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THE "NAIBEA" ACCELERATOR: AN IMPROVED INVERSE FREE ELECTRON LASER

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THE "NAIBEA" ACCELERATOR: AN IMPROVED INVERSE FREE ELECTRON LASER*

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ABSTRACT

The basic principles of linearly accelerating electrons with the aid of a combined laser and static $E\&M$ field are discussed. The NAIBEA accelerator is then detailed. Predictions are made for future developments in the field of relativistic particle accelerators.

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I. INTRODUCTION

It has been a common practice to design particle accelerators using the concept of *gradiant* (G). G-accelerators, however, grow in size quite fast with the increase in the desired final energy of the particle. An example of this is the planned SSC which aims at proton CM energy of 40 TeV and has an overall circumference of 54 miles! It is therefore of great interest, for future research in nuclear and particle physics, to develop alternative means of accelerating particles, using smaller machines[1)].

Recently we proposed a method of linearly accelerating electrons with the aid of a powerful laser coupled with a variable static electric field[2)]. A variable static magnetic field can also be used[3)]. Our numerical calculation showed that the Lorentz factor, γ, grows almost linearly with the length of the accelerator tube, thus allowing reaching GeV energies with ~ 10m tubes! We coined the new machine as "NAIBEA".

The aim of this paper is to supply the full details summarized in our recent letter[2)]. In Section II we give some general discussion of laser accelerator. In Section III we lay out the equations that govern our accelerator. In Section IV we compare the NAIBEA with the Inverse Free Electron Laser (IFEL). In Section V we consider the motion of a relatistic electron in a static electric field. In Section VI we discuss the IBEA of Kawata[4)]. In Section VII we discuss in details the NAIBEA accelerator. In Section VIII we give several examples of NAIBEA. In Section IX we discuss several important effects related to NAIBEA. Finally, in Section X we present our conclusions.

II. GENERAL CONSIDERATIONS

The motivation for seeking alternatives to conventional acceleration concepts is clear. The available electric field usually employed to accelerate particles is about 10 MeV/m.

Thus to reach e.g. the CEBAF energy (4 GeV) one needs a tube of about at 400 meters in length (reduced when using superconductivity).

It is clear that to reach higher energies bigger and bigger machines are required and one naturally starts seeking alternatives to the conventional concept.

A laser with a power of say $P(W/\text{cm}^2)$ supplies an electric field intensity of

$$e\,E_{\text{laser}} = \left[\frac{4\pi\, e^2}{c}\, P\right]^{1/2} \quad . \tag{1}$$

If we take for P, say, 10^{16} W/cm^2, we obtain

$$e\,E_{\text{laser}} \cong 2.0 \times 10^5 \ \text{MeV/m} = 0.2 \ \text{TeV/m} \quad . \tag{2}$$

If one were to utilize even as little as 1% of the electric field intensity of the laser, one would end up with an accelerator which is 100 times smaller than the conventional ones. The problem one faces here in that direction of the electric field, to which the particle velocity must couple, is perpendicular to the direction of propagation of both the laser (the Poynting vector) and eventually the particle.

To be able to accelerate the particle along the laser Poynting vector, one must have a very small component of the particle velocity along the laser electric field and be sure to have the particle motion transverse to the laser Poynting vector well contained: oscillatory. The applied alternating static electric or magnetic field is the needed degree of freedom to guarantee a transverse oscillatory motion of the particle which must be within the transverse extension of the laser pulse.

The above considerations allow us to make some useful estimates.

Let me call the wavelength of the laser λ_0. The wavelength seen in the particle rest frame is obtained from Doppler shift argument to be

$$\lambda = \lambda_0 \left(\frac{1+\beta_0}{1-\beta_0}\right)^{1/2} \quad , \qquad \beta_0 = \frac{v_0}{c} \tag{3}$$

where v_0 is the initial velocity of the particle. The velocity of the particle in the laboratory is γv_0, $\gamma = \sqrt{1-\beta_0^2}$. Thus the distance in the Lab. traveled by the particle during a time lapse of $\dfrac{\lambda}{c}$ is

$$\Delta Z = \frac{\lambda}{c}\,\gamma\, v_0 = \lambda_0\, \frac{\beta_0}{1-\beta_0} \quad . \tag{4}$$

Therefore if λ_0 is several microns, ΔZ could be macroscopic (several tens of cm's) if β_0 is close enough to unity (relativistic injected particles). The above observations allow the laser accelerator to be macroscopic.

