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IFUSP/P-1025

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Janeiro/1993

IFUSP-P/1025

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December, 1992

Abstract

We present a topological version of two dimensional dilaton supergravity. It is obtained by gauging an extension of the super-Poincaré algebra in two space-time dimensions. This algebra is obtained by an unconventional contraction of the super de Sitter algebra. Besides the generators of the super de Sitter algebra it has one more fermionic generator and two more bosonic generators one of them being a central charge. The gauging of this algebra is performed in the usual way. Unlike some proposals for a dilaton supergravity theory we obtain a model which is non-local in the gravitino field.

[1] Partially supported by CNPq

Two dimensional gravity theories have been studied with the purpose of gaining conceptual and technical insights in order to handle the more difficult problem of treating four dimensional gravity [1]. Another point of interest is that effective string theories in four dimensions show that Einstein gravity should be corrected by coupling to a dilaton field [2]. In particular the process of black-hole evaporation [3] should then be studied in this new framework [4]. In this case the problem can be reduced to a two-dimensional dilaton gravity model coupled to topological matter. When we consider only the graviton and the dilaton fields the resulting theory is a topological gravity theory[5].

The introduction of supersymmetry in dilaton gravity theories was made in a non topological way. It was motivated in order to have a positive energy theorem for the two dimensional case [6] and also for the study of two-dimensional black holes [7]. On the other hand topological supergravity theories have been obtained. The construction of $N=1$ [8] and $N=2$ [9] topological supergravities have been made by gauging the groups $OSp(2 \mid 1)$ and $OSp(2 \mid 2)$, respectively. By a contraction of the gauge group the corresponding super-Poincaré theories are then obtained.

As stated above the dilaton gravity theory can have a gauge formulation leading to a topological theory [5]. When the gauge algebra is the Poincaré algebra the fields have unusual transformation properties under Poincaré gauge transformations [10]. This can be overcome if we take a central extension of Poincaré algebra as the gauge algebra [11]. This central extension consists in changing the usual commutator of the translation generators of the Poincaré algebra to $[P_a, P_b] = \epsilon_{ab} Z$ where Z belongs to the center of the algebra. This new algebra can be obtained by an unconventional contraction of the de Sitter algebra $SO(2,1)$ where the Lorentz generator J is replaced by $J + Z/\lambda$, where λ is the cosmological constant, and the limit $\lambda \to 0$ is taken [11].

In this paper we generalize this unconventional contraction to the super de Sitter $OSp(2 \mid 1)$ algebra to obtain the supersymmetric version of the central extension of the Poincaré algebra. We will show that besides the supersymmetry generator Q_α it is necessary another fermionic generator U_α which does not generate any supersymmetry. It is also necessary the introduction of a scalar generator K besides the central charge of the non supersymmetric case. We find the quadratic

Casimir operator starting from that of the $SO(2,1)$ algebra showing the existence of a non-degenerated metric. We then gauge this new algebra and build up the corresponding topological supergravity theory.

In two dimensions the commutation relations

$$\begin{aligned} [P_a, P_b] &= \lambda\epsilon_{ab}J \\ [J, P_a] &= \epsilon_a{}^b P_b \end{aligned} \tag{1}$$

define the $SO(2,1)$ and $SO(1,2)$ algebras for $\lambda > 0$ and $\lambda < 0$ respectively. This is independent of the conventions for the flat Minkowski metric h_{ab} and the antisymmetric tensor ϵ_{ab}. For definiteness (which is needed in the supersymmetric case) we will adopt the conventions $h_{ab} = diag(-1,+1)$ and $\epsilon^{01} = 1$. The supersymmetric extension of the algebra (1) with $\lambda > 0$ and with $N = 1$ supersymmetry is [8]

$$\begin{aligned} [P_a, P_b] &= \lambda\epsilon_{ab}J \\ [J, P_a] &= \epsilon_a{}^b P_b \\ [J, Q_\alpha] &= -\frac{1}{2}(\gamma_2 Q)_\alpha \\ [P_a, Q_\alpha] &= \frac{1}{2}\lambda^{\frac{1}{2}}(\gamma_a Q)_\alpha \\ \{Q_\alpha, Q_\beta\} &= (\gamma^a C)_{\alpha\beta}P_a - \lambda^{\frac{1}{2}}(\gamma_2 C)_{\alpha\beta}J \end{aligned} \tag{2}$$

