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INCLUSIVE BREAK-UP OF EXOTIC NUCLEI

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Abstract

The inclusive break-up of neutron-rich nuclei is discussed. Using an extension of the Kox empirical formula for the total reaction cross-section of heavy ions, the nuclear component of the break-up of exotic nuclei is calculated as a function of bombarding energy. Results are presented for $^{11}\text{Li} + ^{12}\text{C}$ and $^{11}\text{Li} + ^{208}\text{Pb}$ systems in the laboratory energy range $B_c \leq E_{\text{Lab}} \leq 60$ MeV/n, where B_c is the height of the Coulomb barrier. Comparison with experimental results for ^8He , ^{11}Li , ^{14}Be fragmentation reactions on Be, Ni and Au targets at 30 MeV/n are made. The one-neutron removal cross-section in ^{11}Be -induced reactions is also calculated.

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The determination of the electromagnetic dissociation cross-section (EDC) of neutron-rich nuclei is of great importance for the inference of the low energy response. The extraction of this response allows the study of the so-called "soft giant resonances" (SGR). In order to obtain this response from the experimental data, it is essential to have a reliable estimate of the nuclear contribution [1]. Having in mind the upsurge of interest in low energy exotic beam physics, a thorough discussion of this question is required.

Several publications have addressed the question of calculating the nuclear component of the $2n$ removal cross-section in ^{11}Li -induced reactions. These range from Glauber, polarization potential and phenomenological. Here, we suggest an extension of the theory of Hussein and McVoy [2] that allows the calculation of σ_{-2n}^N within an optical model description. Similar ideas were developed by Ogawa [3] and collaborators. Our method, to be described below, allows the calculation of σ_{-2n}^N in the low energy range $B_c \leq E \leq 60$ MeV/n, where the Glauber model, used by Ogawa et al [3], would not work.

In order to simplify the presentation we present in what follows a qualitative account of the HM theory. Denoting the survival probability of a fragment i by $P_i(b)$, where b is the impact parameter, we can write the inelastic nuclear inclusive probability, $P_b^{N,inel}(b)$, of detecting fragment b in the break-up of the projectile a into $x + b$

$$P_b^{N,inel}(b) = (1 - P_x(b)) P_b(b) \quad (1)$$

Eq.(1) measures the probability that b survives where as x interacts non-elastically with the target.

The other important break-up process is the elastic nuclear break-up, whose probability is given by

$$P_b^{N,el}(b) = P_b(b) P_x(b) - P_a(b) \quad (2)$$

Summing (1) and (2) gives the total nuclear including break-up probability

$$P_b^N(b) = P_b^{N,el}(b) + P_b^{N,inel}(b) \quad (3)$$

$$= P_b(b) - P_a(b) \quad (4)$$

The corresponding cross-section is just

$$\sigma_b^N = 2\pi \int b db (P_b(b) - P_a(b)) \quad (5)$$

$$\equiv \sigma_{Int}(a) - \sigma_{Int}(b) \quad (6)$$

In Eq.(6), $\sigma_{Int}(i)$ is the interaction cross-section of fragment i . Eq.(6) has been derived within the Glauber theory by Ref.3). Here, we suggest that this equation is valid in general. The next step is to decide upon a Model for $\sigma_{Int}(i)$. We now assume that $\sigma_{Int}(a) - \sigma_{Int}(b)$ is equal to $\sigma_R(a) - \sigma_R(b)$, where $\sigma_R(i)$ is the total reaction cross-section of fragment i . Such an approximation implies that the inelastic excitations of the target by a and b are equal. Then a very simple recipe can be devised for σ_{-2n}^N based on known facts about $\sigma_R(i)$.

