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DEFORMED GAUSSIAN ORTHOGONAL ENSEMBLE DESCRIPTION OF ISOSPIN MIXING AND SPECTRAL FLUCTUATION PROPERTIES

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DEFORMED GAUSSIAN ORTHOGONAL ENSEMBLE DESCRIPTION OF ISOSPIN MIXING AND SPECTRAL FLUCTUATION PROPERTIES*

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Abstract

Analysis of isospin symmetry breaking in the case of pure spectral observables is made within the recently developed Deformed Gaussian Orthogonal Ensemble (DGOE). The low energy spectrum of ^{26}Al recently studied by Mitchell et al is used to perform the analysis. Both the spacing and eigenvector distributions are considered. The value of the Coulomb matrix elements that causes the isospin breaking is extracted and found to be consistent with other measurement.

Recently, the influence of isospin mixing on the spectral observables in the low energy spectrum of ^{26}Al has been experimentally studied by Mitchell et al [1]. The data has been subsequently analysed by Guhr and Weidenmuller [2] using a random matrix model through which the root mean square Coulomb matrix element was deduced and found consistent with other data. The

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model used by Ref.2 involves two coupled Gaussian Orthogonal Ensemble (GOE), one for the T=0 state (all together 75 of them) and the other for the T=1 states (25 states).

In the present paper we develop a different model to study isospin mixing and apply it to the data of Ref.1. Our model is based on the recently developed Deformed Gaussian Orthogonal Ensemble (DGOE) [3, 4]. The DGOE is constructed using the maximum entropy principle applied to generic random matrices subjected to appropriate constraints. We show that our theory can easily account for the data of Ref.1 in so far the level spacing distribution and Δ_3 statistic are concerned. We also suggest that a more stringent test of the nature of the physics contained in the data can be supplied through the concomitant study of the transition amplitude distribution, which is not necessarily of the Porter-Thomas form. The DGOE supplies a means to study all statistical features in a straight forward way.

The starting point of our discussion is the maximum entropy principle stated below

$$\delta[S - \lambda_0 < 1 > - < TrH^2 > \alpha_0] = 0 \tag{1}$$

where

$$S = \int dH P(H) \ell n P(H)$$

$$< 1 >= \int dH P(H) = 1$$

$$< TrH^2 >= \int dH \overset{P(H)}{\vphantom{x}} TrH^2 = \mu_0$$

The distribution P(H) of the Hamiltonian ensemble H is the obtained from (1)

$$P(H)_{GOE} = exp[-\lambda_0 - 1 - \alpha_0 TrH^2], \tag{2}$$

$\alpha_0 = \frac{N(N+1)}{4\mu_0}$ and $exp(-\mu_0 - 1) = 2^{\frac{-N}{2}}(\frac{\pi}{2\alpha_0})^{\frac{-N(N+1)}{4}}$

Eq. (2) is the Gaussian Orthogonal Ensemble GOE which describes very well nuclear spectra when all symmetries are obeyed. The dimension of H above is taken to be N.

To discuss isospin mixing we introduce the Deformed Gaussian Orthogonal Ensemble DGOE which is discussed in Ref. 3-4. The idea is to divide H into 4 block matrices vis

$$H = PHP + QHQ + PHQ + QHP \tag{3}$$

$$P \equiv \sum_{i=1}^{M} \mid i >< i \mid, Q = 1 - P$$

$$P^2 = P, Q^2 = Q, PQ = QP = 0$$

The matrix PHP represents, e.g, the T = 0 states while QHQ, the T = 1 states. The non-diagonal block matrix PHQ accounts for the isospin mixing. A constraint is now imposed on the quantity Tr PHQHP, namely.

$$< TrPHQHP >= \nu \tag{4}$$

Calling the Lagrange multiplier associated with (4), β, we obtain for P(H) (Eqs. 1 and 4)

$$P_{DGOE}(H) = P_{GOE}(H) exp[-\beta Tr PHQHP](1 + \frac{\beta}{2\alpha})^{\frac{M(N-M)}{2}} \tag{5}$$

where M and N-M are the dimensions of the symmetric block matrices PHP and QHQ, respectively. It is easy to show that the above procedure is equivalent to the one followed by Guhr and Weidenmuller (GW) [2], who write for H

