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STATISTICS OF COLOR VARIABLES AND QUARK CONFINEMENT

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Summary

Based on rigorous results derived from Gentile statistics we present a convincing proof of quark confinement in hadrons. Color variable is taken as a dynamical variable of 3-quarks system. The representations of gentilionic elements of $\mathcal{A}\,S^{(3)}$ are given in two contexts: occupation number color space and symplectic space. A general method of obtaining new representations of $S^{(N)}$ is given into details. Some drawbacks of QFT and Parastatistics are pointed out, and possible solutions for these puzzles are suggested. Some analogies between physical optics and particle physics are also given. Finally, the possibility of applying usual Drell-Yan models to quark-quark hadron interactions is discussed.

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1. INTRODUCTION

In the last few years[1–4] we have developed, within the framework of unrelativistic quantum mechanics, the concept of general statistics, first introduced by Gentile in the 40's[5–8]. Besides the two usual Bose and Fermi statistics, we have considered the possibility of Gentile statistics being applicable to some kinds of systems. Since the beginning we were aware of the difficulties we would have to encounter in adapting these new concepts to known physical systems. In the case of elementary particles, the exhaustive analysis carried out since Greenberg and Messiah's set of papers[9–10] has led to the inescapable conclusion of the non-obervability of particles which were neither bosons or fermions. On the other hand, during the last three decades, the quark model of strongly interacting particles has grew in importance and, by now, seems to be the most important candidate to explain the known properties of hadrons. But, in spite of the great successes exhibited by the standard model $(SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y)$, in accounting for almost all known and virtually doable particle physics, some fundamental problems, inherent to the model, remain unsolved: the statistical problem, the quark confinement, the coalescence of hadrons, and so on. Generally speaking, the dynamics governing the behavior of quarks and gluons inside hadrons is poorly understood. We have some good quantitative results such as the energy evolution of cross sections for deep inelastic scattering, for electron-positron annihilation, for the production of massive lepton pairs and other processes. But, when we deal with transitions of quarks and gluons to hadron states and with processes which involve low transverse momenta, we met significant difficulties. Useless to mention is the fundamental problem of quark confinement. To remedy the situation, theorists have resorted to phenomenological constructions which do not follow directly from the fundamentals of QFT, such as QCD. For these reasons, methods based on the fundamentals of QFT rate highly in importance.

Since Gentile's theory is not yet a QFT, it is not our intention, in this paper, to give complete answers to these very difficult problems. Here, we shall be mainly concerned with the possibility of ascribing to quarks the properties derived from Gentile's statistics applied to color variables. Nevertheless, until now, we have developed the theory in a rigorous fashion. In our works quoted above, it was assumed the principle of indistinguishability, requiring that Hilbert space vectors differing only by a permutation of particles represent the same physical state. Thus, every physical state is a basis for a representation of the permutation group and must, consequently, belong to one irreducible subspace of the Hilbert space. It is worthwhile to mention that for $N = 3$, which is the relevant case in quark theory, there is a 2×2 irreducible representation, named Gentile representation. In this work, following the standard treatment[11] of N- particle systems in quantum mechanics, let us denote by G_H the full symmetry group associated with the hamiltonian H of the N particles. Hence, the permutation group $S(N)$ of order $N!$ is properly a subgroup of G_H. It is also convenient to write $G_H = G'_H \cdot S(N)$, where G'_H is the subgroup of G_H not involving permutations. Now, due to the irreducibility of each representation of $S(N)$, every corresponding subspace of the Hilbert space must be stable under the operations of G'_H. This is accomplished if all elements of G'_H commute with the permutations. With this restriction, the full symmetry group of H can now be written as the direct product $G_H = G'_H \times S(N)$. In geometrical language, this assumption amounts to say that there exists a trivialisation of the covering[12]. This mode of considering the full symmetry group acting on the space, spin and flavor coordinates is far from being trivial. The fact that the elements of G'_H commute with all permutations restricts their form considerably. Thus, for interaction terms, like the coulombian, which remain invariant only if the coordinates of each particle are changed in the same way, G'_H automatically commutes with the permutation group and the Pauli principle plays no role. However, this is not a general situation.

For hamiltonians with interaction terms enabling each particle to move independently of the others, the Pauli principle becomes essential in avoiding the possibility of elements of G'_H mapping other states onto totally antisymmetric ones. But the quark standard model for hadrons takes into account these possibilities and assumes also for color variables the antisymmetrization of the state vector. At this point, it is important to note that no rigorous theoretical demonstration of the confinement hypothesis has yet been given by QCD, but it is believed that the necessary elements are already contained in QCD. It is widely held[13] that color degree of freedom is the only responsible for quark confinement. On the other hand, if color is taken as a single label for quarks, the antisymmetry of the state vector alone does not allow one to assess confinement. But this is not our position in the present work. A complete description of a quantum state needs the specification of a great set of dynamical variables, like position, angular momentum, spin, label, etc. Truly speaking, in a complete axiomatic theory, the number of dynamical variables, due to Gödel's theorem, amounts to infinity. In our approach, color is considered as a dynamical variable, arising from the algebra $\mathcal{A}S^{(3)}$ derived from the permutation group $S^{(3)}$ which represents the effect of indistinguishability. Then, when we speak about quarks, we hypothesize three color fields with non-zero color triality. When we build up hadrons: baryons and mesons, we must construct particles of zero color triality. So, we must take combinations like

$$q^{\bar{\alpha}}(1)\, q_{\alpha}(2)$$

where the $q'2$ are the usual individual quark states and the color indices α, β, and γ each have only three possible values, to get mesons, or like

