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MAXWELL ELECTROMAGNETIC THEORY, PLANCK'S RADIATION LAW AND BOSE-EINSTEIN STATISTICS

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ABSTRACT

We give an example in which it is possible to reconcile quantum statistics with classical concepts. This is done by studying the interaction of charged matter oscillators with the thermal and zeropoint electromagnetic fields characteristic of quantum electrodynamics and classical stochastic electrodynamics. Planck's formula for the spectral distribution is interpreted without resorting to any discontinuities whatsoever. We conclude that $\hbar\omega$ is the *average* value of the energy quanta which the oscillators exchange with the radiation field in order to achieve thermal equilibrium.

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The general proposal of Tersoff and Bayer[1], when adapted to the blackbody radiation phenomenon, provides a clue for bringing quantum and classical physics to a closer relation. The similarity between quantum electrodynamics (QED) and the classical theory called stochastic electrodynamics (SED) has been stressed by several authors in recent years[2–7]. It should be mentioned that to obtain a closer relation between the classical and quantum approaches was one of the goals of Planck and Einstein research in the first and second decades of this century. It is well known that Einstein himself never gave up to obtain a deeper understanding of the light-quantum concept within the realm of classical electrodynamics[8, 9]. Following then the proposal of Tersoff and Bayer, we show that it is possible to reconcile the statistics of Bose and Einstein with classical electrodynamics. As a natural byproduct of our analysis, we interpret Planck's radiation law for the blackbody radiation, without using concepts strange to classical physics, like discontinuities in the energy of the charged particle during the emission or absorption processes. This reinforces the attempts to interpret the photon model using only the undulatory aspect of the electromagnetic field, thus providing a much simpler picture of the light-quantum.

For a single one-dimensional nonrelativistic matter oscillator (mass m, charge e and frequency ω such that $e^2\omega/mc^2 \ll 1$) interacting with a cavity radiation with spectral distribution $\rho(\omega,T) = \rho_0(\omega) + \rho_T(\omega)$, there is a simple relation (valid within both QED and SED) between the average oscillator energy $\langle\epsilon\rangle$ and $\rho(\omega,T)$, namely[10]

$$\langle\epsilon\rangle = \frac{4\pi}{3}\frac{e^2}{m}\omega^2\int_0^\infty \frac{d\omega'\left[\rho_0(\omega')+\rho_T(\omega')\right]}{\left(\omega'^2-\omega^2\right)^2+\left(\frac{2}{3}\frac{e^2}{mc^3}\omega'^3\right)^2} \cong \frac{\pi^2c^3}{\omega^2}\rho(\omega,T) =$$

$$= \frac{\hbar\omega}{2} + \frac{\pi^2c^3}{\omega^2}\rho_T(\omega) \equiv \langle\epsilon_0\rangle + \langle\epsilon_T\rangle \quad . \tag{1}$$

In (1) we have considered that $\langle \epsilon_0 \rangle = \hbar\omega/2$ is the average zeropoint (or zero temperature) energy of the oscillator, $\langle \epsilon_T \rangle$ is its average thermal energy and

$$\rho_0(\omega) = \frac{\hbar\omega^3}{2\pi^2 c^3} \quad , \tag{2}$$

is the spectral distribution of the zeropoint electromagnetic field (which exist in both SED and QED). We are denoting by $\rho_T(\omega)$ the spectral distribution of thermal radiation with temperature T. As far as we know, formula (1) was firstly obtained by Planck (see ref. [8], p. 369) with $\rho_0(\omega) = 0$. The inclusion of the zeropoint field contribution to the oscillator average energy was made later, in 1911, also by Planck[7].

Let us consider a system of A identical matter oscillators. We shall assume that they do not interact directly with each other, but are exchanging energy with the zeropoint electromagnetic field and the thermal radiation existing inside a cavity. This implies that the thermal energy of the system of oscillators plus the radiation field is *not conserved.* If we denote the thermal energy of the A oscillators by U, and the remaining thermal energy by u, the total thermal energy in the cavity can be written as

$$E \equiv U + u \quad . \tag{3}$$

Therefore, u represents the radiation energy associated with the thermal electromagnetic fields.

If we accept the zeropoint field (classical or quantum) as a real radiation field which pervades the physical space, we can consider that each oscillator absorbs both the thermal and the zeropoint energy from the radiation field existing inside the cavity. It is worthwhile to stress that in any comprehensible theory, like QED or SED for instance,

these zeropoint electromagnetic fields *must be considered real* since they affect the matter in many different ways which can be observed experimentally[4–7] as for instance the Lamb shift, the Casimir effect, spontaneous emission and atomic stability[6] .

