermitian operators of coordinates X^μ ; the corresponding canonically conjugated operators of momenta are P_μ , so that:

$$\begin{aligned} X^\mu |x\rangle &= x^\mu |x\rangle, & \langle x|y\rangle &= \delta^4(x - y), & \int |x\rangle \langle x| dx &= I, \\ [P_\mu, X^\nu]_- &= -i\delta_\mu^\nu, & P_\mu |p\rangle &= p_\mu |p\rangle, & \langle p|p'\rangle &= \delta^4(p - p'), \\ \int |p\rangle \langle p| dp &= I, & \langle x|P_\mu|y\rangle &= -i\partial_\mu \delta^4(x - y), & \langle x|p\rangle &= \frac{1}{(2\pi)^4} e^{ipx}, \\ [\Pi_\mu, \Pi_\nu]_- &= -igF_{\mu\nu}(X), & \Pi_\mu &= -P_\mu - gA_\mu(X). \end{aligned}$$

The equation (5) implies the formal solution for the operator \tilde{S}^c :

$$\tilde{S}^c = \left[\Pi_\nu \gamma^\nu - m\gamma^5 - \frac{\mu}{2} \gamma^5 \sigma^{\alpha\beta} F_{\alpha\beta} \right]^{-1}.$$

The operator in square brackets is a pure Fermi one, if one reckons γ -matrices as Fermi operators. In general case the inverse operator to a Fermi operator A can be presented by means of an integral over the super-proper time (λ, χ) of an exponential with an even exponent [9],

$$A^{-1} = \int_0^\infty d\lambda \int e^{i(\lambda(A^2 + ic) + \chi A)} d\chi,$$

where λ is an even and χ is an odd (Grassmann) variable, the latter anti-commutes with A by definition. In our case

$$A = \Pi_\nu \gamma^\nu - m\gamma^5 - i\frac{\mu}{2} \gamma^5 F_{\alpha\beta} \gamma^\alpha \gamma^\beta,$$

$$A^2 = \Pi^2 - m^2 - i \left(m\mu + \frac{g}{2} \right) F_{\alpha\beta} \gamma^\alpha \gamma^\beta + \frac{\mu^2}{4} (F_{\alpha\beta} \gamma^\alpha \gamma^\beta)^2 - i\mu\gamma^5 \gamma^\alpha [F_{\alpha\beta}, \Pi^\beta]_+,$$

where was used $[\Pi_\nu \gamma^\nu, i\frac{\mu}{2} \gamma^5 F_{\alpha\beta} \gamma^\alpha \gamma^\beta]_+ = i\mu\gamma^5 \gamma^\alpha [F_{\alpha\beta}, \Pi^\beta]_+$. So we get:

$$\tilde{S}^c = \int_0^\infty d\lambda \int e^{-i\hat{H}(\lambda, \chi)} d\chi,$$

where:

$$\hat{H}(\lambda, \chi) = \lambda \left\{ m^2 - \Pi^2 + i \left(m\mu + \frac{g}{2} \right) F_{\alpha\beta} \gamma^\alpha \gamma^\beta - \frac{\mu^2}{4} (F_{\alpha\beta} \gamma^\alpha \gamma^\beta)^2 + i\mu\gamma^5 \gamma^\alpha [F_{\alpha\beta}, \Pi^\beta]_+ \right\} + \left(\Pi_\nu \gamma^\nu - m\gamma^5 - i\frac{\mu}{2} \gamma^5 F_{\alpha\beta} \gamma^\alpha \gamma^\beta \right) \chi.$$

Thus, the Green's function (6) takes the form:

$$\tilde{S}^c(x_{\text{out}}, x_{\text{in}}) = \int_0^\infty d\lambda \int \langle x_{\text{out}} | e^{-i\hat{H}(\lambda, \chi)} | x_{\text{in}} \rangle d\chi. \quad (7)$$

Now we are going to present the matrix element entering in the expression (7) by means of a path integral. In spite of the operator $\hat{H}(\lambda, \chi)$ has the γ -matrix structure, one can do this similar to the usual way, namely, first write $\exp -i\hat{H} = (\exp -i\hat{H}/N)^N$ and then insert $(N-1)$ resolutions of identity $\int |x\rangle \langle x| dx = I$ between all the operators $\exp -i\hat{H}/N$. Besides, we introduce N additional integrations over λ and χ to transform then the ordinary integrals over these variables into the corresponding path-integrals:

$$\tilde{S}^c(x_{\text{out}}, x_{\text{in}}) = \lim_{N \rightarrow \infty} \int_0^\infty d\lambda_0 \int \prod_{k=1}^N \langle x_k | e^{-i\hat{H}(\lambda_k, \chi_k) \Delta\tau} | x_{k-1} \rangle \delta(\lambda_k - \lambda_{k-1}) \delta(\chi_k - \chi_{k-1}) d\chi_0 d x_1 \dots d x_{N-1} d\lambda_1 \dots d\lambda_N d\chi_1 \dots d\chi_N, \quad (8)$$

where $\Delta\tau = 1/N$, $x_0 = x_{\text{in}}$, $x_N = x_{\text{out}}$.