$$q_{\alpha}(1)\, q_{\beta}(2)\, q_{\gamma}(3)$$

to get baryons. In the sequel, a possible construction of these states according to gentilionic theory, will be shown, without imposing "ab initio" the antisymmetry represented by the tensor $E^{\alpha\beta\gamma}$(14). Introducing this antisymmetric tensor, we agree that starting from the colored quark hypothesis, the Pauli principle requirements can be met both for spin quarks as well hadron spectroscopy but we can not agree that quark confinement is not unconditional and that in principle quarks can be freed if the hadrons acquire a sufficiently high energy. For us, these conclusions are the result of an underestimation of the essential dynamical role played by color in the description of hadronic structure. Also, we impose a slight modification in the notation to show, in a more consistent way, the analogies envisaged. In Section 2, we construct the occupation number space for the 3-particles system, analysing the effect of the color Casimir operator $H_{\{2,1\}}^{[2,1]}$ on the color conservation, a very strong result which assures quark confinement. The detailed picture of the occupation number space allows us to think about the origin of only two statistics for observables particles: bosons and fermions. It is reasonable to suppose that the symmetry imposed by the indistinguishability principle does not depend on the particular description of a physical system and thus, this description can be classical or quantal. On the other, hand, many years ago(15), Schönberg has shown that the boson and the fermion algebras of the creation and annihilation operators have the same structures of the commutative and anti-commutative Grassman algebras. Since every boson algebra leads to a symplectic algebra and every fermion algebra leads to a Clifford algebra, it is natural to expect that some properties of quantum algebras can be inferred from their classical counterparts. This is indeed so. In their generalization of the Penrose twistor theory to a Clifford algebra(16), Bohm and Hiley have stressed the possibility of imposing a fixed helicity at every spatial point. We, in Section 2, follow an identical reasoning. There, we show that some kind of helicity is broken when gentileons are created in the occupation number color space. The same does not occur for bosons and fermions. Their helicities are kept unchanged during the creation process. All these considerations refer to color variables, without imposing

any restriction on spin variables. Constructing the state vectors explicitely, we show the respectives helicities. We also discuss the choice of two generators of SU(3) made by Gell-Mann and compare them with the two generators of $S^{(3)}$. An interesting analogy between color variables and physical optics is given.

In Section 3, we give the representation of $S^{(3)}$ in symplectic space. After an explicit calculation, we show how it is possible to interpret, in the symplectic context, the removal of degeneracies of the intermediate (gentilionic) representation of $S^{(3)}$. A generalization for other symmetry groups is given.

Finally, in Section 4, which can be considered as the core of the present paper, some very sound questions are discussed. We analyse some aspects of the two usual alternatives used in high energy physics: QCD and Parastatistics. It becomes clear that Gentile statistics offers a new possibility in constructing a more satisfactory theory for hadrons.

Some new and exciting suggestions are given toward the elaboration of gentilionic quantum field theory. Of course, there is much work to be done in pursuing such lines of reasoning.

2. GENTILIONIC STATES AND THE CONSERVATION OF COLOR

Let us consider a baryon composed of three identical quarks. Associated with the permutation group $S^{(3)}$ it is possible to consider the permutation algebra $\mathcal{A}S^{(3)}$. Since the elements of $\mathcal{A}S^{(3)}$ commute with the hamiltonian of the 3-particles system, they can be taken as belonging to the operator algebra of quantum mechanics. We have also shown(4) that this operator algebra, when represented by 2×2 matrices, has only one invariant which has been called the color Casimir operator $K^{[2,1]}_{(2,1)}$. By using appropriate symmetry adapted kets in the carrier space of representation(3–4) it was shown that $K^{[2,1]}_{(2,1)}$ has eigenvalue zero. A simple geometrical image(4) shows that it is possible to interpret this result as a vectorial conservation law

$$\vec{R}+\vec{B}+\vec{G}=\vec{0} \tag{1}$$

where these symbols are attached to the usual red, blue and green colors. A very precise meaning can be given to this vectorial law. In the introduction, we have said that color is not a single label. The vectorial character of color expressed by $K^{[2,1]}_{(2,1)}$, imposes the existence of a plane where (1) is satisfied. Since the orientation of this plane is not totally arbitrary because we have 6 color configurations dictated by $S^{(3)}$, it is easy to imagine the interior color space as being that of color occupation number space. Later on, we will identify this space in the context of symplectic geometry. On the other hand, the plane fixed by (1) reminds us closely the usual interpretation of states α and β of mixed symmetries used in several contexts like nuclear states classification and SU(3) states classification(17–18).

A long time ago(19–20), Schönberg has shown that the second quantization methods are a general mathematical technique applicable to formalisms involving linear equations of change, differential with respect to the time variable. In this manner, the ordinary second quantization formalism for systems of bosons or fermions, and the "second quantization" of the classical theory developed by him are obtained as particular cases of the general

methods. Also, there are several ways of applying the second quantization methods to the same linear problem, which lead to different formalisms. Thus, second quantization methods are independent of the quantum description of a system. But we can apply them to wave mechanics without resorting to the specific differential structure in study. Of course, Schrödinger equation can be an arena for such a discussion. As a consequence of the above statements, we can construct for the color dynamical variables, an occupation number color space, with the following conditions: we fix three axis with unit versors $\vec{i}, \vec{j}$ and $\vec{k}$, corresponding to the three basic colors red, blue and green. On each axis of the occupation number color space, a color creation operator means one step in the positive sense, whereas an annihilation operator, a step in the negative sense. In this space, the plane defined by (1) is orthogonal to the fermionic state vector which, when conveniently understood, is identified to the gentilionic - with all colors distinct - state vector. In terms of these vectors, the Casimir color operator can be written as

$$n_R+n_B+n_G=3 \tag{2}$$

which is the plane equation, with each n meaning an occupation number. In terms of the unit vectors, (2) becomes

$$3\vec{i}+3\vec{j}+3\vec{k}=3\vec{r_1} \tag{3}$$

where $\vec{r_1}$ is a unit vector orthogonal to plane (2). It is also seen that all allowable states for 3 particles can be pictured on the sides of the triangle formed by the intersections of plane (2) with the coordinates planes. Thus, the choice of this plane is not only a convenient way to classify the states, but is also very important for the interpretation of the Casimir color operator. Now, we make explicit the construction of the several states, starting from the fermionic ones:

a) Fermionic states

1) $a_R^\dagger a_B^\dagger a_G^\dagger \phi(0,0,0) = a_R^\dagger a_B^\dagger \phi(0,0,1) = a_R^\dagger \phi(0,1,1) = \phi(1,1,1)$