According to SED (and also QED) the energy in the electromagnetic field will be very large due to the high frequency behaviour of $\rho_0(\omega)$. We shall therefore assume that (2) is valid up to a very high frequency which we are not going to estimate[4, 6]. This means that the zeropoint electromagnetic field will be considered as a very large energy reservoir interacting with the A oscillators. Therefore, even when the temperature is very small ($T \to 0$), there is a large amount of energy available for the oscillators. We shall also assume, in accordance with SED and QED, that the thermal energy E, introduced in (3), is a *random variable* with a large range of variation ($0 < E < \infty$).

The probability distribution for the thermal energy ϵ_i, of a single oscillator, can be obtained (within SED or QED) from the assumption that the thermal radiation is composed by Gaussian electromagnetic fields[3]. Since the equation of motion for the charged oscillator is linear, it is easy to conclude that the position and the momentum of the charged particle are also Gaussian variables[2–4]. A natural consequence of this fact is that the probability $d\xi_i$ for finding the i-th oscillator with thermal energy between ϵ_i and $\epsilon_i + d\epsilon_i$ is

$$d\xi_i = \frac{e^{-\epsilon_i/\langle \epsilon_T \rangle}}{\langle \epsilon_T \rangle} d\epsilon_i \quad . \tag{4}$$

In the absence of the zeropoint field, ($\rho_0(\omega) = 0$) and assuming that $\langle \epsilon_T \rangle = kT$, expression (1) will lead to the Rayleigh-Jeans law for the spectral distribution $\rho_T(\omega)$, whereas (4) will lead to the Maxwell-Boltzmann distribution for the oscillator energies. Here, however, we shall avoid the equipartition assumption. Instead, we intend to calculate $\langle\epsilon_T\rangle$ from other principles. In order to achieve this goal, we shall use the proposal suggested by Tersoff and Bayer and the fact that the oscillators are in contact with a large energy reservoir, even when the temperature is arbitrarily close to zero.

The approach of Tersoff and Bayer requires the assignment of different (random) probability weights (for the absorption of thermal energy) to identical oscillators. So, in order to use their approach, the first step is to justify this assumption. This apparently paradoxical task can be achieved if we recognize that the probability weights are proportional to the probability of having some thermal energy u (see (3)) available in the fluctuating radiation fields inside the cavity.

Regarding this point, our approach will be similar to that proposed by Ehrenfest[11] in 1911. Ehrenfest generalized Boltzmann's concept of "a priori probability" and used probability weights which vary with the energy interval. In the following we shall present a simple model which makes an explicit use of this idea.

Let us consider, for instance, a situation where the total thermal energy of the cavity is E and the thermal energy of the i-th oscillator has the ***arbitrary*** value ϵ_i. The other matter oscillators only interact with the radiation field whose thermal energy u may vary ($0 \le u \le E - \epsilon_i$). Therefore, the probability weight $\alpha_i(E, \epsilon_i)$ that the system (A oscillators plus radiation with total thermal energy E) is found in this state will be defined by

$$\alpha_i(E, \epsilon_i) \equiv \frac{\int_0^{E-\epsilon_i} du\, Q(u)}{\sum_{j=1}^{A} \int_0^{E-\epsilon_j} du\, Q(u)} \quad , \tag{5}$$

where $Q(u)\,du$ is the probability that the energy in the thermal radiation fields is between u and $u + du$.

We have assumed, with definition (5), that these probability weights $\{\alpha_i\}$ are subjected to the restriction

$$\sum_{i=1}^{A} \alpha_i(E, \epsilon_i) = 1 \quad , \tag{6}$$

and that the set of energies $\{\epsilon_i\}$ are such that

$$U = \sum_{i=1}^{A} \epsilon_i \quad . \tag{7}$$

A few remarks are necessary in order to clarify better our approach. The first one is that when the energies ϵ_i are much smaller than E, we get $\alpha_i = 1/A$. However, $\alpha_i = 0$ if $\epsilon_i = E$. These results are expected on physical grounds. Another important point, which we want to stress, is that according to the definition (5), it is expected a variation of the probability weight α_i when the energy difference $E - \epsilon_i$ varies, that is, $d(E - \epsilon_i) \neq 0$ implies $d\alpha_i \neq 0$. In other words, the *continuous variation* of the α_i's is related to the *continuous variation of the oscillators energies* ϵ_i and the total thermal energy E.