Bearing in mind the limiting process, one can, as usual, restrict himself by calculation of the matrix elements from (8) approximately,

$$\langle x_k | e^{-i\hat{H}(\lambda_k, \chi_k) \Delta\tau} | x_{k-1} \rangle \approx \langle x_k | 1 - i\hat{H}(\lambda_k, \chi_k) \Delta\tau | x_{k-1} \rangle, \quad (9)$$

using the resolutions of identity $\int |p\rangle \langle p| dp$. In this connection it is important to notice that the operator $\hat{H}(\lambda_k, \chi_k)$ has the symmetric form in the operators \hat{x} and \hat{p} by origin. Indeed, the only terms in $\hat{H}(\lambda_k, \chi_k)$, which contain products of these operators are $[\hat{p}_\alpha, A^\alpha(\hat{x})]_+$ and $[\hat{p}_\alpha, F^{\alpha\beta}(\hat{x})]_+$. One can verify that these are maximal symmetrized expressions, which can be combined from entering operators (see also remark in ref. [12], p. 388). Thus, one can write

$$\hat{H}(\lambda, \chi) = \text{Sym}_{(\hat{x}, \hat{p})} \mathcal{H}(\lambda, \chi, \hat{x}, \hat{p}),$$

where $\mathcal{H}(\lambda, \chi, x, p)$ is the Weyl symbol of the operator $\hat{H}(\lambda, \chi)$ in the sector of x, p ,

$$\mathcal{H}(\lambda, \chi, x, p) = \lambda \left(m^2 - \mathcal{P}^2 + i \left(m\mu + \frac{g}{2} \right) F_{\alpha\beta} \gamma^\alpha \gamma^\beta + 2i\mu\gamma^5 F_{\alpha\beta} \gamma^\alpha \mathcal{P}^\beta - \frac{\mu^2}{4} (F_{\alpha\beta} \gamma^\alpha \gamma^\beta)^2 \right) + \left(\mathcal{P}_\nu \gamma^\nu - m\gamma^5 - i\frac{\mu}{2} \gamma^5 F_{\alpha\beta} \gamma^\alpha \gamma^\beta \right) \chi,$$

where $\mathcal{P}_\nu = -p_\nu - gA_\nu(x)$.

That is a general statement [13], which can be easily checked in that concrete case by direct calculations, that the matrix elements (9) are expressed in terms of the Weyl symbols in the middle point $\bar{x}_k = (x_k + x_{k-1})/2$. Taking all that into account, one can see that in the limiting process the matrix elements (9) can be replaced by the expressions

$$\int \frac{d p_k}{(2\pi)^4} \exp i \left[p_k \frac{x_k - x_{k-1}}{\Delta\tau} - \mathcal{H}(\lambda_k, \chi_k, \bar{x}_k, p_k) \right] \Delta\tau, \quad (10)$$

which all are noncommuting due to the γ -matrix structure and are situated in (8) so that the numbers k increase from the right to the left. For the two δ -functions, accompanying each matrix element (9) in the expression (8), we use the integral representations

$$\delta(\lambda_k - \lambda_{k-1}) \delta(\chi_k - \chi_{k-1}) = \frac{i}{2\pi} \int e^{i(\pi_k(\lambda_k - \lambda_{k-1}) + \nu_k(\chi_k - \chi_{k-1}))} d\pi_k d\nu_k,$$

where ν_k are odd variables. Then we attribute formally to the γ -matrices, entering into (10), index k , and then we attribute to all quantities the "time" τ_k , according the index k they have, $\tau_k = \Delta\tau k$, so that $\tau \in [0, 1]$. Introducing the T-product, which acts on γ -matrices, it is possible to gather all the expressions, entering in (8), in one exponent and deal then with the γ -matrices

like with odd (Grassmann) variables. Taking into account all was said, we get for the right side of (8):

$$\begin{aligned} \tilde{S}^c(x_{\text{out}}, x_{\text{in}}) = & \quad (11) \\ & \text{T} \int_0^\infty d\lambda_0 \int \exp \left\{ i \int_0^1 \left[\lambda \left(\mathcal{P}^2 - m^2 - i \left(m\mu + \frac{g}{2} \right) F_{\alpha\beta} \gamma^\alpha \gamma^\beta \right. \right. \right. \\ & - 2i\mu\gamma^5 F_{\alpha\beta} \gamma^\alpha \mathcal{P}^\beta + \frac{\mu^2}{4} (F_{\alpha\beta} \gamma^\alpha \gamma^\beta)^2 \left. \left. \left. + \left(m\gamma^5 - \mathcal{P}_\nu \gamma^\nu + i \frac{\mu}{2} \gamma^5 F_{\alpha\beta} \gamma^\alpha \gamma^\beta \right) \chi \right. \right. \right. \\ & \left. \left. \left. + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi} \right] d\tau \right\} d\chi_0 \mathcal{D}x \mathcal{D}p \mathcal{D}\lambda \mathcal{D}\pi \mathcal{D}\chi \mathcal{D}\nu, \end{aligned}$$

where x, p, λ, π , are even and χ, ν , odd trajectories, obeying the boundary conditions $x(0) = x_{\text{in}}, x(1) = x_{\text{out}}, \lambda(0) = \lambda_0, \chi(0) = \chi_0$. The operation of T-ordering acts on the γ -matrices, which suppose formally to depend on time τ .