This is the algebra $OSp(2 \mid 1)$ written in a Lorentz covariant form. The conventions for the Dirac matrices are the following $\gamma^0 = i\sigma_2, \gamma^1 = \sigma_1, \gamma_2 = \sigma_3$ and $C = i\sigma_2$ where σ_i are the Pauli matrices. For $\lambda < 0$ the chiral components of Q_α acquire different signs in the commutator with P_a and J and they give rise to the algebra $OSp(1 \mid 2)$.

The quadratic Casimir operator of the algebra (2) is

$$C = P^2 + \lambda J^2 + \frac{1}{2}\lambda^{\frac{1}{2}}Q_\alpha C^{\alpha\beta}Q_\beta \tag{3}$$

which gives rise to the following non-degenerated metric on the algebra $< P_a, P_b >= h_{ab}, < J, J >= \lambda$ and $< Q_\alpha, Q_\beta >= \frac{1}{2}\lambda^{\frac{1}{2}}C_{\alpha\beta}$.

We will now perform in a systematic way the unconventional contraction [11] which leads to the central extension of the algebra (1) in order to extended it to the supersymmetric case later on. Due to the

well known ambiguity of the angular momentum in two dimensions we first introduce a new generator Z through the replacement

$$J \to J + Z/\lambda \tag{4}$$

To get all commutation relations involving this new generator Z we will perform the replacement (4) in (1) and equate the terms with the same power of λ which become singular when we take the limit $\lambda \to 0$. From the commutation relation $[J, P_a]$ we find $[J, P_a] = \epsilon_a{}^b P_b$ and $[Z, P_a] = 0$ and from the commutation relation $[P_a, P_b]$ we find $[P_a, P_b] = \epsilon_{ab}Z$. Taking into account the ambiguity in the angular momentum we impose that the old commutation relation $[J, J] = 0$ should be replaced by $[J, J + Z/\lambda] = 0$ and we find that $[J, J] = [J, Z] = 0$. We then conclude that Z belongs to the center of the algebra and we get the central extension of the Poincaré algebra [11]

$$\begin{aligned} [P_a, P_b] &= \epsilon_{ab}Z \\ [J, P_a] &= \epsilon_a{}^b P_b \\ [P_a, Z] &= [J, Z] = 0 \end{aligned} \tag{5}$$

We will now apply the same procedure to the supersymmetric case (2). In the anticommutation relation $\{Q_\alpha, Q_\beta\}$ there will appear a term proportional to $\lambda^{-\frac{1}{2}}Z$ on the right hand side of it. This means that a replacement of Q_α involving powers of $\lambda^{-\frac{1}{2}}$ is needed which implies the introduction of a new fermionic generator. Since powers of $\lambda^{-\frac{1}{2}}$ will appear in the commutators they must also be allowed in (4) which means the introduction of a new bosonic generator. So we start with the following replacements

$$\begin{aligned} J &\to J + \lambda^{-1}Z + \lambda^{-\frac{1}{2}}K \\ Q_\alpha &\to Q_\alpha + \lambda^{-\frac{1}{2}}U_\alpha \end{aligned} \tag{6}$$

where K is the new bosonic generator and U_α is the new fermionic generator. The replacements (6) just reflect the ambiguity of the angular momentum extended to the supersymmetry generators. It is not difficult to find out the non-vanishing commutation relations according to the rules stated above.

$$[P_a, P_b] = \epsilon_{ab}Z$$

$$
\begin{aligned}
[J, P_a] &= \epsilon_a{}^b P_b \\
[P_a, Q_\alpha] &= \frac{1}{2}(\gamma_a U)_\alpha \\
[J, Q_\alpha] &= -\frac{1}{2}(\gamma_2 Q)_\alpha \\
[J, U_\alpha] &= -\frac{1}{2}(\gamma_2 U)_\alpha \\
[K, Q_\alpha] &= -\frac{1}{2}(\gamma_2 U)_\alpha \\
\{Q_\alpha, Q_\beta\} &= (\gamma^a C)_{\alpha\beta} P_a - (\gamma_2 C)_{\alpha\beta} K \\
\{Q_\alpha, U_\beta\} &= -(\gamma_2 C)_{\alpha\beta} Z
\end{aligned} \tag{7}
$$