2)$a_B^\dagger a_R^\dagger a_G^\dagger \phi(0,0,0) = -a_R^\dagger a_B^\dagger a_G^\dagger \phi(0,0,0) = -\phi(1,1,1)$

3)$a_R^\dagger a_G^\dagger a_B^\dagger \phi(0,0,0) = -a_R^\dagger a_B^\dagger a_G^\dagger \phi(0,0,0) = -\phi(1,1,1)$

4) $a_G^\dagger a_R^\dagger a_B^\dagger \phi(0,0,0) = +a_R^\dagger a_B^\dagger a_G^\dagger \phi(0,0,0) = +\phi(1,1,1)$

5) $a_B^\dagger a_G^\dagger a_R^\dagger \phi(0,0,0) = +a_R^\dagger a_B^\dagger a_G^\dagger \phi(0,0,0) = +\phi(1,1,1)$

6) $a_G^\dagger a_B^\dagger a_R^\dagger \phi(0,0,0) = -a_R^\dagger a_B^\dagger a_G^\dagger \phi(0,0,0) = -\phi(1,1,1)$

where the $a^\dagger$'s are creation operators acting on the vacuum state $\phi(0,0,0)$. The respective amplitude probabilities are the last terms. Since all colors are distinct, no mixed symmetries appear, but two helicities for the color variables are manifest. Thus, amplitudes corresponding to processes 1), 4) and 5) appear with + sign, whereas amplitudes corresponding to processes 2), 3) and 6) appear with − sign. Assuming that the (+) sign defines the right orientation (helicity) and the (−) sign defines left orientation (helicity), we can write the total amplitude of probability of the fermion state as

$$\phi_{\text{FERMI}}(R,B,G) = (1/\sqrt{6})\,[\phi_{\text{Right}}(R,B,G) - \phi_{\text{Left}}(R,B,G)]$$

clearly showing the two possibilities for fermionic helicities for color variables. Clearly speaking, there is a repetition of the usual spin with two handedness. Nevertheless, some additional cares must be taken since we can not guarantee color conservation for fermionic colored particles since this irreducible representation of $S^{(3)}$ is subject to Pauli Principle. Always an "ad hoc" proposition is needed to assure colorlessness. Even the possibility of choosing a preferred orientation is theoretically ruled out, but this difficulty is also found when dealing with gentileons. It may be that the deepest reason of this uncertainty is found in Gödel's theorem. The algebraic structure with which is endoved the quantum operators is isomorphic to the arithmetic scheme studyied by Gödel and this circumstance leads to undecidable propositions. But this research is outside the scope of the present work. These questions are been answered in a forthcoming paper[21].

b) Gentilionic states

To construct the probability amplitudes, we start from the vacuum $\phi(0,0,0)$ and first apply one red creation operator

$$a_R^\dagger \phi(0,0,0) = \phi(1,0,0)$$

then, a second red operator

$$a_R^\dagger a_R^\dagger \phi(0,0,0,) = \phi(2,0,0)$$

Finally, a blue operator

1) $a_B^\dagger a_R^\dagger a_R^\dagger \phi(0,0,0) = \phi(2,1,0)$

Permuting the colors we get

2) $a_R^\dagger a_B^\dagger a_R^\dagger \phi(0,0,0) = -\phi(2,1,0)$

and

3) $a_R^\dagger a_R^\dagger a_B^\dagger \phi(0,0,0) = \phi(2,1,0)$

It is easy to see that in the plane (R,B), the amplitudes 1) and 3) have well defined helicities, whereas 2) shows a broken helicity. When we add 1), 2) and 3) to get the partial probability amplitude, and repeat the same procedure for the two remaining coordinates planes, we arrive at a result similar to the fermionic case, with the exception of the presence of a term corresponding to mixed symmetries:

$$\phi_G = 1/\sqrt{9}\,[\phi_{\text{Right}} + \phi_{\text{Left}} - \phi_{\text{Mixed}}]$$

where the color indices were ommited.

c) Bosonic states

Here, the result is trivial and the resulting probability amplitude is the same found in the literature.

Now, we are able to establish a connection between the two groups: $S^{(3)}$ and $SU^{(3)}$. If we define a pair of orthogonal basic vectors $\vec{r}_2$ and $\vec{r}_3$ on the gentilionic plane by the equations

$$\vec{r}_2 = \frac{\sqrt{6}}{6}(\vec{i} - 2\vec{j} + \vec{k})$$
$$\vec{r}_3 = \frac{\sqrt{2}}{2}(\vec{i} - \vec{k}) \tag{4}$$

we can easily establish a correspondence with the two generators of $SU^{(3)}_{\text{color}}$ group, $\tilde{T}_3$ and $\tilde{Y}$:

$$\vec{r}_2 = \frac{\tilde{T}_3}{2} - \frac{\sqrt{3}}{2}\tilde{Y}$$
$$\vec{r}_3 = -\frac{\sqrt{3}}{2}\tilde{T}_3 - \frac{1}{2}\tilde{Y} \tag{5}$$

completing the connection between the common parts of the groups. It must be emphasized that $SU(3)_{\text{color}}$ is a non-relativistic group attached to every point in space time. It has 8 generators which have been identified with 8 gluons fields. We also know that by Schur's Lemma, every permutation group admits a unitary representation. But this representation is not unique. For instance, $S^{(3)}$ admits an irreducible non-unitary representation non equivalent to the unitary representation. Since $S^{(3)}$ has only two generators, they can be identified in a straightforward way, with the 2 preferred generators of $SU^{(3)}$ chosen by Gell-Mann to make a classification of the particle spectra of hadrons. But $S^{(3)}$ has only two generators and we need interpret them as gluons fields. This is easy if we admit that each gluon carries a color and an anti-color which are constantly exchanged among the quarks, always obeying the conservation law imposed by $K^{[2,1]}_{(2,1)}$:

$$ab + ba = -1$$

As another implication of Gentile statistics, we have

$$\phi(3,0,0) = \phi(0,3,0) = \phi(0,0,3) = 0$$

independently of the vacuum state. This occurs as a consequence of the action of $K^{[2,1]}_{[2,1]}$:

$$\vec{R} + \vec{B} + \vec{G} = \vec{0}$$

since, to have the vectorial equality, we need 3 or 2 vectors:

$$2\vec{R} + \vec{B} = \vec{0}$$

when $\vec{R} = \vec{G}$. But if we have $\vec{R} = \vec{B} = \vec{G}$, we get

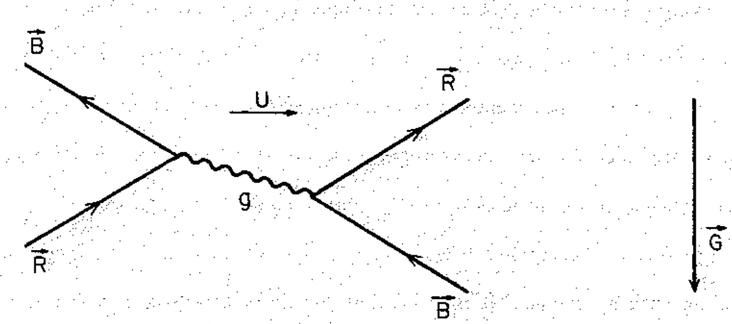
$$3\vec{R} = \vec{0}$$

and this implies $\vec{R} = \vec{0}$ and we have not spin color. Without this degree of freedom, the system becomes purely bosonic. This reasoning also applies to mesonic states

$$\vec{R} + \overline{\vec{R}} = \vec{0}$$

and suggests an explanation of the non-observability of isolated quarks. One isolated quark would lead to a violation of the conservation law imposed by $K^{[2,1]}_{(2,1)}$.

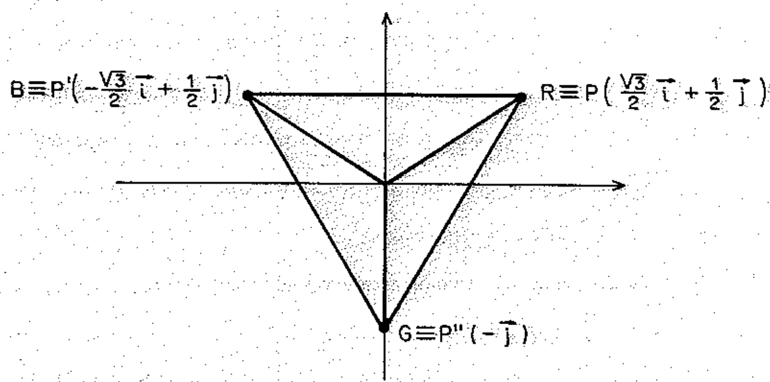
To show how a color gluon is build up in our theory, let us examine the following figure



F I G. 1

Where $\vec{g} = \vec{U} - \vec{B} = a_R^\dagger a_B^\dagger$, and $|\vec{U}| = \sqrt{3}\,|\vec{R}|$, showing the continuous change of colors.

To end this section we should like to illustrate, in a beautiful manner, the analogy between the color triangle of physical optics and the quantum mechanical interpretation of the operator $K_{(2,1)}^{[2,1]}$. Let us sketch the three colored quarks at the vertices of the triangle in the plane (x, y).



F.I.G. 2

In physical optics, the 3 fundamental colors are arbitrarily chosen to give the white at the centre of the color triangle. In the gentilionic triangle depicted above in the occupation number color space, the same situation is found again: the system composed of three colors remains colorless. Now, this color system, associated with a hadron, leads to the inescapable conclusion of colorlessness. Let us show how this occurs[22].

1) The vibrations are decomposed on each coordinate axis

$$\begin{aligned} x &= x_0 \sin(\omega t + \alpha) \\ y &= y_0 \sin(\omega t + \beta) \end{aligned}$$

where (x_0, y_0) are the respective amplitude, ω is the angular frequency and (α, β) are phase differences. From P to P' there is a phase difference given by

$$x' = x_0 \sin(\omega t + \alpha + \delta) = -x_0 \sin(\omega t + \alpha)$$

Calling $\alpha + \delta = \alpha'$, we get, $\delta = \pi$.

When only P'' is vibrating, we compose the oscillations on x, from P to P', and we get

$$A = 2\, x_0 \cos\left(\frac{\alpha - \alpha'}{2}\right) = 2\, x_0 \cos\left(\frac{\pi}{2}\right) = 0$$

where A is the total amplitude of oscillation.