If, for simplicity, we take ΔZ to be $n\,\lambda$, where n is the number of cycles within the laser pulse (in fact, because of the acceleration, $\Delta Z > n\,\lambda$), and call the diameter of the laser d_0 (which is *not* Doppler shifted), then after $n\,\lambda$ encounters with the laser, the particle will have traversed (through the action of the applied alternating static field) the laser n times. The gain in energy, $\Delta\varepsilon$, of the particle after traveling a distance of ΔZ is then given by

$$\Delta\varepsilon = e\,E_{\text{laser}}(n\,d_0) \quad .$$

To be able to make sensible comparison with conventional accelerators, it is useful to introduce an effective laser electric field intensity E_{eff}, such that

$$\Delta\varepsilon = e\,E_{\text{eff}}(\Delta Z) \quad . \tag{5}$$

We therefore obtain

$$e\,E_{\text{eff}} = e\,E_{\text{laser}}\,\frac{n\,d_0}{\Delta Z}$$

or

$$e\;E_{\text{eff}} = e\,E_{\text{laser}} \left(\frac{1-\beta_0}{\beta_0}\right)\left(\frac{d_0}{\lambda_0}\right) \tag{6}$$

If we take e.g. $d_0 = 1$ mm, $\lambda_0 = 10^{-2}$ mm, $\beta_0 = 0.9999$, then, for the laser of Eq. (1), we find

$$e\,E_{\text{eff}} = 2\ \text{GeV/m} \quad .$$

More realistic calculation, to be described below, gives a smaller value for $e\,E_{\text{eff}}$.

Thus, we can say that Eq. (7) supplies an *upper* limit to the effective laser electric field intensity. Multiplying this intensity by the tube length supplies us with an upper limit to the gain in energy. It is an upper limit since: a) the distance $n\,\lambda$ is much smaller than the actual distance travelled by the particle during n encounters with the laser, and b) the laser field intensity E_{laser} has its maximum value in the center of the laser beam. It decays in the transversal plane, usually, as $\exp\left[-|x^2+y^2|/w_0^2\right]$, where w_0 is the spot size of the presumed Gaussian beam[5]. We now turn to a general formulation of NAIBEA[2].

III. THE EQUATIONS OF MOTION

The starting point of our analysis is the Lagrangian of the electron EM field

$$\mathcal{L} = -mc^2\sqrt{1-\frac{v^2}{c^2}} + \frac{e}{c}\,\vec{v}\cdot\vec{A} - e\,\Phi \tag{7}$$

where $\vec{v} = (v_x, v_y, v_z)$ is the velocity vector $\vec{A}$ and Φ are the vector and scalar EM fields, respectively. From (1) we can immediately derive the equations of motion of the electron (see figure (1))

$$\frac{dp_z}{dt} = \frac{e}{c}\left(\vec{v}\times\vec{B}\right)_x \tag{8}$$

$$\frac{dp_y}{dt} = e\,E_y + \frac{e}{c}\left(\vec{v}\times\vec{B}\right)_y \tag{9}$$

$$\frac{dp_x}{dt} = 0 \tag{10}$$

$$E_y = -E_{y_0}\sin\varphi + E^{\text{app}} \tag{11}$$

$$\left(\vec{v}\times\vec{B}\right)_z = v_y\,B_z = +v_y\,E_{y_0}\sin\varphi \tag{12}$$

$$\left(\vec{v}\times\vec{B}\right)_y = v_x\,B_z = +v_x\,E_{y_0}\sin\varphi \quad . \tag{13}$$

The momenta, $p_i = m\,\gamma\,v_i$ where γ is the Lorentz factor $\left(1-\frac{v^2}{c^2}\right)^{-1/2}$. Equations (2) and (3) can be rewritten as

$$\frac{dp_x}{dt} = +e\,\beta_y\,E_{y_0}\sin\varphi \tag{14}$$

$$\frac{dp_y}{dt} = e\,(1-\beta_x)\,E_{y_0}\sin\varphi - e\,E_{\text{app}} \tag{15}$$

where $\beta_i = \frac{v_i}{c}$ and φ is the phase of the laser wave given by

$$\varphi = k(ct - x) \quad . \tag{16}$$

Equations (8) and (9) are fully relativistic. The rate of change of energy is given by

$$\frac{d\varepsilon}{dt} = e\,v_y\,E_y \quad . \tag{17}$$

In the absence of the applied field, the equations of motion are easily solved. For $E_y = E_{y_0}\sin\varphi$,

$$\frac{d\varepsilon}{dt} = e\,v_y\,E_{y_0}\sin\varphi \tag{18}$$

changing the variable from t to φ, we get for Eq. (9) ($E_{\text{app}} = 0$)

$$\frac{dp_y}{d\varphi} = \frac{e\,E_{y_0}}{kc}\sin\varphi \tag{19}$$

Thus

$$p_y = p_y(0) + \frac{e}{kc}\,E_{y_0}(1-\cos\varphi) \equiv m\gamma c\,\beta_y = mc\,\frac{\beta_y}{\sqrt{1-\beta_x^2-\beta_y^2}} \quad . \tag{20}$$