We can check that all the Jacobi identities are satisfied. Then the generator U_α transform as a spinor under a Lorentz transformation but does not generate any supersymmetry since its anticommutator vanishes. K behaves like a momentum generator in a third direction regarding the fermionic sector, while in the bosonic sector it behaves like a central charge. Z remains a central charge. We can set $Z = K = U_\alpha = 0$ consistently and we then obtain the $N = 1$ super-Poincaré algebra. Therefore the algebra (7) is a non-trivial extension of the super-Poincaré algebra.

We can find the quadratic Casimir operator by making the replacement (6) in (3). We find besides the regular term given below terms proportional to λ^{-1} and $\lambda^{-\frac{1}{2}}$. These terms are also invariants of the algebra but they give rise to a degenerated metric on the algebra. The regular term gives the quadratic Casimir operator

$$
C = P^2 + K^2 + JZ + ZJ + \frac{1}{2}C^{\alpha\beta}(Q_\alpha U_\beta + U_\beta Q_\alpha) \tag{8}
$$

from which we can read off the metric $< P_a, P_b >= h_{ab}, < J, Z >= 1$, $< K, K >= 1, < Q_\alpha, U_\beta >= \frac{1}{2}C_{\alpha\beta}$ which is non-degenerated.

The topological gauge theory associated to this algebra follows in the usual way [12]. The one-form gauge potential A has the following expansion in terms of the generators of the algebra

$$
A = e^a P_a + wJ + vK + \psi^\alpha Q_\alpha + \xi^\alpha U_\alpha + AZ \tag{9}
$$

The two-form field strength $F = dA + A^2$ can be used to write the action $S = \int Tr(\eta F)$ where η is in the coadjoint representation. We

then get

$$
\begin{aligned}
S &= \int [\eta_a F^a(P) + \eta F(J) + \eta' F(K) + \\
&+ \eta'' F(Z) + \chi_\alpha F^\alpha(Q) + \chi'_\alpha F^\alpha(U)]
\end{aligned} \tag{10}
$$

where the field strengths are

$$
F^a(P) = de^a + we^b\epsilon_b{}^a - \frac{1}{2}\psi\gamma^a\psi \tag{11}
$$

$$
F(J) = dw \tag{12}
$$

$$
F(K) = dv + \frac{1}{2}\psi\gamma_2\psi \tag{13}
$$

$$
F(Z) = dA + \frac{1}{2}e^a e^b \epsilon_{ab} + \psi\gamma_2\xi \tag{14}
$$

$$
F^\alpha(Q) = D\psi^\alpha \equiv d\psi^\alpha - \frac{1}{2}w(\psi\gamma_2)^\alpha \tag{15}
$$

$$
F^\alpha(U) = D\xi^\alpha + \frac{1}{2}e^a(\psi\gamma_a)^\alpha - \frac{1}{2}v(\psi\gamma_2)^\alpha \tag{16}
$$

The field equations obtained from the variation of η are then $F = 0$ and those obtained from the variation of the gauge fields are

$$
D\eta_a + \epsilon_{ab}e^b\eta'' - \frac{1}{2}\psi\gamma_a\chi' = 0 \tag{17}
$$

$$
d\eta + e^a\epsilon_a{}^b\eta_b + \frac{1}{2}\psi\gamma_2\chi + \frac{1}{2}\xi\gamma_2\chi' = 0 \tag{18}
$$

$$
d\eta' + \frac{1}{2}\psi\gamma_2\chi' = 0 \tag{19}
$$

$$
d\eta'' = 0 \tag{20}
$$

$$
D\chi + \eta_a\gamma^a\psi - \eta'\gamma_2\psi - \eta''\gamma_2\xi - \frac{1}{2}e^a\gamma_a\chi' + \frac{1}{2}v\gamma_2\chi' = 0 \tag{21}
$$

$$
D\chi' - \eta''\gamma_2\psi = 0 \tag{22}
$$

Equation (11) allows us to solve for the spin connection

$$
w = -(dete)^{-1}e^a\epsilon^{\mu\nu}(\partial_\mu e^b_\nu h_{ab} - \frac{1}{2}\psi_\mu\gamma_a\psi_\nu) \tag{23}
$$