$$H = H_U + \lambda H_C \tag{6}$$

where H_U is just PHP + QHQ, and λH_C, represents the symmetry-breaking Coulomb interaction (and thus $\lambda H_C \equiv$ PHQ + QHP). Guhr and Weidenmuller treat H in the following way. For a fixed value of λ they create an ensemble of interacting spectra with the help of random number generator. They then calculate the spacing distribution P(s) and the spectral rigidity $\Delta_3(L)$ of the ensemble. The equivalence of our treatment with that of GW is made firm by the observation that λ is just [3]

$$\lambda = (1 + \frac{\beta}{2\alpha})^{\frac{-1}{2}} \tag{7}$$

We note here that α measures the second moments of PHP and QHQ whereas β measures the second moment of QHP vis

$$< H_{ij} H_{k\ell} >= (\delta_{ik}\delta_{j\ell} + \delta_{jk}\delta_{i\ell}) \frac{1}{4\alpha + 2\beta \mid \delta_{ki} - \delta_{\ell i} \mid} \tag{8}$$

We further note that $1 + \frac{\beta}{2\alpha}$ measures the information content, I, of the DGOE relative to the GOE vis

$$I = \frac{M(N-M)}{N(N+1)} \ell n(1 + \frac{\beta}{2\alpha}) \tag{9}$$

and thus, for large N and $M/N \equiv n$,

$$I = 2n(1-n)\ell n(1/\lambda) \tag{10}$$

Note that λ of Eq.(6) has the range of values $0 < \lambda < 1$; $\lambda = 0$ representing two decouped GOE'S , $\lambda = 1$ representing one GOE.

From our DGOE one can calculate the spacing distribution P(s) as well as the amplitude distribution P(c), both of which are affected by the constraint (4). It is easy to show that for $\beta = 0$, ($\lambda = 1$), our ensemble reduces to GOE , P(s) becomes the Wigner distribution and $P(c^2)$ the Porter-Thomas distribution. For larger values of β (weaker coupling), the DGOE supplies distributions that deviates appreciably from the GOE. The data of Ref.1, seem to indicate the need for such distributions. We turn now to a detailed analysis of the data reported in Ref. 1.

The spacing distribution P(s) is plotted in fig.1 (histogram). Also shown are the data of Ref.1. In the calculation of P(s), we considered N = 100, M = 25. The experimental spacing distribution, shown as the dotted histogram, describes 75 levels with T = 0 and 25 levels with T = 1 in ^{26}Al in the excitation energy range between 0 and 8 MeV. The dotted curve is the Wigner distribution which obtains when $\beta = 0$. The value of β we used in our calculation is 636α which gives $\lambda = 0.056$, similar to Guhr and Weidenmuller. Another quantity that is calculated is $\Delta_3(L)$ shown in Fig.2. The value of $\beta = 636\alpha$ accounts very well for the experimental $\Delta_3(L)$. Also shown in Fig.3 is the GOE $\Delta_3(L)$ and the Poissonian $\Delta_3(L)$. The case of two uncoupled

GOE ($\beta = \infty$, $\lambda = 0$), which represents no isospin mixing, is not shown and can be found in GW[2].

From the above we conclude that the data of Ref.1, is very well accounted for with the DGOE with $\frac{\beta}{\alpha} = 636$. This corresponds to the same value of the Coulomb mixing matrix element extracted by Guhr and Weidenmuller, namely $< H_c > \simeq 20KeV$. Futher, from Eq.(10) we can state that the information content of the spectrum studied by Mitchell et al [1] relative to the GOE is I = 1.28.

To complete the analysis, we present in fig.3 the theoretical intensity distribution P(y) (histogram) which, should be compared to the Poster-Thomas (GOE), shown as the dashed curve. P(y) above while was calculated within the DGOE described above the full curve was obtained with a χ^2 - distribution (see below). It is thus clear that the deviation from the PT distribution is quite large indicating a rather strong sensitivity to the strength of the mixing situation. In cases of weak mixing (β very large) we have indicated in Ref.3) that the intensity distribution is better described by a sum of two χ^2 distributions,

$$P(y) = aP_{\nu}(y) + bP_{\mu}(y) \tag{11}$$

$$P_{\eta}(y) = [\frac{\eta}{2 < y >}]^{\frac{\eta}{2}} \frac{y^{\frac{\eta}{2}-1} exp[-\eta y/2 < y >]}{\Gamma(\eta/2)},$$

$$a + b = 1$$

where the values of ν and μ are determined using the surprisal analysis of Alhassid and Levine [5]. A typical case is shown in Fig.3b with $\beta = 10000\alpha$. Weak mixing is thus characterized by large devition of P(s) from the Wigner surmise and an amplitude distribution of the type given in Eq.(11). The spectral rigidity $\Delta_3(L)$ would lie between the lower GOE $\Delta_3(L)$ and the higher two uncoupled GOE's $\Delta_3(L)$. We have also verified a strong dependence of P(y) on the dimension of PHP relative to that of H. It would be interesting to verify the above experimentally. We also encourage the experimentalists of Ref.1 to verify our P(y), fig.3a, using their data.