The expression (11) can be reduced to:

$$\begin{aligned} \tilde{S}^c(x_{\text{out}}, x_{\text{in}}) = & \\ & \int_0^\infty d\lambda_0 \int \exp \left\{ i \int_0^1 \left[\lambda \left(\mathcal{P}^2 - m^2 - i \left(m\mu + \frac{g}{2} \right) F_{\alpha\beta} \frac{\delta_l}{\delta\rho_\alpha} \frac{\delta_l}{\delta\rho_\beta} \right. \right. \right. \\ & - 2i\mu F_{\alpha\beta} \mathcal{P}^\beta \frac{\delta_l}{\delta\rho_5} \frac{\delta_l}{\delta\rho_\alpha} + \frac{\mu^2}{4} \left(F_{\alpha\beta} \frac{\delta_l}{\delta\rho_\alpha} \frac{\delta_l}{\delta\rho_\beta} \right)^2 \left. \left. \left. + \left(m \frac{\delta_l}{\delta\rho_5} - \mathcal{P}_\nu \frac{\delta_l}{\delta\rho_\nu} \right. \right. \right. \right. \\ & \left. \left. \left. + i \frac{\mu}{2} F_{\alpha\beta} \frac{\delta_l}{\delta\rho_5} \frac{\delta_l}{\delta\rho_\alpha} \frac{\delta_l}{\delta\rho_\beta} \right) \chi + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi} \right] d\tau \right\} d\chi_0 \mathcal{D}x \mathcal{D}p \mathcal{D}\lambda \mathcal{D}\pi \mathcal{D}\chi \mathcal{D}\nu \\ & \times \text{T} \exp \int_0^1 \rho_n(\tau) \gamma^n d\tau \Big|_{\rho=0}, \end{aligned}$$

where five Grassmannian sources $\rho_n(\tau)$ are introduced, which anticommute with the γ -matrices by definition.

Using the representation (46) for $\text{T} \exp \int_0^1 \rho_n(\tau) \gamma^n d\tau$ in the form of a Grassmannian path-integral (see Appendix), we get the Hamiltonian path-integral representation for the Green's function in question:

$$\begin{aligned} \tilde{S}^c(x_{\text{out}}, x_{\text{in}}) = & \quad (12) \\ & \exp \left(i\gamma^n \frac{\partial_l}{\partial\theta^n} \right) \int_0^\infty d\lambda_0 \int \text{ex} \left\{ i \int_0^1 \left[\lambda \left(\mathcal{P}^2 - 8i\mu\psi^5 F_{\alpha\beta} \mathcal{P}^\alpha \psi^\beta \right. \right. \right. \\ & + 2ig F_{\alpha\beta} \psi^\alpha \psi^\beta - M^2 \left. \left. \left. + 2i \left(\mathcal{P}_\nu \psi^\nu - M\psi^5 \right) \chi - i\psi_n \dot{\psi}^n + p\dot{x} \right. \right. \right. \\ & \left. \left. \left. + \pi\dot{\lambda} + \nu\dot{\chi} \right] d\tau + \psi_n(1)\psi^n(0) \right\} d\chi_0 \mathcal{D}x \mathcal{D}p \mathcal{D}\lambda \mathcal{D}\pi \mathcal{D}\chi \mathcal{D}\nu \mathcal{D}\psi \Big|_{\theta=0}, \end{aligned}$$

where: $\mathcal{P}_\nu = -p_\nu - gA_\nu(x)$, $M = \tau - 2i\mu F_{\alpha\beta} \psi^\alpha \psi^\beta$; x, p, λ, π , are even and χ, ν, ψ^n , odd trajectories, obeying the boundary conditions $x(0) = x_{\text{in}}, x(1) = x_{\text{out}}, \lambda(0) = \lambda_0, \chi(0) = \chi_0, \psi^n(1) + \psi^n(0) = \theta^n$, and θ^n are some Grassmannian variables.

Integrating over momenta p , we come to the corresponding path-integral in the Lagrangian form:

$$\begin{aligned} \tilde{S}^c(x_{\text{out}}, x_{\text{in}}) = & \quad (13) \\ & \exp \left(i\gamma^n \frac{\partial_l}{\partial\theta^n} \right) \int_0^\infty de_0 \int \exp \left\{ i \int_0^1 \left[-\frac{\dot{x}^2}{2e} - e \frac{M^2}{2} - \dot{x}^\alpha (gA_\alpha \right. \right. \\ & + 4i\mu\psi^5 F_{\alpha\beta} \psi^\alpha \psi^\beta) + ig e F_{\alpha\beta} \psi^\alpha \psi^\beta + i \left(\frac{\dot{x}_\alpha \psi^\alpha}{e} - M^* \psi^5 \right) \chi - i\psi_n \dot{\psi}^n \\ & \left. \left. \left. + \pi\dot{e} + \nu\dot{\chi} \right] d\tau + \psi_n(1)\psi^n(0) \right\} \mathcal{M}(e) d\chi_0 \mathcal{D}x \mathcal{D}e \mathcal{D}\pi \mathcal{D}\chi \mathcal{D}\nu \mathcal{D}\psi \Big|_{\theta=0}, \end{aligned}$$

where $M^* = m + 2i\mu F_{\alpha\beta} \psi^\alpha \psi^\beta$, and the measure $\mathcal{M}(e)$ has the form:

$$\mathcal{M}(e) = \int \mathcal{D}p \exp \left[\frac{i}{2} \int_0^1 e p^2 d\tau \right]. \quad (14)$$

The discussion of the role of the measure (14) can be found in [9].