Also

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon}{d\varphi}(kc)(1-\beta_x) \tag{21}$$

$$\frac{d\varepsilon}{d\varphi} = \frac{1}{kc(1-\beta_x)}\frac{d\varepsilon}{dt} = \frac{e\,E_{y_0}}{k}\,\frac{\beta_y}{1-\beta_x}\,\sin\varphi = \frac{e\,E_{y_0}}{2\pi}\left(\frac{\lambda}{1-\beta_x}\right)\beta_y\,\sin\varphi \tag{22}$$

$$\frac{dp_z}{d\varphi} = \frac{e\,E_{y_0}}{kc}\,\frac{\beta_y}{(1-\beta_x)}\,\sin\varphi \quad . \tag{23}$$

Thus $\varepsilon - p_z\,c \equiv u_0$ is a constant and given by $\gamma(0)\,mc^2(1-\beta_x(0))$. Therefore

$$\frac{d\varepsilon}{d\varphi} = \frac{e\,E_{y_0}}{k}\,\frac{p_y\,c}{u_0}\,\sin\varphi \quad , \qquad \text{or}$$

$$\frac{d\varepsilon}{d\varphi} = \frac{e\,E_{y_0}}{ku_0}\left[p_y(0)c\,\sin\varphi + \frac{e}{k}\,E_{y_0}(1-\cos\varphi)\,\sin\varphi\right] \quad . \tag{24}$$

Upon integration, we find

$$\varepsilon(\varphi) = \varepsilon(0) + \frac{e\,E_{y_0}}{ku_0}\left[\left(p_y(0)c + \frac{e}{k}\,E_{y_0}\right)\,(1-\cos\varphi)\,\frac{e\,E_{y_0}}{2k}\,(1-\sin^2\varphi)\right] \quad . \tag{25}$$

Thus, as a function of φ, ε oscillates. At $\varphi = \pi$, $\varepsilon(\pi)$ is given by

$$\varepsilon(\pi) = \varepsilon(0) + \frac{2e\,E_{y_0}}{ku_0}\left(p_y(0)c + \frac{e}{k}\,E_{y_0}\right) \quad . \tag{26}$$

Thus the maximum energy gain by the particle during a half wave length encounter with the laser is

$$\Delta\varepsilon = \frac{2e\,E_{y_0}\,\lambda}{2\pi\,\gamma(0)(1-\beta_x(0))\,mc^2}\left(p_y(0)c + \frac{e\,E_{y_0}}{2\pi}\,\lambda\right) \tag{27}$$

Now

$$\begin{aligned} p_y^2(0)c^2 &= \gamma^2(0)\,m^2c^4 - p_z^2(0)\,c^2 \\ &= \varepsilon^2(0) - p_z^2(0)\,c^2 \\ &\equiv (\varepsilon(0) + p_z(0)\,c)\,u_0 \;\approx\; 2\varepsilon(0)\,u_0 \quad . \end{aligned} \tag{28}$$

Thus

$$\Delta\varepsilon = \frac{2e\,E_{y_0}\,\lambda}{2\pi u_0}\left(\sqrt{2}\,\varepsilon^{1/2}(0)\,u_0^{1/2} + \frac{e\,E_{y_0}\,\lambda}{2\pi}\right) \quad . \tag{29}$$

Therefore we have approximately

$$\Delta\varepsilon \sim 2\,\frac{\left(e\,E_{y_0}\,\lambda\right)}{(2\pi)\,u_0^{1/2}}\,\sqrt{2}\,\varepsilon^{1/2}(0) \quad . \tag{30}$$

It is clear that for a laser with $e\,E_{y_0} \sim 10^3$ MeV/m, $\lambda \sim 10^5$ m, $\beta_0 \sim 0.9999$, $\gamma_0 = 70$, get $u_0 = 3.5\times10^{-3}\,\varepsilon_0 = 35$ MeV, $2e\,E_{y_0}\,\lambda/2\pi = 0.5$, get

$$\Delta\varepsilon \sim 110 \text{ MeV} \quad .$$

The distance travelled by the particle during a $\frac{\lambda}{2}$ encounter with the laser is $\sim \frac{\lambda}{2(1-\beta_x)} = \frac{\lambda\,\gamma_0\,mc^2}{u_0} = \frac{\lambda\varepsilon_0}{u_0} = \frac{10^{-5}\cdot 35}{3.5\times10^{-3}} = 10^{-1}\,\text{m} = 10\,\text{cm}$. Thus the particle energy is increased from 35 MeV to 145 MeV within a distance of 10 cm. Of course, if y is contained somehow, this energy increase is given back to the laser during the next $\frac{\lambda}{2}$ encounter. The question we consider now is how to make the particle continue gaining energy from the laser.