The explicit knowledge of the functional relation between the set of probability weights $\{\alpha_i\}$ and the set of energies $\{\epsilon_i\}$ is not necessary in order to apply the Tersoff and Bayer method, as we shall see in a moment. However, for the sake of completeness and clarity, we would like to say that a simple expression for $Q(u)$, namely

$$Q(u) = \text{const. } \exp\left[-\frac{\left(E - \epsilon_i - u - (A-1)\langle\epsilon_T\rangle\right)^2}{2(A-1)\langle\epsilon_T\rangle^2}\right] \quad , \tag{8}$$

can be obtained from the assumption that the oscillators thermal energy, $\sum_{j\neq i} \epsilon_j$ (see (7)), is a "random walk" with a large number of statistically independent "steps" ϵ_j. It is worthwhile to notice that for small ϵ_i ($A \gg \epsilon_i/\langle\epsilon_T\rangle$), it is possible to show that (8) and (5) lead to $\alpha_i \simeq \epsilon_i / \sum_{j=1}^{A} \epsilon_j$, which is a very simple and physically appealing result (notice that in this limit equations (6) and (7) are equivalent).

We need one step more in order to apply the Tersoff and Bayer's method and to connect it with the quantum picture. Let us divide the total thermal energy U of the matter oscillators into arbitrarily small fractions (quanta), U/N, with $N \gg 1$. Therefore, since the oscillator energies can take arbitrary values ϵ_i such that (7) is valid, we can introduce a set of numbers $\{n_i\}$ such that

$$\epsilon_i \equiv n_i \frac{U}{N} \quad , \quad N \equiv \sum_{i=1}^{A} n_i \quad . \tag{9}$$

Then, each n_i represents the amount of fractions U/N (or quanta) which an oscillator has. We shall assume that these fractions of energy, U/N, besides being *arbitrarily* small, are *distinguishable* within the context of our classical approach.

In (9) we are assuming that the numbers n_i are fixed. In other words, the thermal energy U may vary continuously from 0 to E, when we take the ensemble average according to the Tersoff and Bayer proposal as we shall see below, but the numbers n_i's are fixed despite the fact that each thermal energy ϵ_i may vary continuously from 0 to $n_i E/N$.

The above considerations will allow us to calculate the probability $P\{n_i\}$, associated to the numbers $\{n_i\}$ of distinguishable quanta, U/N, which are distributed amongst A distinguishable oscillators according to (9). This probability, as proposed by Tersoff and Bayer, is

$$P\{n_i\} = N! \left\langle \prod_{i=1}^{A} \frac{\alpha_i^{n_i}}{n_i!} \right\rangle = N! \prod_{i=1}^{A} \left(\int_0^1 d\alpha_i \frac{\alpha_i^{n_i}}{n_i!} \right) \delta \left(1 - \sum_{j=1}^{A} \alpha_j \right) \quad , \tag{10}$$

where the continuous and random variation of each α_i is related to the *continuous and random variation of the energies* ϵ_i and E.

The exact result for this expression, is simply[1]

$$P\{n_i\} = \frac{N!\,(A-1)!}{(N+A-1)!} \quad , \tag{11}$$

independently of the numbers $\{n_i\}$ and the value of the random thermal energy E (see (3)). This remarkable result of Tersoff and Bayer is valid within the realm of both SED and QED. It allows the derivation of Planck's radiation law without resorting to any discontinuities in the emission or absorption of radiation by the matter oscillators, and to interpret Planck's quanta in a simpler way using only the undulatory properties of the electromagnetic field.

Considering that A and N were assumed to be very large numbers ($A \gg 1$ and $N \gg 1$) and using the Boltzmann relation between the entropy and probability, the entropy per oscillator is given by

$$S \equiv -\frac{k}{A} \ln P\{n_i\} \cong k\left[(1+n)\ln(1+n) - n \ln n\right] \quad , \tag{12}$$

where $n \equiv N/A \equiv n(\langle\epsilon\rangle)$ will be considered a function of the total average energy $\langle\epsilon\rangle$ of a single oscillator (see (1)).

Since we are not assuming the equipartition of thermal energy ($\langle\epsilon\rangle - \langle\epsilon_0\rangle \neq kT$) we must take the zeropoint energy into account. Therefore, following Boyer[12], we take

$$\frac{\partial^2 S}{\partial\langle\epsilon\rangle^2} = -\frac{k}{\langle\epsilon\rangle^2 - \left(\dfrac{\hbar\omega}{2}\right)^2} \tag{13}$$

because $\langle\epsilon_0\rangle = \hbar\omega/2$.