3 Action of Spinning Particle with Anomalous Magnetic Momentum

The exponent in the integrand (13) can be considered as an effective and nondegenerate Lagrangian action of a spinning particle with an anomalous magnetic momentum. It consists of two principal parts. The first one, which

unifies two summands with the derivatives of e and χ , can be treated as a gauge fixing term S_{GF} ,

$$S_{GF} = \int_0^1 (\pi \dot{e} + \nu \dot{\chi}) d\tau,$$

and corresponds, in fact, to the gauge conditions

$$\dot{e} = \dot{\chi} = 0. \quad (15)$$

The rest part of the effective action can be treated as a gauge invariant action of a spinning particle with an anomalous magnetic momentum. It has the form

$$S = \int_0^1 \left[-\frac{\dot{x}^2}{2e} - e \frac{M^2}{2} - \dot{x}^\alpha (g A_\alpha + 4i\mu\psi^5 F_{\alpha\beta}\psi^\beta) + ig e F_{\alpha\beta}\psi^\alpha\psi^\beta + i \left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - M^*\psi^5 \right) \chi - i\psi_n\dot{\psi}^n \right] d\tau. \quad (16)$$

Indeed, one can verify that there are two types of gauge transformations, under which the actions is invariant, in accordance with two gauge conditions (15). The first one coincide with reparametrizations (2) and the second one is a generalization of supertransformations (3),

$$\begin{aligned} \delta x &= i\psi\epsilon, & \delta e &= i\chi\epsilon, & \delta\chi &= \dot{\epsilon}, & \delta\psi^\alpha &= \frac{1}{2e}(\dot{x}^\alpha + i\chi\psi^\alpha)\epsilon, \\ \delta\psi^5 &= \left[\frac{M}{2} - \frac{i}{m\epsilon}\psi^5 \left(\dot{\psi}^5 - 2\mu F_{\alpha\beta}\dot{x}^\alpha\psi^\beta - \frac{M^*}{2}\chi \right) \right] \epsilon. \end{aligned} \quad (17)$$

Let us analyze the theory with the action (16). The Lagrangian equations of motion have the form:

$$\frac{\delta S}{\delta e} = \frac{1}{e^2} \left(\dot{x}^2 - i\dot{x}_\alpha\psi^\alpha\chi \right) - \frac{M^2}{2} + ig F_{\alpha\beta}\psi^\alpha\psi^\beta = 0, \quad (18)$$

$$\frac{\delta_r S}{\delta\chi} = i \left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - M^*\psi^5 \right) = 0, \quad (19)$$

$$\begin{aligned} \frac{\delta_r S}{\delta\psi^\alpha} &= 2i\dot{\psi}_\alpha + 2ie(g + 2M\mu)F_{\beta\alpha}\psi^\beta - 4\mu\psi^5(i\dot{x}^\beta - \chi\psi^\beta)F_{\beta\alpha} \\ &\quad - i\frac{\dot{x}_\alpha}{e}\chi = 0, \end{aligned} \quad (20)$$

$$\frac{\delta_r S}{\delta\psi^5} = -2i\dot{\psi}^5 + 4i\mu\dot{x}^\alpha F_{\alpha\beta}\psi^\beta + iM^*\chi = 0, \quad (21)$$

$$\begin{aligned} \frac{\delta S}{\delta x^\alpha} &= \frac{d}{d\tau} \left(\frac{\dot{x}_\alpha}{e} \right) + g\dot{x}^\beta F_{\beta\alpha} - 4i\mu\psi^5\dot{x}^\beta\psi^\gamma F_{\beta\alpha,\gamma} + ie(g + 2M\mu)F_{\beta\gamma,\alpha}\psi^\beta\psi^\gamma \\ &\quad + 2\mu\psi^5\chi F_{\beta\gamma,\alpha}\psi^\beta\psi^\gamma = 0. \end{aligned} \quad (22)$$

Because of the existence of the gauge transformations (2) and (17) there are two identities between all these equations. That means that the number of independent equations is less than the number of the variables, and we can fix two of the latter by imposing of some gauge conditions. One can choose (it is also seen from the Hamiltonian analysis which follows) the gauge conditions $\chi = 0$ and $e = 1/m$ to simplify the analysis of the equations (18-22). Our aim is to show that equations (18-22) describe a spinning particle with anomalous momentum μ . To this end we need to use the nonrelativistic approximation and consider the case of a weak magnetic field. In such a case $F_{0i} = 0$, and $F_{ij} = -\epsilon_{ijk}B_k$, where B_k are components of a magnetic field \vec{B} and ϵ_{ijk} is the tree dimensional Levi-Civita symbol. It is easy to see from the equations (18,19,21) that in the gauge and in the above mentioned approximation we have:

$$\dot{x}^0 \approx 1, \quad \dot{x}^i \approx v^i = \frac{dx^i}{dx^0}, \quad (23)$$

Introducing the three dimensional spin vector [1]

$$s_k = i\epsilon_{kjl}\psi_l\psi_j,$$

and using (23), we get from the equations (20) and (22):

$$\begin{aligned} \dot{\vec{s}} &= 2 \left(\frac{g}{2m} + \mu \right) \vec{s} \times \vec{B}, \\ m\dot{\vec{v}} &= g\vec{v} \times \vec{B} + 2 \left(\frac{g}{2m} + \mu \right) \nabla (\vec{s} \cdot \vec{B}), \end{aligned} \quad (24)$$

The equations (24) describe [14] a nonrelativistic motion of a particle with total spin momentum \vec{s} , and total magnetic momentum $2 \left(\frac{g}{2m} + \mu \right) \vec{s}$. Taking into account that the giromagnetic ratio for electron equals 2, one can conclude that μ corresponds to the anomalous magnetic momentum of electron, what confirms the interpretation of the action (16).