In conclusion, we have analysed in this paper the problem of isospin symmetry breaking in the case of pure spectral observables. The Deformed

Gaussian Orthogonal Ensemble of Ref. 3-4 is used for the purpose. The experimental data was taken from Ref. 1. We find that the DGOE accounts very well for the data, and the resulting Coulomb matrix element in ^{26}Al was found to be 26 KeV consistent with that extracted by Ref. 2. We also discussed the intensity distribution and discovered that it deviates considerably from the Porter-Thomas one. This should be easily verified experimentally.

References

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Figure Captions

Fig.1 Experimental spacing distribution P(s) for 75 levels with T=0 and 25 levels with T=1 in ^{26}Al in the excitation energy range $0 < E^{*} < 8MeV$, plotted as dashed histogram 1. The DGOE histogram with $\beta = 636\alpha$ is shown as full histogram. The spacing distributions of a single GOE and the Poisson distribution are also shown as the dotted and dashed-dotted curves, respectively.

Fig.2 The spectral rigidity $\Delta_3(L)$ calculated with the DGOE (full curve) GOE (lower full) and Poisson (upper dashed-dotted). The experimental values are taken from Ref. 1.

Fig.3 The intensity distribution P(y), a) $\beta = 636\alpha$, b) $\beta = 10^4\alpha$. See text for details.