Before presenting our numerical results we first present a derivation of Eq.(6) using the theory of inclusive break-up reaction developed by Hussein and McVoy (HM). We also extend this to take into account the elastic break-up contribution not considered in HM. According to HM, the inclusive enelastic break-up reaction



$$a = b + z$$

is described by the following cross-section

$$\begin{aligned} \frac{d\sigma}{d\Omega_b dE_b} &= -\frac{2}{\hbar v_a} \rho(E_b) (\psi_z^{(+)} | W_{zA} | \psi_a^{(+)}) \\ \rho(E_b) &= \frac{M_b k_b}{(2\pi)^3 \hbar^2} \end{aligned} \quad (8)$$

where W_{zA} is the imaginary potential of the $z - A$ system and the overlap wave function $|\psi_z^{(+)}\rangle$ is just $\langle \chi_b^{(-)} | \phi_a \chi_a^{(+)} \rangle$. If the internal wave function of a, ϕ_a , is taken to be a Gaussian, then the integration of (2) gives the primary inelastic yield of the observed spectator fragment b ,

$$\sigma_b^{N,inel} = \frac{\pi}{\sigma^2} \sum_{j=0}^{\infty} |S_j^b|^2 (1 - |S_j^z|^2) \quad (9)$$

where σ is the momentum width of ϕ_a and $|S_j^z|^2$ is given by

$$|S_j^z|^2 \equiv \frac{(\sigma^2)^{j+i}}{j!} \int_0^{\infty} b_i^{2j} db_i^2 e^{-\sigma^2 b_i^2} |S_i(b_i)|^2 \quad (10)$$

In (4) $S_i(b_i)$ is the elastic S-matrix of fragment i . Therefore $|S_j^z|^2$ measures the Fermi motion modified survival probability of fragment i . Note, if σ is very small, as would be the case of a loosely bound $z - b$ system,

$$\begin{aligned} \sigma_b^{N,inel}(\sigma \rightarrow 0) &= 2\pi \int b db |S_b(b)|^2 (1 - |S_z(b)|^2), \\ P_i(b) &\equiv |S_i(b)|^2 \end{aligned} \quad (11)$$

Eq.(5) is the formula used by Esbensen and Bertsch [4] and by Ogawa et al [3] to represent the inclusive inelastic break-up yield. It is obvious from the discussion above that such an expression is reasonable only when σ is very small or, equivalently, the binding energy of the cluster is very small, such as in $^{11}\text{Li}(E_{2n} \simeq 0.3 \text{ MeV})$.

We turn now to elastic break-up contribution. The cross-section for this process in the prior representation is given by

$$\begin{aligned} \frac{d^2 \sigma_b^{N,el}}{d\Omega_b dE_b} &= \frac{2\pi \rho(E_b)}{\hbar v_a} \sum_{k_x} |\langle \chi_x^{(-)} \chi_b^{(-)} | (U_{xA} + U_{bA} - U_{aA}) | \phi_a \chi_a^{(+)} \rangle|^2 \\ &\cdot \delta(E - E_x - E_b - \epsilon_0) \end{aligned} \quad (12)$$

In Eq.(12), U_{iA} is the complex optical for the $i + A$ system and ϵ_0 is the Q-value (separation energy) that determines the spatial extension of the internal wave function of the projectile ϕ_a . The total elastic break-up is obtained easily by writing

$$\sigma_b^{N,el} \equiv \int \frac{d^2 \sigma_b^{N,el}}{d\Omega_b dE_b} d\Omega_b dE_b = \int \frac{d^2 \sigma_b^{N,el}}{d\Omega_b dE_b} \frac{d\vec{k}_b}{(2\pi)^3} \frac{1}{\varrho(E_b)} \quad (13)$$

thus, approximately

$$\begin{aligned} \sigma_b^{N,el} &= \frac{2\pi}{\hbar v_a} \int \frac{d\vec{k}_b}{(2\pi)^3} \int \frac{d\vec{k}_x}{(2\pi)^3} |\langle \chi_x^{(-)} \chi_b^{(-)} | U_{xA} + U_{bA} | \chi_x^{(+)} \chi_b^{(+)} \phi_a \rangle \\ &- \langle \chi_a^{(-)} | U_{aA} | \chi_a^{(+)} \phi_a \rangle|^2 \delta(E - E_x - E_b - \epsilon_0) \end{aligned} \quad (14)$$