Going over to the Hamiltonian formalism, we introduce the canonical momenta:

$$\begin{aligned} p_\alpha &= \frac{\partial L}{\partial \dot{x}^\alpha} = -\frac{1}{e}(\dot{x}_\alpha - i\psi_\alpha \chi) - gA_\alpha - 4i\mu\psi^5 F_{\alpha\beta}\psi^\beta, \\ P_e &= \frac{\partial L}{\partial \dot{e}} = 0, \quad P_\chi = \frac{\partial L}{\partial \dot{\chi}} = 0, \quad P_n = \frac{\partial L}{\partial \dot{\psi}^n} = -i\psi_n. \end{aligned} \quad (25)$$

It follows from the equation (25) that there exist primary constraints $\Phi_A^{(1)} = 0$,

$$\Phi_A^{(1)} = \begin{cases} \Phi_1^{(1)} = P_\chi, \\ \Phi_2^{(1)} = P_e, \\ \Phi_{3n}^{(1)} = P_n + i\psi_n. \end{cases} \quad (26)$$

We construct the Hamiltonian $H^{(1)}$, according to the standard procedure (we use the notations of the book [16]),

$$H^{(1)} = H + \lambda_A \Phi_A^{(1)}, \quad \text{where } H = \left(\frac{\partial_r L}{\partial \dot{q}} \dot{q} - L \right) \Big|_{\substack{\partial_r L = P \\ \partial_t L = P}}, \quad q = (x, e, \chi, \psi^n),$$

and get for H :

$$\begin{aligned} H &= -\frac{e}{2} \left(\mathcal{P}^2 - 8i\mu\psi^5 F_{\alpha\beta} \mathcal{P}^\alpha \psi^\beta + 2ig F_{\alpha\beta} \psi^\alpha \psi^\beta - M^2 \right) \\ &\quad - i \left(\mathcal{P}_\alpha \psi^\alpha - M\psi^5 \right) \chi. \end{aligned}$$

From the conditions of the conservation of the primary constraints $\Phi_{1,2}^{(1)}$ in time τ , $\dot{\Phi}_{1,2}^{(1)} = \{ \Phi_{1,2}^{(1)}, H^{(1)} \} = 0$, we find the secondary constraints $\Phi_{1,2}^{(2)} = 0$,

$$\Phi_1^{(2)} = \mathcal{P}_\alpha \psi^\alpha - M\psi^5 = 0, \quad (27)$$

$$\Phi_2^{(2)} = \mathcal{P}^2 - 8i\mu\psi^5 F_{\alpha\beta} \mathcal{P}^\alpha \psi^\beta + 2ig F_{\alpha\beta} \psi^\alpha \psi^\beta - M^2 = 0, \quad (28)$$

and the same conditions for the constraints $\Phi_{3n}^{(1)}$ give equations for the determination of λ_{3n} . Thus, the Hamiltonian H appears to be proportional to constraints, as one can expect in the case of a reparametrization invariant theory,

$$H = i\chi \Phi_1^{(2)} - \frac{e}{2} \Phi_2^{(2)}.$$

No more secondary constraints arise from the Dirac procedure, and the Lagrange's multipliers λ_1 and λ_2 remain undetermined, in perfect correspondence with the fact that the number of gauge transformations parameters equals two for the theory in question [16]. One can go over from the initial set of constraints $(\Phi^{(1)}, \Phi^{(2)})$ to the equivalent one $(\Phi^{(1)}, T)$, where:

$$T = \Phi^{(2)} + \frac{i}{2} \frac{\partial_r \Phi^{(2)}}{\partial \psi^n} \Phi_{3n}^{(1)}. \quad (29)$$

The new set of constraints can be explicitly divided in a set of the first-class constraints, which is $(\Phi_{1,2}^{(1)}, T)$ and in a set of the second-class constraints, which is $\Phi_{3n}^{(1)}$.

4 Quantization

First we consider an operator quantization, expecting to get in this procedure the Dirac-Pauli equation (4).

In our case we perform only a partial gauge fixing, by imposing the supplementary gauge conditions $\Phi_{1,2}^G = 0$ to the primary first-class constraints $\Phi_{1,2}^{(1)}$,

$$\Phi_1^G = \chi = 0, \quad \Phi_2^G = e = 1/m, \quad (30)$$

which coincide with those we used in the Lagrangian analysis. One can check that the conditions of the conservation in time of the supplementary constraints (30) give equations for determination of the multipliers λ_1 and λ_2 . Thus, on this stage we reduced our Hamiltonian theory to one with the first-class constraints T and second-class ones $\varphi = (\Phi^{(1)}, \Phi^G)$. After that we will use the so called Dirac method for systems with first-class constraints [15], which, being generalized to the presence of second-class constraints, can be formulated as follow: the commutation relations between operators are calculated according to the Dirac brackets with respect to the second-class constraints only; second-class constraints operators equal zero; first-class constraints as operators are not zero, but, are considered in sense of restrictions on state vectors. All the operator equations have to be realized in some Hilbert space.