Then equation (12) implies that $R = 0$, where R is curvature scalar, while (15) implies the vanishing of the gravitino field strength. From (20) we obtain $\eta'' = \lambda = constant$ which plays the role of the cosmological constant as in the bosonic case [11]. The remaining fields may have a geometric interpretation when coupled to matter (like in the bosonic case [13]). In order to get the corresponding supergravity theory we solve the equations for v, η_a, η' and ξ obtaining

$$\begin{aligned} v &= \frac{1}{2\eta''}\chi'\psi \\ \eta_a &= -\epsilon_a{}^b e_b^\mu(\partial_\mu\eta + \frac{1}{2}\psi_\mu\gamma_2\chi + \frac{1}{2}\xi_\mu\gamma_2\chi') \\ \eta' &= \frac{1}{4\eta''}\chi'\chi' \\ \xi &= \frac{1}{2\eta''}e^a\chi'\gamma_a\gamma_2 + \frac{1}{8(\eta'')^2}(\chi'\chi')\psi \end{aligned} \tag{24}$$

We then find that the action (10) reduces to

$$S = \int[\eta dw + \chi_\alpha D\psi^\alpha + \frac{1}{2}\eta'' e^a e^b\epsilon_{ab} - \frac{1}{2}e^a\psi\gamma_a\chi' + \frac{1}{8}\eta''\chi'\chi'\psi\gamma_2\psi] \tag{25}$$

The supersymmetry transformations can be obtained from the transformations generated by Q and using the solutions (24). The resulting supersymmetry transformations which leave the action (25) invariant are

$$\begin{aligned} \delta e^a = \epsilon\gamma^a\psi, &\quad \delta\eta = -\frac{1}{2}\epsilon\gamma_2\chi \\ \delta\psi = D\epsilon, &\quad \delta\chi = -\eta_a\gamma^a\epsilon \\ \delta\chi' = \eta''\gamma_2\epsilon & \end{aligned} \tag{26}$$

where the equation relating χ' and ψ (22) has been used. If we eliminate χ' from the action by using the field equation (22) then the resulting action would be non local. We then have a non local version of $N = 1$ topological supergravity theory. The non-locality appears only in the fermionic sector and it may give rise to some new class of induced supergravity theory. Other aspects, like the relation to matrix models and the presence (or absence) of black holes remains to be investigated.

Notice however that there exists a local version for $N = 1$ dilaton supergravity [6, 7]. It is given by the action (after some field redefinitions and elimination of the auxiliary fields)

$$\begin{aligned} S &= \int[\eta dw + \chi_\alpha D\psi^\alpha + \eta'' e^a e^b\epsilon_{ab} + \\ &+ (\eta'')^2(\eta^{\frac{1}{2}}\psi\gamma_2\psi + \frac{1}{2}\eta^{-\frac{1}{2}}e^a\psi\gamma_a\psi + \frac{1}{16}\chi\chi e^a e^b\epsilon_{ab})] \end{aligned} \tag{27}$$

This action is invariant under the following supersymmetry transformations:

$$\begin{aligned} \delta e^a = \epsilon\gamma^a\psi, &\quad \delta\eta = -\frac{1}{2}\epsilon\chi \\ \delta\psi = D\epsilon - \frac{1}{2}(\eta'')^{\frac{1}{2}}e^a\epsilon\gamma_a, &\quad \delta\chi = -\eta_a\gamma^a\epsilon + 2(\eta'')^{\frac{1}{2}}\eta^{\frac{1}{2}}\gamma_2\epsilon \end{aligned} \tag{28}$$

However, as it is easily seen, the field equations do not imply the zero curvature condition neither the vanishing of the gravitino field strength. So, although it is a possible extension of the dilaton gravity theory it does not share the topological properties presented by (25).

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