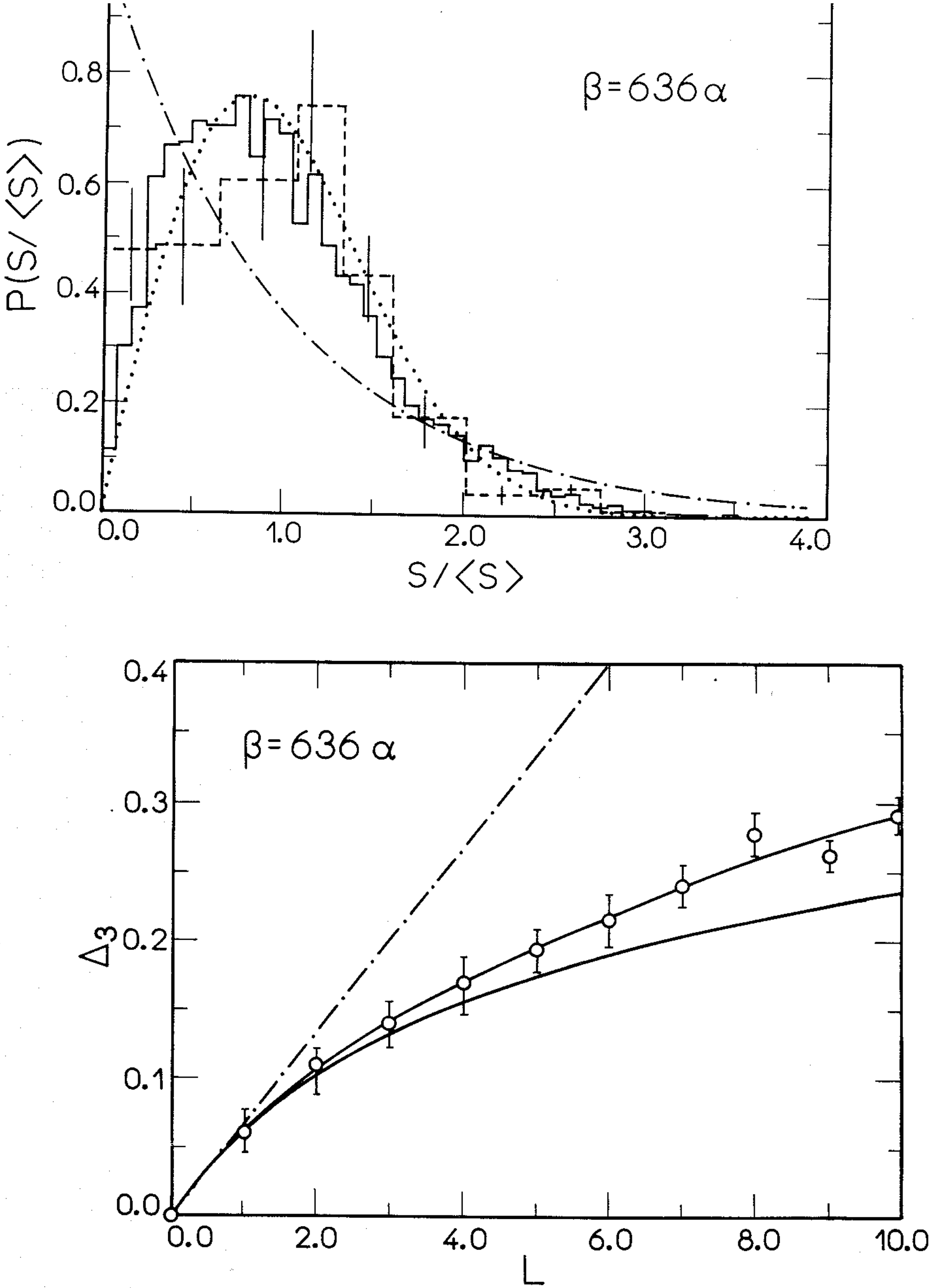
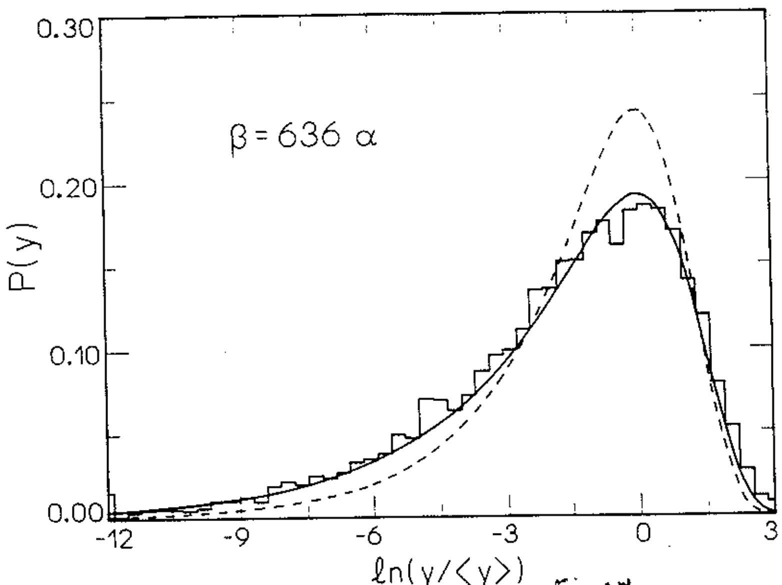
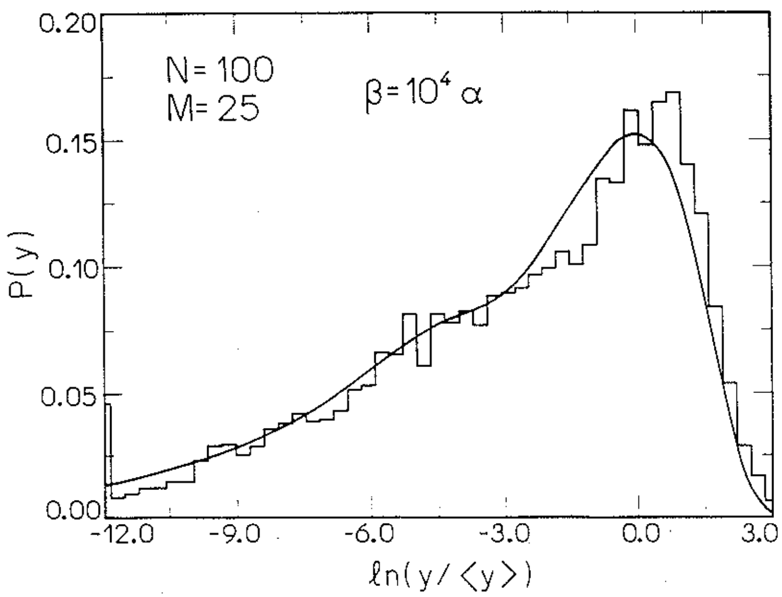


Fig. 2



0.30
0.20
0.10
0.00
P(y)
β = 636 α
-12
-9
-6
-3
0
3
ln(y/⟨y⟩)

Fig 3a



0.20
0.15
0.10
0.05
0.00
P(y)
N = 100
M = 25
β = 10^4 α
-12.0
-9.0
-6.0
-3.0
0.0
3.0
ln(y/⟨y⟩)

Fig 3b