In Eq.(14), we have made use of the fact that U_{aA} depends on \bar{r}_{aA} , U_{xA} depends on \bar{r}_{xA} and U_{bA} on \bar{r}_{bA} . With the aid of the Glauber-type

approximation employed by HM, Eq.(14) can be reduced to the form (for very small momentum width of Φ_a).

$$\sigma_b^{N,c,d} = 2\pi \int b db [|S_b(b)|^2 |S_x(b)|^2 - |S_a(b)|^2] \quad (15)$$

Having verified the reaction theoretical basis of Eq.(6), with σ_{int} replaced by σ_R , (see discussion following Eq.6) we now turn to applications. In order to calculate Eq.(6) we use the well-known Kox formula for σ_R [5]. We have

$$\sigma_R = \frac{\hbar\pi r_0^2 \omega}{2\pi E} \left\{ A_1^{1/3} + A_2^{1/3} + b \frac{A_1^{1/3} + A_2^{1/3}}{A_1^{1/3} A_2^{1/3}} - a + D \right\} \ell n(1 + \exp \frac{2\pi}{\hbar\omega} (E - B_c)) \quad (16)$$

In Eq.(16) we have replaced the barrier factor $(1 - \frac{E_c}{E})$ in the original Kox formula by the more exact Wong factor $\frac{\hbar\omega}{2\pi E} \ell n(1 - \exp[\frac{2\pi}{\hbar\omega} (E - B_c)])$ with $\hbar\omega \sim 3.0$ MeV.

When applied to non-exotic nuclei Eq.(16) gives the following values for the parameters $r_0 = 1.1$ fm, $b = 1.85$, $a = 0.65$, $D = 0$ and $B_c = \frac{Z_1 Z_2 e^2}{1.3(A_1^{1/3} + A_2^{1/3})}$. In the calculation we present below for the break-up of neutron-rich nuclei, we use the exact values for their radii.

In figure 1 we show σ_{-2n}^N vs. E_{Lab} for the system $^{11}\text{Li} + ^{12}\text{C}$ in the energy range 6.5 MeV/n $< E_{Lab} < 60$ MeV/n. We believe our model to be adequate in this range since mostly geometrical features of the system dictates the value of σ_{-2n}^N . At high energies, a description using the nucleon-nucleon cross-section is more appropriate. We see in the figure that σ_{-2n}^N rises with energy and reaches a maximum at about 10 MeV/n after which it starts slowly decreasing. The average value of the cross-section in the energy range 10 MeV/n $< E_L < 50$ MeV/n is roughly $\pi(R_{^{11}\text{Li}} + R_{\text{Target}})^2 - \pi(R_{^{12}\text{C}} + R_{\text{Target}})^2$. The Coulomb dissociation cross-section for this system is calculated using the formulation of Ref.6 within the usual Weissacher-Williams picture, and assuming a cluster structure of the exotic nucleus [7] with $E_{2n} = 0.35$ MeV. We show this also in Fig.1. The Coulomb contribution also peaks but at a much smaller energy, $E \cong 2$ MeV/n. Then it starts dropping, much faster than the nuclear contribution. At $E \sim 58$ MeV/n it amounts to just about 3% of σ_{-2n}^N , as one expects due to the small charges involved. We mention here that our results for σ_{-2n}^N at $E_{Lab} \sim 60$ MeV/n is very close to those

Ref.3 and to Canto et al.[8]. The result of Ref.8 at lower energy, however, overestimates σ_{-2n}^N .

In figure 2 we show the result of our calculation for the system $^{11}\text{Li} + ^{208}\text{Pb}$. Here the Coulomb dissociation cross-section is quite large and dominant. Again we see the peaking in σ_{-2n}^N . We should remind the reader that our calculated Coulomb dissociation cross-section within the cluster model is an overestimate [1],[9]. We include it here for completeness.