2) Now, we are able to establish the amplitude of the triangular vibrations (baryons). We assume three plane oscillations:

$$\begin{aligned} A^2 &= \left(\sum_{i=1}^{3} x_{0i}\right)^2 - 4 \sum_{i,j} x_{0i}\, x_{0j} \sin^2\left(\frac{(\alpha_i - \alpha_j)}{2}\right) \\ &= (x_{01} + x_{02} + x_{03})^2 - 4 \left\{ x_{01}\, x_{02} \sin^2 \frac{(\alpha_1 - \alpha_2)}{2} \right. \\ &+ \left. x_{01}\, x_{03} \sin^2 \frac{(\alpha_1 - \alpha_3)}{2}) + x_{02}\, x_{03} \sin^2 \frac{(\alpha_2 - \alpha_3)}{2} \right\} \end{aligned}$$

If we want $A \equiv 0$ (white light) we remember that $x_{01} = x_{02} = x_{03} = x_0$ (which is imposed by the principle of indistinguishability) and we arrive at

$$\begin{aligned} 0 &= 9\, x_0^2 - 4\, x_0^2 \left\{ \sin^2 \frac{(\alpha_1 - \alpha_2)}{2} \right. \\ &+ \left. \sin^2 \frac{(\alpha_1 - \alpha_3)}{2} + \sin^2 \frac{(\alpha_2 - \alpha_3)}{2} \right\} \end{aligned}$$

On the other hand, the conservation color Casimir $K_{(2,1)}^{[2,1]}$ requires that

$$\alpha_1 - \alpha_2 = \alpha_1 - \alpha_3 = \alpha_2 - \alpha_3$$

leading to $\frac{3}{4} = \sin^2 \frac{(\alpha_1 - \alpha_2)}{2}$. This last equality amounts to say that

$$\alpha_1 - \alpha_2 = \frac{2\pi}{3} \quad \text{c.q.d.}$$

In the mesonic case, the exact composition is given by

$$A = 2\, x_0 \cos \frac{(\alpha - \alpha')}{2}$$

and, when we have $\alpha' = \alpha + \pi$, the polarization of the meson becomes manifest, with

$$\begin{aligned} x &= x_0 \sin(\omega t + \alpha) \\ \bar{x} &= x_0 \sin(\omega t + \alpha + \pi) \end{aligned}$$

and the oscillations are conjugate. In general, the phase difference is given by $\alpha - \alpha' = (2m+1)\pi$, with m integer. For the space variation, $k\, x (k =$ wave vector), we have

$$k\Delta = \frac{2\pi}{\lambda} \quad \Delta = (2m+1)\pi$$

and, thus, $\Delta = (2m+1)\frac{\lambda}{2}$, where λ is the wavelenght.

Summarising these results, we get

i) Baryon

$$\vec{A} = \vec{R} + \vec{B} + \vec{G} = \vec{0} \quad (\text{White} \quad \vec{A} = \vec{0})$$

ii) Meson

$$\vec{A} = \vec{R} + \vec{\bar{R}} = \vec{0} \quad (\text{White})$$

iii) Individual quark

$$\vec{A} \neq \vec{0} \quad (\text{always colored or polarized})$$

It is clear that if the direction and the ratio of the axes of elliptical vibrations are continuosly changed, every variation will have the effect of changing the corresponding

wavelenght. In our quantum mechanical interpretation of color, the same interpretation occurs: gluons are continuosly exchanged.

Many years ago(23), Landau has stressed the importance of establishing new connections among several non-related branches of physical theories. Indeed, for him, the task of theoretical physics would exactly be this ordering of new uncorrelated phenomena. In this section, we think we have been partially successful in this job, when re-establishing and old bridge between physical optics and wave mechanics. But this can not be taken as a surprise, since we have been working with the permutation group, a mathematical structure which is permeating the most scattered fields of research in physics and mathematics(24). As examples, we quote Hopf algebra, quantum groups, knot theory, topology of 3- manifolds and, in special, dynamics of strong interactions.

But some fundamental questions remain unanswered. If color is taken as a dynamical variable of the system, there would be associated to it, a canonical variable, according to the rules of hamiltonian mechanics. What kind of variable must be introduced in an ambiguous question? First of all, we must fix the physical dimensions of color. If we try to choose energy as fundamental, the canonical conjugate variable must have dimensions of time. Taking into account our results on confinement we prefer to consider the Principle of Complementarity as the responsible for the most plausible explanation of conjugation of variables. From a holistic point of view, there is complementarity beteween dynamics and geometry, preventing thus, a complete description of the system. To this point we will return in another paper(21).

3. THE $S^{(3)}$ ALGEBRA AS A SYMPLECTIC SPACE

As we have seen, the $S^{(3)}$ algebra $\mathcal{A}S^{(3)}$, has two generators (a, b) and is of order 6. From an algebraic-geometric point of view, the vectors of $\mathcal{A}S^{(3)}$ belonging to the irreducible intermediate gentilionic representation, span a four-dimensional vector space V^4 defined over a field F_2 of characteristic 2. Let us define over V^4 a skew-symmetrical bilinear form $q(x, y)$ with the following properties:

1) $q(x, x+z) = q(x, y) + q(x, z)$
2) $q(x+y, z) = q(x, z) + q(y, z)$
3) $q(\alpha x, y) = q(x, \alpha y) = \alpha q(x, y)$
4) $q(x, x) = 0$

where $x, y, z \in V^4$ and $\alpha \in F_2$. With these definitions(25–26), we have endowed V^4 with a symplectic structure. If we define a basis of V^4 as

$$\mathbb{X}_V 4 = \{x_1, x_2, x_3, x_4\}$$

a connection with the irreducible (2×2) matrix representation of $S^{(3)}$ is established. The (2×2) matrices can be written in terms of a suitable basis of four (2×2) matrices, using Pauli matrices σ_i and the unit I matrix. With this choice, the elements of $S^{(3)}$ are written as

$\eta_1 = I$

$\eta_2 = -\frac{I}{2} + i\frac{\sqrt{3}}{2}\sigma_2$

$\eta_3 = -\frac{I}{2} - i\frac{\sqrt{3}}{2}\sigma_2$

$\eta_4 = \sigma_3$

$\eta_5 = \frac{\sqrt{3}}{2}\sigma_1 - \frac{1}{2}\sigma_3$

$\eta_6 = -\frac{\sqrt{3}}{2}\sigma_1 - \frac{1}{2}\sigma_3$

The basis matrices are related to $\mathbb{X}_V 4$ by

$$\overline{\mathbb{X}}_V 4 = \left\{-\frac{I}{2}, i\frac{\sqrt{3}}{2}\sigma_2, \frac{\sqrt{3}}{2}\sigma_1, \frac{1}{2}, \sigma_3\right\}$$

where we have changed from a hyperbolic basis $\mathbb{X}_V 4$ to a symplectic basis $\overline{\mathbb{X}}_V 4$. In general, a rearrangement of the basis vectors changes one basis into another. This is a common feature of all non-null symplectic spaces(27). This property can become very important for gentileons. It can tell us something about rearranging $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ or $\begin{pmatrix} y_1 \\ y_3 \end{pmatrix}$ where $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ has mixed symmetries and $\begin{pmatrix} y_1 \\ y_3 \end{pmatrix}$ not(1–2). From the symplectic point of view, gentileons are really hybrids and the correspondence between two bases are given by:

$$\underset{\text{Hyperbolic basis}}{\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}} \rightarrow \underset{\text{Symplectic basis}}{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}}$$