The definition of temperature T will allow us to write (13) in the following (equivalent) form:

$$\frac{1}{T} = \frac{\partial S}{\partial\langle\epsilon\rangle} = \frac{k}{\hbar\omega} \ln \left(\frac{\langle\epsilon\rangle + \dfrac{\hbar\omega}{2}}{\langle\epsilon\rangle - \dfrac{\hbar\omega}{2}} \right) \quad . \tag{14}$$

Planck's formula for the spectral distribution of thermal radiation $\rho_T(\omega)$ can be derived from (14) and (1) as is well known[12]. Here, however, we want to use (12) and (14) in order to interpret $n = n(\langle\epsilon\rangle)$ introduced above.

Taking into account (12) and (14) we obtain the following differential equation for $n = n(\langle\epsilon\rangle)$:

$$\hbar\omega \, \ln\left(\frac{1+n}{n}\right) \frac{dn}{d\langle\epsilon\rangle} = \ln\left(\frac{\langle\epsilon\rangle + \dfrac{\hbar\omega}{2}}{\langle\epsilon\rangle - \dfrac{\hbar\omega}{2}}\right) \quad , \tag{15}$$

which has the solution

$$n(\langle\epsilon\rangle) = \frac{\langle\epsilon\rangle - \dfrac{\hbar\omega}{2}}{\hbar\omega} \quad . \tag{16}$$

Since $n = N/A$, we conclude that

$$\hbar\omega = \frac{A}{N}\left(\langle\epsilon\rangle - \frac{\hbar\omega}{2}\right) = \frac{\langle U\rangle}{N} \tag{17}$$

where $\langle U\rangle$ is the average thermal energy of the A oscillators. In other words, $\hbar\omega$ is the *average* value of the energy quanta which the matter oscillators exchange with the radiation field in order to achieve thermal equilibrium. Identical conclusion was obtained by Barranco and França[9] in a recent paper, although through a different reasoning.

We have shown that according to SED and QED the thermal photons can be described using only the wave aspect of the radiation field. This simplified view of thermal radiation is important because it is also well known that the photoelectric and the Compton effects can be explained without the particle model of the photon[13]. These facts are known since the pioneering works by Richardson (1914), Wentzel (1927), Schrödinger (1927) and Klein and Nishina (1929) among many others[14]. This suggests to us that the particle model of the photon may be an obsolete model and that

QED essentially provides the rules of the interaction without explicitly invoking the corpuscular character of the photon[13, 14].

We conclude with a few more comments. It is well known[7] that Planck used equations (11) and (14) in order to derive $\rho_T(\omega)$. However, his justification of (11) was based on an entirely different reasoning. Planck assumed that the quantities $\hbar\omega$ should be considered *indivisible and indistinguishable energy quanta.* We have seen, however, that $\hbar\omega$ can be interpreted as the *average* value of the energy quanta which the matter oscillators exchange with the radiation field, in order to achieve thermal equilibrium. This result, which we obtained using the statistical method of Tersoff and Bayer, has a microscopic justification (in both SED[15] and QED[6]) based on the interference of the zeropoint electromagnetic fields and radiation reaction fields during the processes of emission and absorption of radiation.

Therefore, a reconciliation with the Planck approach may be possible, provided that we recognize that the *average* amounts of energy, $\hbar\omega$, cannot be distinguished from each other, if they are distributed amongst A identical but distinguishable oscillators (see ref. [11], p. 256). This reconciliation with classical electromagnetic theory was a desire of Planck because his *second blackbody theory*[7] is considered a step towards Maxwell theory. Furthermore, there are some indications that the distinguishability of photons interacting with optical instruments has been experimentally verified. This important discovery, which was reported by at least two different groups very recently[16, 17], did not receive the attention it deserves.

It should be mentioned that some authors[18] have interpreted the Tersoff and Bayer work (namely eq. (6)) as some kind of "non-local action at-a-distance". Nevertheless we think that this interpretation does not apply to the case of blackbody radiation phenomenon due to the finite size of the cavity and the large masses of the oscillators[19].

Finally we would like to mention other attempts to obtain a derivation of Planck's radiation law. As far as we know, Einstein and Stern[20] in 1913, and Nernst[21] in 1916 were the first physicists who claimed to have derived Planck's formula without assuming discontinuities (see also ref. [7]). More recently Boyer[22] and others[23-25] have proposed different ways to derive the blackbody radiation spectrum without quantum assumptions. These approaches were based on Planck's second theory which introduced the concept of zeropoint energy of the electromagnetic field.

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