We have also calculated σ_{-2n}^N for other systems such as those studied by Rüsager et al. at $E_{Lab} = 30$ MeV/n [10]. We present in table 1 some representative cases. The measured values if the total 2n-removal cross-section are also given. The agreement between our model calculation and the experimental data is reasonable. The two-neutron separation energies in ^8He , ^{11}Li and ^{14}Be were taken to be 2.137 MeV, 0.35 MeV and 1.34 MeV, respectively. The experimental value of σ_{-2n} for $^{11}\text{Li} + \text{Au}$, namely 10.8 ± 1.9 barns would yield a Coulomb dissociation cross-section of $\cong 9.0 \pm 1.9$ barns if we subtract our calculated nuclear component. We find this value a bit too large considering the result of Ref.9 which gives for σ_{-2n}^C of $^{11}\text{Li} + ^{208}\text{Pb}$ at $E_{Lab} = 28$ MeV/n, a quite similar system, the value of 3.6 ± 0.4 b. Finally, we have performed calculation of the one-neutron removal cross-section in ^{11}Be -induced reactions. Here, the one-neutron separation energy is about 0.5 MeV and one would expect similar behaviour for σ_{-n} as that of σ_{-2n} . At $E_{Lab} = 30$ MeV/n, we obtain for the system $^{11}\text{Be} + ^{28}\text{Si}$ $\sigma_{-n}^N = 568$ mb, $\sigma_{-n}^C = 82$ mb, where as for $^{11}\text{Be} + ^{238}\text{U}$ we find $\sigma_{-n}^N = 1050$ mb and $\sigma_{-n}^C = 2463$ mb.

In conclusion, we have developed in this paper a model for the nuclear component of the inclusive break-up cross-section of exotic nuclei valid at low energies where geometrical features dominate. We applied our model to the $^{11}\text{Li} + ^{12}\text{C}$ and $^{11}\text{Li} + ^{208}\text{Pb}$ systems in the energy range $B_c \leq E_{Lab} < 60$ MeV/n. The cross-section is found to rise sharply as the energies is increased above these barrier then it saturates at the value $\pi(R_{^{11}\text{Li}} + R_{\text{Target}})^2 - \pi(R_{^{12}\text{C}} + R_{\text{Target}})^2$. At energies above 50 MeV/n the cross-section starts dropping. The one-neutron removal cross-section in ^{11}Be -induced reactions is also calculated. We also compared our results with available data at $E_L = 30$ MeV/n. The systems considered are ^8He , ^{11}Li and ^{14}Be on Be, Ni and Au targets. The agreement with the data was found to be quite reasonable.

Acknowledgements

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Table Captions

Table 1. Comparison between calculated cross-sections and the experimental data taken from Ref.10. See text for details.

Figure Captions

Figure 1: The two-neutron removal cross-section for $^{11}\text{Li} + ^{12}\text{C}$ vs. E_{Lab} , see text for details.

Figure 2: Same as in figure 1 for $^{11}\text{Li} + ^{208}\text{Pb}$.

Table 1

Beam	Calculated Cross-Section			Measured Cross-Section		
	Be	(mb) Ni	Au	Be	(barns) Ni	Au
⁸ He	Nuclear:	310.295	555.23	832.50		
	Coulomb:	0.564	14.78	45.83		
	Sum :	310.858	570.01	878.33	0.41 ± 0.15	1.5 ± $\frac{1.0}{0.5}$
¹¹ Li	Nuclear:	693.14	1181.02	1702.45		
	Coulomb:	15.61	645.55	4529.87		
	Sum :	708.75	1826.57	6212.32	0.47 ± 0.1	2.1 ± 0.4
¹⁴ Be	Nuclear:	400.2	685.53	994.79		
	Coulomb:	1.91	55.40	219.78		
	Sum :	402.1	740.93	1214.57	0.44 ± 0.1	1.0 ± 0.3