The field F_2 is defined by the tables

+	0	1
0	0	1
1	1	0

×	0	1
0	0	0
1	0	1

where $2 \equiv 0$ (characteristic 2).

Using these notations, the elements η_i become

$$\eta_1 = 2\,x_1 = (2, 0, 0, 0)$$

$$\eta_2 = 1\,x_1 + 1\,x_2 = (1, 1, 0, 0)$$

$$\eta_3 = 1\,x_1 - 1\,x_2 = (1, -1, 0, 0)$$

$$\eta_4 = 2\,x_4 = (0,0,0,2)$$

$$\eta_5 = 1\,x_3 - 1\,x_4 = (0,0,0,-1)$$

$$\eta_6 = 1\,x_3 - 1\,x_4 = (0,0,0,-1,-1)$$

Returning to the skew-symmetrical bilinear form $q(x,y)$:

$$\begin{aligned} q(x,y) &= (\alpha_1,\alpha_2,\alpha_3,\alpha_4)\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} \\ &= \alpha_1\beta_2 - \alpha_2\beta_1 + \alpha_3\beta_4 - \alpha_4\beta_3 \end{aligned}$$

in the hyperbolic basis. The same result is found in the symplectic basis.

Now, we can easily see that $\{x_1, x_2\} = \{-\frac{I}{2}, i\frac{\sqrt{3}}{2}\sigma_2\}$ constitutes a hyperbolic pair. The same is true for $\{x_3, x_4\} = \{\frac{\sqrt{3}}{2}\sigma_1, \frac{1}{2}\sigma_3\}$. We have two hyberbolic planes H_{12} and H_{34}:

$$H_{12} = F_2\,x_1 + F_2\,x_2$$

and

$$H_{34} = F_2\,x_3 + F_2\,x_4$$

with the lines:

1st line $\eta_1 = 0\,x_1 + 0\,x_2 = 2\,x_1 + 0 \quad (2 \equiv 0)$

2nd line $\eta_2 = 1\,x_1 + 1\,x_2$

3rd line $\eta_3 = 1\,x_1 - 1\,x_2 \quad (1 \equiv -1)$

in H_{12} and

1st line $\eta_4 = 0\,x_3 + 2\,x_4$

2and line $\eta_5 = 1\,x_3 - 1\,x_4$

3rd line $\eta_6 = -1\,x_3 - 1\,x_4$

in H_{34}. If we look at the definition of $q(x,y)$ we note that V^4 is spanned by two symplectic orthogonal subspaces H_{12} and H_{34}:

$$V^4 = H_{12} \oplus H_{34}$$

because $q(H_{12}, H_{34}) = 0$.

This result is very important, showing that the decomposition of $S^{(3)}$ into two hyperbolic planes is symplectic and not euclidian. The degeneracy of the intermediate (gentilionic) representation of $S^{(3)}$ is removed at a symplectic level, where we have a splitting of operations of $S^{(3)}$: in H_{12} the identity induces true rotations whereas in H_{34}, also inversions are present. These interchanges of the actions of the elements of $S^{(3)}$ are responsible for the gluonic interpretation of colors and anti-colors. But this is a natural result, since the Pauli matrices form a non-abelian algebra and all (2×2) matrices representing $S^{(3)}$ belong to Sp (2), since the factor det $= \pm\,1$ does not reverse the sign of the skew-symmetrical bilinear form because $(+1) \equiv (-1)$. Thus,

$$q(x,y) = q(S^{(3)}\,x, S^{(3)}\,y)$$

and the symplectic structure is preserved.

To perform simple calculations in symplectic geometry, it may be useful to impose some euclidean structure on the symplectic space. We fix a symplectic coordinate system and introduce an euclidian structure using the coordinate scalar product $(\vec{x}, \vec{x})$. In this euclidian structure, the symplectic basis is orthonormal. The skew scalar product, like every bilinear form, can be expressed in terms of the scalar product by:

$$q(\vec{x}\vec{y}) = (\mathcal{I}\vec{x}, \vec{y})$$

where $\mathcal{I} : R^2 \to R^2$ is some operator. It follows from the skew-symmetry of the skew-scalar product that the operator $\mathcal{I}$ is skew-symmetric. Of course, in the symplectic basis, the matrix of the operator $\mathcal{I}$ is

$$\mathcal{I} \to \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

where I is the identify. As an example, in the hyperbolic planes of V^4, $\mathcal{I}$ is simply rotation by $\frac{\pi}{2}$.

Since $\mathcal{I}^2 = -I$, we can introduce into our space not only a symplectic structure and an euclidian structure, but also a complex structure by defining multiplication by $i = \sqrt{-1}$ as being the action of $\mathcal{I}$. In this way, our space is identified with a complex space. As a result, we conclude that the linear transformations which preserve the euclidian structure form the orthogonal group 0(2n); and those preserving the complex structure form the complex linear group GL(n,C).