The sub-set of the second-class constraints $(\Phi_{1,2}^{(1)}, \Phi^G)$ has a special form [16], so that one can use it for eliminating of the variables e, P_e, χ, P_χ , from

the consideration, then, for the rest of the variables x, p, ψ^n , the Dirac brackets with respect to the constraints φ reduce to ones with respect to the constraints $\hat{\Phi}_{3n}^{(1)}$ only and can be easily calculated,

$$\{x^\alpha, p_\beta\}_{D(\hat{\Phi}_{3n}^{(1)})} = \delta_\beta^\alpha, \quad \{\psi^n, \psi^m\}_{D(\hat{\Phi}_{3n}^{(1)})} = \frac{i}{2}\eta^{nm},$$

while others Dirac brackets vanish. Thus, the commutation relations for the operators $\hat{x}, \hat{p}, \hat{\psi}^n$, which correspond to the variables x, p, ψ^n respectively, are

$$\begin{aligned} [\hat{x}^\alpha, \hat{p}_\beta]_- &= i\{x^\alpha, p_\beta\}_{D(\hat{\Phi}_{3n}^{(1)})} = \delta_\beta^\alpha, \\ [\hat{\psi}^m, \hat{\psi}^n]_+ &= i\{\psi^m, \psi^n\}_{D(\hat{\Phi}_{3n}^{(1)})} = -\frac{1}{2}\eta^{mn}. \end{aligned} \quad (31)$$

Besides, the operator equations hold:

$$\hat{\Phi}_{3n}^{(1)} = \hat{P}_n + i\hat{\psi}_n = 0. \quad (32)$$

The commutation relations (31) and the equations (32) can be realized in a space of four columns $\Psi(x)$ dependent on x^α as: \hat{x}^α are operators of multiplication, $\hat{p}_\alpha = -i\partial_\alpha$, $\hat{\psi}^\alpha = \frac{i}{2}\gamma^5\gamma^\alpha$, and $\hat{\psi}^5 = \frac{i}{2}\gamma^5$, where γ^n are the γ -matrices (γ^α, γ^5). The first-class constraints \hat{T} as operators have to annihilate physical vectors; in virtue of (32), (29) these conditions reduce to the equations:

$$\hat{\Phi}_{1,2}^{(2)}\Psi(x) = 0, \quad (33)$$

where $\hat{\Phi}_{1,2}^{(2)}$ are operators, which correspond to the constraints (27), (28). There is no ambiguity in the construction of the operator $\hat{\Phi}_1^{(2)}$, according to the classical function $\Phi_1^{(2)}$ from (27). Thus, taking into account the realizations of the commutation relations (31), one easily can see that the first equation (33) reproduces the Dirac-Pauli equation (4). As to the construction of the operator $\hat{\Phi}_2^{(2)}$, according to the classical function $\Phi_2^{(2)}$ from (28), we meet here an ordering problem since the constraint $\Phi_2^{(2)}$ contains terms with products of the momenta and functions of the coordinates, namely terms of the form $p_\alpha A^\alpha, p_\alpha F^{\alpha\beta}$. For such terms we choose the symmetrized form of the corresponding operators,

$$p_\alpha A^\alpha \rightarrow \frac{1}{2}[\hat{p}_\alpha, A^\alpha(\hat{x})]_+, \quad p_\alpha F^{\alpha\beta} \rightarrow \frac{1}{2}[\hat{p}_\alpha, F^{\alpha\beta}(\hat{x})]_+, \quad (34)$$

which, in particular, provides the hermiticity of the operator $\hat{\Phi}_2^{(2)}$. But the main reason is, the correspondence rule (34) provides the consistency of the two equations (33). Indeed, in this case we have

$$\hat{\Phi}_2^{(2)} = (\hat{\Phi}_1^{(2)})^2, \quad (35)$$

and the second equation (33) appears to be merely the consequence of the first equation (33), i.e. of the Dirac-Pauli equation (4). To verify the validity of (35), one needs only to take into account that the operator, which corresponds to the term $8i\mu\psi^5 F_{\alpha\beta} \mathcal{P}^{\alpha\beta}$ in the constraint $\hat{\Phi}_2^{(2)}$ (28), in virtue of the structure of the γ -matrices, can be written in the form:

$$\begin{aligned} 8i\mu\psi^5 F_{\alpha\beta} \mathcal{P}^{\alpha\beta} &\rightarrow i\mu [F_{\alpha\beta}(\hat{x}), \hat{P}^\alpha]_+ \gamma^\beta = \\ 2i\mu F_{\alpha\beta}(\hat{x}) \hat{P}^\alpha \gamma^\beta + \mu \delta^\alpha F_{\alpha\beta}(\hat{x}) \gamma^\beta &= [\hat{P}_\alpha \gamma^\alpha, \frac{\mu}{2} \sigma^{\alpha\beta} F_{\alpha\beta}(\hat{x})]_- \end{aligned}$$

To complete the operator quantization, one has to present an inner product in the space of realization of commutation relations. The general method of its construction, in the frame of the Dirac method we used, is unfortunately still unknown. Nevertheless, in this concrete case, the space of physical vectors, obeying the condition (33), can be transformed into a Hilbert space, if one takes for the inner product ordinary scalar product of solutions of the Dirac equation, which does not depend on x^0 , in spite of the integration is fulfilled in it over x^i only. It is not difficult to verify that the introduced operators, obey of natural properties of hermiticity, which are known from the Dirac relativistic mechanics. In particular, the operator \hat{p}_0 , which has to be considered on the same foot with \hat{p}_i , is also hermitian on the solutions of the Dirac-Pauli equation (4), in virtue of the above mentioned independence of the scalar product on x^0 .