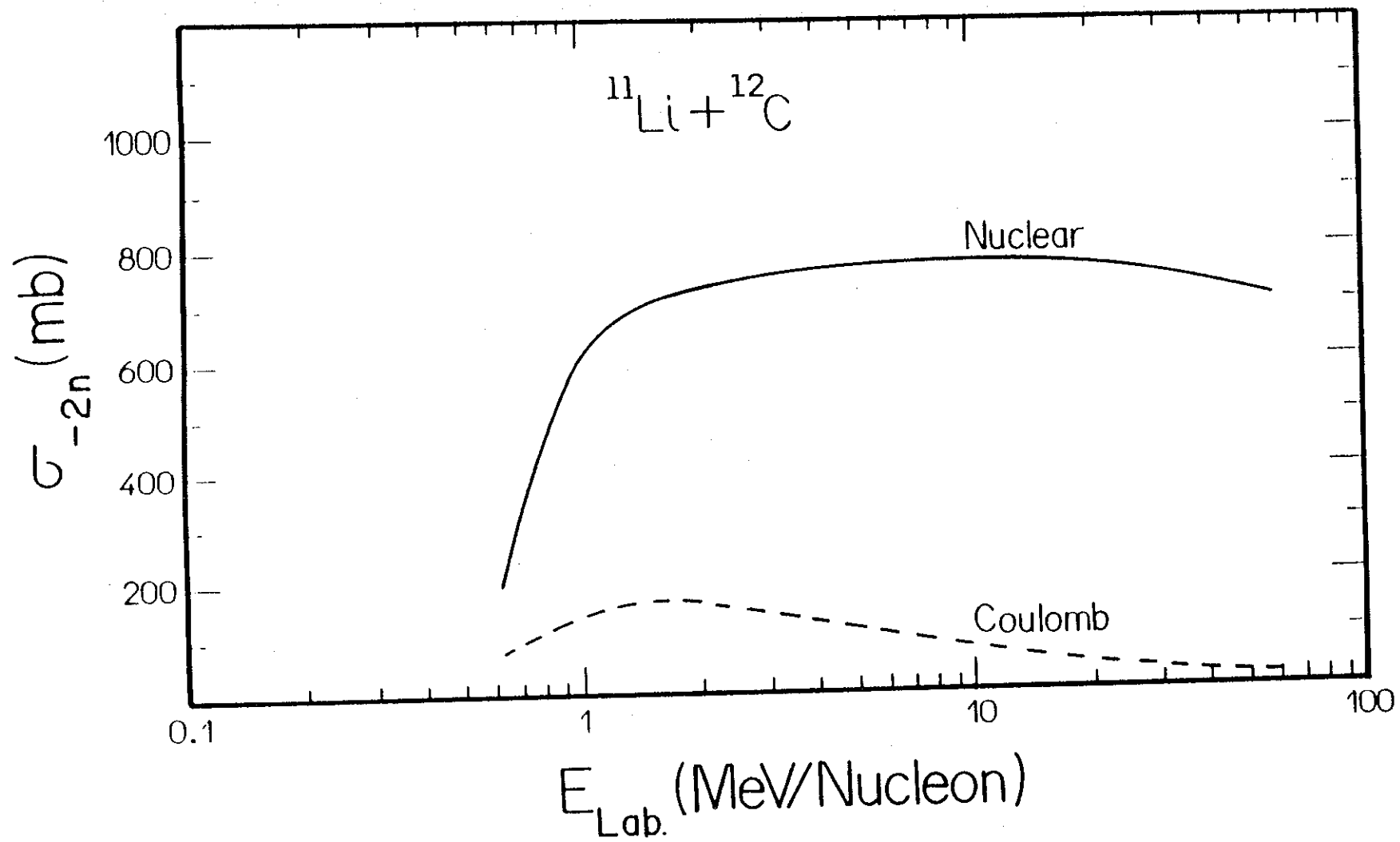


Fig. 2