Another conclusion is that transformations which are both orthogonal and symplectic are complex; those which are both complex and orthogonal are symplectic and those which are both symplectic and complex are orthogonal; thus, the intersection of two of three groups is equal to the intersection of all three:

$$0(2n) \cap \mathrm{Sp}(2n) = \mathrm{Sp}(2n) \cap \mathrm{GL}(n, C) = \mathrm{GL}(n, C) \cap 0(2n) \quad .$$

The intersection is called the unitary group $U(n)$. Unitary transformations preserve the hermitian scalar product $(x, y) + $ q(x,y); the scalar and skew-scalar products on R^{2n} are its real and imaginary parts.

Our previous discussion opens the way to a very deep question. Let us take an example: we know that SU(2) is the weak interactions: the physical fermions are doublets of the global SU(2). They are all singlets under the gauge $\mathrm{SU}(2)_L$. Transitions between them are affected by W boson emission or absorption, which is allowed because the $W's$ are isospin 1. This global SU(2) is in some ways one of the deepest mysteries of know particle physics because as a symmetry it is only approximate. It is broken by electromagnetism and

fermion doublet mass splittings and the non existence of right-handed neutrinos. While exact symmetries are symmetries, approximate symmetries usually arise from underlying physics. What kind of physics underlies the exact $\mathrm{S}^{(3)}$ symmetry? The answer here is clearly tomorrow. The answer might even be yesterday, with the serious possibilities offered by Gentile statistics and the corresponding permanent confinement of quarks. But what we should like to emphasize here is the richness of possible physical effects with explanation based on the invariance of the system under $\mathrm{S}^{(3)}$ (or $\mathrm{S}^{(N)}$). $\mathrm{S}^{(N)}$ is at the centre of the Principle of indistinguishability and even the definition of a quantum system can be questioned, as occurs in the EPR paradox. Each symmetry corresponding to each group contained in the geometry must exhibit a new aspect of physical theories.

4. CONCLUSIONS

Some years ago, Hawking[28], in his celebrated lecture "Is the end in sight for theoretical physics?", has answered affirmatively for the question. This answer was very sad. We know that the end of theoretical physics means the end of all basic physics. Fortunately, this point of view might well have been refuted by experiment. Needless to say, no one has suggested a way in which experiments might shed light on the issue of quark confinement. The present theories (QCD and or parastatistics) are only a crude phenomenology, we leave open the question of which of their properties we should take seriously. We do not deny legacy of these attempts to solve the statistical problem and the corresponding confinement. We understand that the quark model has created an almost unbearable situation in theoretical physics. It is difficult to reconcile the known statistics with the existence of three fermions in the same state. In the case of QCD, the problem has been partially solved by imposing $SU(3)_{\text{color}}$ as the color symmetry. But this is not a natural choice, since it is not required by the foundations of quantum mechanics. What we have said in the present text is that SU(3) is only a possible symmetry which serves as a solution for one step of the confinement problem. Nevertheless, it does not explain confinement. Such divergent views are held by the most respected of theorists. On the other hand parastatistics is an almost unintelligible anzats, firstly announced by Okayama[29] and latter explored by Green[30] and followed by others, culminating with the enormous effort of Kamefuchi and Ohnuki[31], aiming to establish a consistent approach to the statistical puzzle. In this subject, we are as guilty as many others for having written a paper, which implied as much in its title[1]. This discussion, although being unsound, has called our attention for a more profound reasoning based on the $S^{(3)}$ symmetry. Among the arguments favorable to our present theory, we can quote the following:

i) Our starting point are the Principles of Quantum Mechanics.

ii) All developments and demonstrations are mathematically rigorous.

iii) No "ad hoc" hypothesis are introduced to explain confinement.

iv) The decomposition shown in Section 3 of the symmetric group $S^{(3)}$ has the advantage of considering quarks, especially for energies corresponding to deep inelastic scattering, as almost free. Under these conditions, Pauli's Principle plays no role, and the quarks can be considered as fermions or bosons (from the point of view of intrinsic angular momentum). Even the bosonisation of quarks arises as possible hypothesis in this limit of energies.

v) Our theory does not admit hadron coalescence without violating the color conservation imposed by $K^{[2,1]}_{(2,1)}$.

vi) From the point of view of usual statistics (fermionic or bosonic) the hadronic quarks must obey the common rules dictated by quantum mechanics. Thus, to be in agreement with experiment, quarks are fermions (intrinsic angular momentum one half). Indeed, fermionic and bosonic representations are included in Gentile statistics. But an expert reader could ask us - "Why only fermions and bosons are accessible to experimental detection?". This is a very delicate question. As we have seen in Section 2, there exists for gentileons (in an abstract space) a new kind of "helicity" which is reversed during the creation of the final state. In Section 3, we have shown that the origin of this "helicity" is not related to metric geometry but to symplectic geometry. Apart from terminology, our world is euclidian. All experimental apparatus is euclidian and it is difficult even imagine another form of testing natural phenomena. But we can not reject the strange possibility of seeing gentileons. The situation shows a close similarity with Bohm-Aharonov's effect. Some years ago, it was impossible to think about detecting pure gauge fields. Now, the situation has drastically changed with the clean tests of Tonomura et al.[32]. Although their result is of fundamental importance, and the proof relatively simple, a complete discussion of this problem is not the purpose of the present paper, and we prefer consider it as an analogy. But, what to do with the problem of detection of gentileons? This is not an easy task because we have not yet established a connection between the internal space and the coordinates of space time. We suggest a very phenomenological approach: since we can not simply mock up the way followed by experimentalists until now, we must collect data on high energy physics over a broad enough range and decide: if free quarks are detected, they are not gentileons; if they are not detected, they surely are gentileons. We believe that quarks will never be detected. For us, this indirect proof suffices.

What we can learn from the reading of the present work is that some new ideas prove to be mistaken, but the harm done in taking them up and working in the directions they suggest is much outweighed by the positive gain resulting from an open and receptive attitude towards original thought. One tendency which has been followed is to build the discussion of a topic around a rigorous theory which has been selected from among many alternatives. This method usually gains commendations on the ground that everything becomes clear, and a mass of experimental data can be coordinated and reduced to a comprehensible unity.