Thus, we see that the operator quantization of the action (16) reproduces the Dirac-Pauli quantum theory. To make the picture complete, one oughts to discuss the path-integral quantization of the theory with the action (16). In fact, the problem reduces to a demonstration that the path-integral (12) for the causal Green's function, which is the propagator for the second quantized theory, can be interpreted in the frame of the known formal methods of quantization of gauge theories. However, a peculiarity of the theory in question is that the Hamiltonian equals zero on the constraints surface; we

have avoided this problem in the operator quantization, choosing the special Dirac method (direct ways of the problem solving in the operator quantization were considered in ref. [8,19]). Nevertheless, the path-integral (12) can be interpreted in the frame of the so called generalized Hamiltonian quantization (BFV method [20]). One can demonstrate that the exponent in path integral (12) appears to be a generalized Hamiltonian action of the BFV method in a special gauge.

We start the BFV consideration with the Hamiltonian formulation of our theory after the partial gauge fixing (30). In this case we have two first-class constraints (29) and the set of second class constraints $\Phi_{3n}^{(1)}$ (26). We introduce new canonical pairs: odd (λ_1, π_1) , and even ghosts $(\theta_1, \bar{\theta}_1)$,

$$\theta_1 = \begin{pmatrix} c_1 \\ B_1 \end{pmatrix}, \quad \bar{\theta}_1 = \begin{pmatrix} \bar{B}_1 \\ \bar{c}_1 \end{pmatrix},$$

for the first-class constraint T_1 ; even (λ_2, π_2) , and odd ghosts $(\theta_2, \bar{\theta}_2)$,

$$\theta_2 = \begin{pmatrix} c_2 \\ B_2 \end{pmatrix}, \quad \bar{\theta}_2 = \begin{pmatrix} \bar{B}_2 \\ \bar{c}_2 \end{pmatrix},$$

for the first-class constraint T_2 ($\text{gh } \theta = -\text{gh } \bar{\theta} = 1$). After that we construct the fermionic generating charge Ω (also called BRST charge) as a solution of the equation

$$\{\Omega, \Omega\}_{D(\Phi_{3n}^{(1)})} = 0,$$

obeying the conditions

$$\begin{aligned} \Omega &= \Omega_a \theta_a, \quad \text{gh } \Omega_a = 0, \\ \Omega_a|_{\theta=\bar{\theta}=0} &= \begin{pmatrix} T_a \\ \pi_a \end{pmatrix}, \quad a = 1, 2. \end{aligned}$$

One can see that such a solution has the form

$$\Omega = T_a c_a + \pi_a B_a + \frac{i}{4} \bar{B}_2 (c_1)^2. \quad (36)$$

In the case of consideration, the Hamiltonian on the constraints surface equals zero, and one can write the BVF Hamiltonian in the form

$$H_{\text{BFV}} = \{\Omega, \Psi\}_{D(\Phi_{3n}^{(1)})},$$

where Ψ is a gauge fermion,

$$\begin{aligned} \Psi &= \Psi_a \bar{\theta}_a, \quad \text{gh } \Psi_a = 0, \\ \Psi_a|_{\theta=\bar{\theta}=0} &= \chi_a, \quad a = 1, 2. \end{aligned}$$

Columns χ_a play the role of gauge conditions. To reproduce the path-integral (12), one needs to choose:

$$\chi_a = \begin{pmatrix} \lambda_a \\ 0 \end{pmatrix}. \quad (37)$$

So we get:

$$\Psi = \lambda_a \bar{B}_a, \quad (38)$$

which leads to:

$$H_{\text{BFV}} = T_a \lambda_a + \bar{B}_a B_a + \frac{i}{2} \bar{B}_2 c_1 \lambda_1, \quad (39)$$

so that the correspondent BFV action has the form:

$$S_{\text{BFV}} = \int dr \left(p\dot{x} + P_n \dot{\psi}^n + \pi_a \dot{\lambda}_a + \bar{\theta}_a \dot{\theta}_a - T_a \lambda_a - \bar{B}_a B_a - \frac{i}{2} \bar{B}_2 c_1 \lambda_1 \right). \quad (40)$$

To construct the path-integral in BFV formulation one needs to use the well known formula for a path-integral of theories with second-class constraints [21], which are in this case $\Phi_{3n}^{(1)}$. Such an integral includes the factor

$$\delta \left(\Phi_{3n}^{(1)} \right) \text{Sdet}^{1/2} \left\{ \Phi_{3a}^{(1)}, \Phi_{3b}^{(1)} \right\}.$$

In the case of consideration the superdeterminant reduces to a constant and the δ -function lifts the integration over the momenta P_n and allows one to replace the constraints T by $\Phi^{(2)}$. The trivial interaction of the ghosts with λ_1 can be eliminated by means of a shift of the variable \bar{B}_1 , then the ghosts path-integral separates. Thus, after the renotations $\lambda_1 \rightarrow -2i\chi$, $\lambda_2 \rightarrow -\lambda$, $\pi_1 \rightarrow \frac{i}{2}\nu$, $\pi_2 \rightarrow -\pi$, we can recognize in the path-integral (12) the BFV path-integral exponent. The operator factor in (12) and the additional integrations over λ_0, χ_0 are connected with the special choice of the Green's function of the Dirac-Pauli equation.