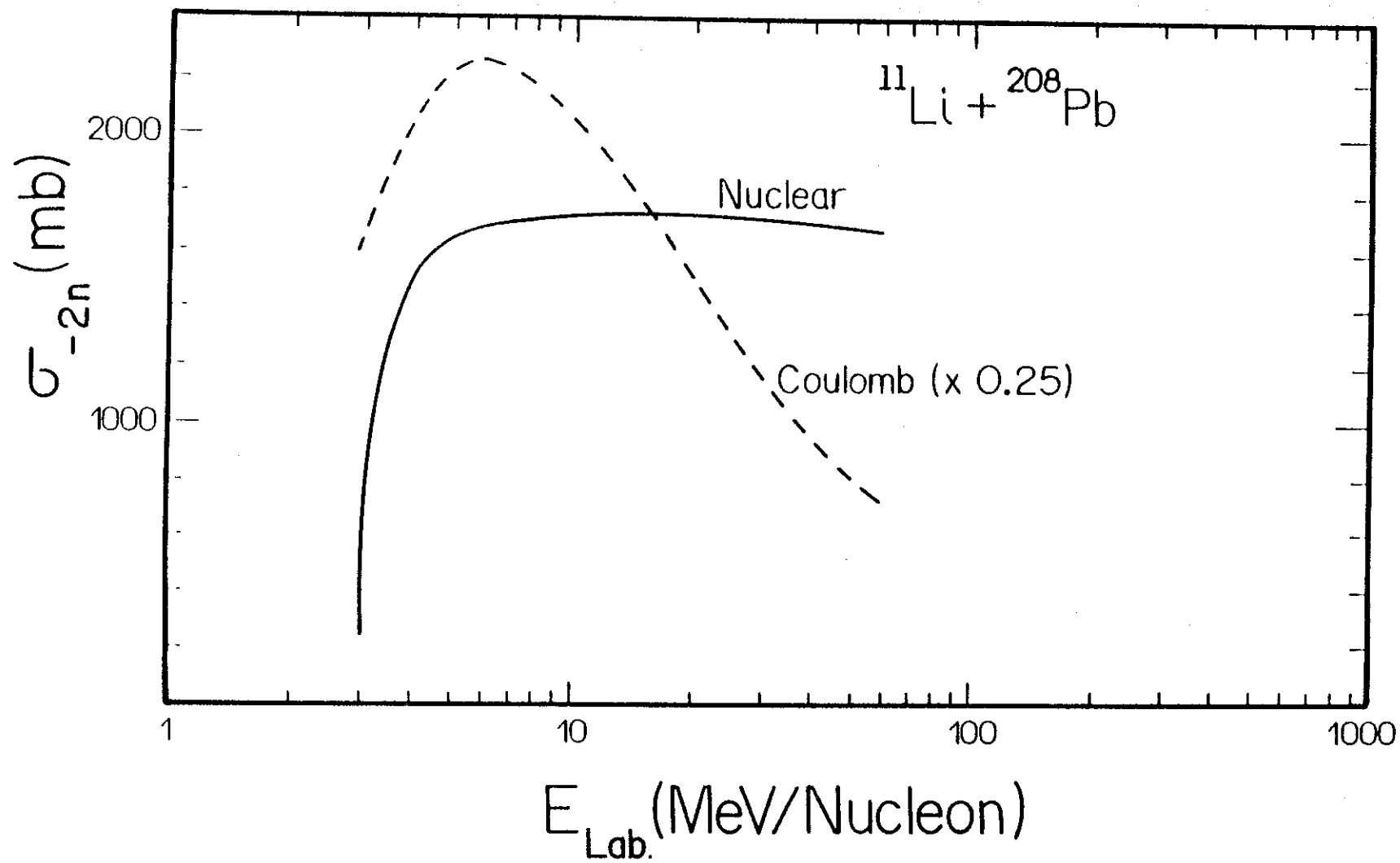


Fig. 2