Until now, we have stressed some positive features of gentilionic theory. This does not mean that we are not aware of the enormous difficulties we shall encounter to establish an alternative theory, in the context of Gentile statistics, aiming at a consistent quantum field theory. For strong interactions, it will not be easy to formulate an alternative QCD. We are working in several directions, with unsuspicious hypothesis, to overcome the usual difficulties. These schemes are woefully naive and incomplete. There are many phenomena of comparable importance that are simply not included. We know that things that we already understand in principle are often ignored in practice. All we have to do is try to avoid such procedures. With this in mind, we propose the following approaches to the formulation of a gentilionic quantum field theory:

i) All consequences of imposing $S^{(N)}$ as a fundamental symmetry (Principle of indistinguishability), should certainly be pursued further because they provide some of our sharpest constraints on speculations of new physics.

ii) In general, the particle and observer can be infinitesimally close together or at opposite ends of universe; the Lorentz trannsformation is still the same. The Lorentz symmetry group of special relativity is an example of "global symmetry". The symmetric group is also a global symmetry. As the particle moves through space time, it traces out a path in the internal space above the space time trajectory. When there is no external gauge potential, the internal space path is completely arbitrary. When the particle interacts with an external gauge field, the path in the internal space is a continuous curve determined by the gauge potential. In the internal color space, the only possible configuration is a color singlet with net charge equal to zero. This means that the external gauge potential does not exert any influence on the particle. One possibility is to consider a bundle where $SU(3)\times S^{(3)}_{\text{color}}$ is the fibre and SU(3) is taken as the structural gauge group. Since the cross-section of the bundle does not depend on the gauge group, the above result is justified. Now, let us remind that

$$R(\theta)\,\psi = \exp(-i\theta L)\,\psi$$

is the three dimensional rotation of a state vector. But this expression has the mathematical form of phase change under a gauge transformation. But, the responsible for the change of the phase in ψ is the gauge potential, and the gauge potential is not a rotation operator. Thus,

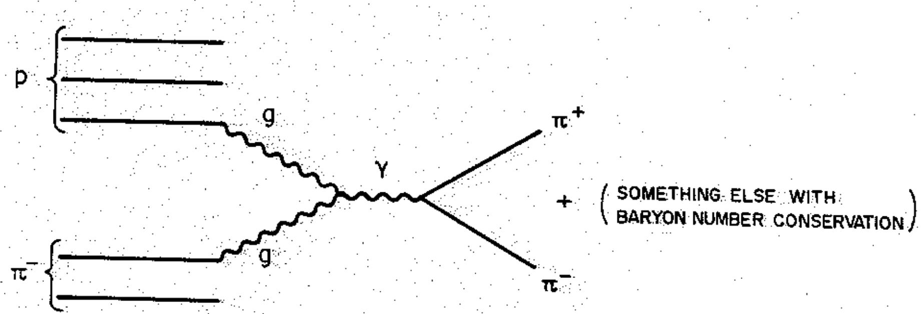
$$\delta\theta = k \quad . \quad \text{gauge potential}$$

Generalizing, we can say that the gauge potential can be written as a linear combination of the generators

$$A_\mu = \sum_{i=1}^{2} A^i_\mu(x)\,(a_i)$$

where A_μ has a dual character, A^i_μ is a field in space time and a_i is a generator (operator in the color space) of $\mathcal{A}S^{(3)}$. The details of this dangerous line of reasoning and its consequence will be published elsewhere.

iii) As a final, but not less important remark, we should like to point out the possibility of applying the usual Drell-Yan model[33] for gentileons. Let us take as an example the $p + \pi^- \rightarrow N^+ + N^- +$ (something else with baryon number conservation) reaction:



F I G. 3

In gentilionic theory, the explanation based on γ emission is absolutely forbidden. No quark can interact with another (hadron-hadron) because this means coalescence of quarks and violation of color conservation. There is no scaling violation since Bjorken scaling is fermionic for angular momenta. This fact changes drastically our views on the constitution of matter at extremely high densities. Recent results at GANIL (France)[34] have shown that, even at low energies, there is, in nuclear reactions with P_b, a long range effect (besides the usual exponential short range) which can be accounted for by emission of di-gluons.

This long range force (van der Waals) is proportional to r^{-7}, giving good agreement with the experimental results. As has been remarked by Hussein and Porto Pato[35], the explanation of the extra-term by di-gluons emission (colorless) is in fairly good agreement with the exponential data. Even waiting for new confirming results, we are faced with a new situation: how can di-gluons be emitted from the nucleons without color conservation? For gentilionic gluons (the two generators of $S^{(3)}$) this possibility, at least in principle, is not understable. It can occur that, in a virtual unknown process the intrinsic helicity of mixed symmetries can be combined among the gluons, giving a net zero helicity in occupation number color space. Although being uncertain, this possibility must be taken carefully. As in the past, several ab-normal properties of nuclear reactions[17–18] have been attributed to parastatistics with continuous order parameter, and after, have being explained by other competing effects, we prefer to wait for new more conclusive results. Nevertheless, at a first glance, in the absence of a Gentilionic QFT, the conclusions of Hussein and Porto Pato seem to be very reasonable, and much work is being done by us in the inspection of the theoretical-experimental suggestions. To sum up, we can say that to reconcile the color spin conservation with the di-gluon emission is an open problem in our scheme. As the Drell-Yan approach manifestly violates color conservation, we can speculate that only at the threshold of GUT, where the proton decay has a non zero probability of occurrence, the di-gluon emission is allowable. But this hypothesis requires a great revision of all fundamentals of quantum mechanics. So, we prefer to postpone its discussion for a next paper.

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FIGURE CAPTIONS

Figure 1. Gluon creation and absorption by quarks.

Figure 2. The color triangle of physical optics.

Figure 3. Schematic Drell-Yan model applied to quark anti-quark interactions.