5 Conclusion

In the conclusion we would like to underline two points. First, that the relationship between the model proposed and the one of spinning particle reproduces on the classical level the relationship between the Dirac-Pauli equation and Dirac equation. Second, the quantization of the model appears to be rather instructive because of the ordering of operators plays here an important role. In this sense the problem is analogous to the well known problem of the quantization of a particle in curved space [12]. It is remarkable that here the Weyl rule follows uniquely from the condition of consistency of the Dirac quantization of theories with first-class constraints. In this connection one can remind the result of the work [22] where it was proved that only the Weyl ordering of operators provides the correspondence principle in the Dirac quantization of theories with second-class constraints.

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Appendix

Here we present the quantity

$$T \exp \int_0^1 \rho_n(\tau) \gamma^n d\tau \quad (41)$$

by a Grassmannian path-integral. Remind that $\rho_n(\tau)$ are odd variables; T-product acts on γ -matrices, $[\gamma^m \gamma^n]_+ = 2\eta^{mn}$, $n = \overline{0, 3, 5}$, which suppose formally to depend on time τ , they anticommute by definition with $\rho_n(\tau)$ and are considered in T-ordering procedure as Fermi operators

First we present (41) in the Sym-form. To this end we use a formula, which is a version of the Weak theorem (see, for example, reference [17]). Let $\hat{\phi}(\tau)$ be some operators and $F(\hat{\phi})$ some functional on them. Then

$$T F(\hat{\phi}) = \text{Sym} \exp \left[\frac{1}{2} \int \frac{\delta_r}{\delta \hat{\phi}(\tau_1)} K(\tau_1, \tau_2) \frac{\delta_r}{\delta \hat{\phi}(\tau_2)} d\tau_1 d\tau_2 \right] F(\hat{\phi}) \Big|_{\hat{\phi}=\hat{\phi}},$$

where

$$K(\tau_1, \tau_2) = T \hat{\phi}(\tau_1) \hat{\phi}(\tau_2) - \text{Sym} \hat{\phi}(\tau_1) \hat{\phi}(\tau_2) = \frac{1}{2} \epsilon(\tau_1 - \tau_2) [\hat{\phi}(\tau_1), \hat{\phi}(\tau_2)]_-,$$

and $\epsilon(\tau) = \text{sgn } \tau$. In the case of consideration, $F(\hat{\phi}) = \exp \int_0^1 \hat{\phi}(\tau) d\tau$, $\hat{\phi}(\tau) = \rho_n(\tau) \gamma^n$, and $K(\tau_1, \tau_2) = -\epsilon(\tau_1 - \tau_2) \rho_n(\tau_1) \rho^n(\tau_2)$. Consequently,

$$T \exp \int_0^1 \rho_n(\tau) \gamma^n d\tau = \exp \left[-\frac{1}{2} \int_0^1 d\tau_1 \int_0^1 d\tau_2 \rho_n(\tau_1) \epsilon(\tau_1 - \tau_2) \rho^n(\tau_2) \right] \exp \int_0^1 \rho_n(\tau) \gamma^n. \quad (42)$$

We have omitted the symbol Sym in the right side of (42) because of the concrete structure of the functional. The quadratic exponential from the right side of (42) can be presented by means of a Gaussian path-integral over Grassmannian trajectories $\xi^n(\tau)$,

$$\exp \left[-\frac{1}{2} \int_0^1 d\tau_1 \int_0^1 d\tau_2 \rho_n(\tau_1) \epsilon(\tau_1 - \tau_2) \rho^n(\tau_2) \right] = \int \exp \left\{ \int_0^1 \left[\frac{1}{4} \xi_n(\tau) \dot{\xi}^n(\tau) - i \rho_n(\tau) \xi^n(\tau) \right] d\tau \right\} \mathcal{D}\xi, \quad (43)$$

where a normalization factor is included in the measure of integration, so that the integral in the right side of (43) equals unit at $\rho = 0$. The trajectories $\xi^n(\tau)$ anticommute with γ -matrices by definition and obey the boundary conditions

$$\xi^n(0) + \xi^n(1) = 0, \quad (44)$$

to make the path-integral in (43) invariant under the shifts of integration variables. One can also check [18] that the representation is available

$$\exp \left[\int_0^1 \rho_n(\tau) \gamma^n d\tau \right] = \exp \left(i \gamma^n \frac{\partial_l}{\partial \theta^n} \right) \exp \left[-i \int_0^1 \rho_n(\tau) \theta^n d\tau \Big|_{\theta=0} \right], \quad (45)$$

where θ^n are odd variables anticommuting with γ -matrices. Gathering (43-45) and making then the change of variables

$$\xi^n(\tau) + \theta^n = 2\psi^n(\tau),$$

we get the path integral representation for (41)

$$\begin{aligned} T \exp \int_0^1 \rho_n(\tau) \gamma^n d\tau = & \quad (46) \\ \exp \left(i \gamma^n \frac{\partial_l}{\partial \theta^n} \right) \int \exp \left[\int_0^1 (\psi_n \dot{\psi}^n - 2i \rho_n \psi^n) d\tau + \psi_n(1) \psi^n(0) \right] \mathcal{D}\psi \Big|_{\theta=0} \end{aligned}$$

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