IV. IMPROVED INVERSE FREE ELECTRON LASER

In order to guarantee that the particle, during its interaction with the laser, continues gaining energy, we have to insist that (see Eq. (17))

$$v_y\,E_y > 0 \quad . \tag{31}$$

Without actually entering into the details of what the applied field should be, we can, at least, predict the general characteristics of a laser machine, by rigorously imposing condition (25). Thus we use for v_y, the following form:

$$v_y = v_y^{\max}\,\cos\frac{\varphi}{2}\,\Theta\,(\pi-\varphi) + v_y^{\max}\,\sin\varphi\,\Theta\,(\varphi-\pi) \quad . \tag{32}$$

Eq. (26) says that at $\varphi = 0$, the starting point, $v_y = v_y^{\max}$. At larger values of φ, v_y behaves in exactly the same manner as E_y, and then makes $\dfrac{d\varepsilon}{dt} \propto \sin^2\varphi > 0$.

For $\varphi > \pi$, the solution for y is immediate

$$v_y = \dot{y} = v_y^{\max}\sin\varphi$$

$$\frac{dy}{d\varphi} = v_y^{\max}\frac{\sin\varphi}{\frac{d\varphi}{dt}} = \beta_y^{\max}\frac{\lambda}{2\pi}\frac{\sin\varphi}{1-\beta_z(\varphi)} \quad . \tag{33}$$

Now since

$$\frac{dt}{c\varphi} = \frac{\lambda}{2\pi}\left(\frac{1}{1-\beta_z} - 1\right) \quad , \tag{34}$$

we have

$$\frac{dy}{d\varphi} = \beta_y^{\max}\left(\frac{dz}{d\varphi} + \frac{\lambda}{2\pi}\right)\sin\varphi$$

$$\approx \beta_y^{\max}\frac{dz}{d\varphi}\sin\varphi \quad , \tag{35}$$

and thus, on the average,

$$y \cong \beta_y^{\max}[z\sin\varphi] \quad . \tag{36}$$

The energy equation becomes

$$\frac{d\varepsilon}{dt} = e\,v_y^{\max}E_{y_0}\sin^2\varphi \quad . \tag{37}$$

Then

$$\frac{d\varepsilon}{d\varphi} \cong \beta_y^{\max}\frac{e\,E_{y_0}}{2\pi}\lambda\sin^2\varphi\frac{dz}{d\varphi} \tag{38}$$

$$\varepsilon(\varphi) \cong \varepsilon(0) + \beta_y^{\max}\left(\frac{e\,E_{y_0}}{2}\right)z(\varphi) \quad . \tag{39}$$

Finally, $z(\varphi)$ is obtained easily from Eq. (28)

$$z(\varphi) = \frac{\gamma_0^2\overline{\beta_y^2}}{1+\gamma_0^2\overline{\beta_y^2}}\frac{\lambda}{\pi\overline{\beta_y^2}}\varphi \quad . \tag{40}$$

Then $\varepsilon(\varphi)$ varies *linearly* with z, z varies linearly with φ, and y oscillates around zero with an amplitude of $\beta_y^{\max}z$. Notice the major difference between the result for $\varepsilon(\varphi)$ above and that of the inverse free electron laser (IFEL). Whereas Eq. (33) indicates that $\dfrac{d\varepsilon}{dz}$ is constant and is given by

$$\frac{d\varepsilon}{dz} = \beta_y^{\max}\frac{e\,E_{y_0}}{2} \approx \frac{\gamma_0^2-1}{\gamma_0^2}\frac{e\,E_{y_0}\theta_0}{2} \tag{41}$$

where θ_0 is the injection angle, assumed very small, the IFEL is characterized by the following equation[6)]

$$\frac{d\varepsilon}{dz} = e\,E_{y_0}\frac{K}{\gamma(z)} \quad , \qquad K \text{ is a constant} \quad . \tag{42}$$

Thus, Eq. (35) results in a continuous constant energy gain, whereas (36) implies a gain which decreases with increasing ε (or γ). Note that Eq. (36) can be integrated to yield the dependence of ε on z of the IFEL,

$$\varepsilon^2(z) = \varepsilon^2(0) + e\,E_{y_0}mc^2Kz \quad . \tag{43}$$

which is to be compared to our Eq. (33),

$$\varepsilon(z) = \varepsilon(0) + \frac{1}{2}e\,E_{y_0}\beta_y^{\max}z \quad .$$

Clearly, a price is paid to get a greater acceleration using the (NAIBEA) mechanism, to be described later, than the IFEL. The transversal motion of the particle is given, according to the former by Eq. (30)

$$y = \beta_y^{\max}z\sin\varphi \quad ,$$

to be compared to the IFEL, which yields a more focused y-motion. However, we feel that if the y-motion expounded upon here, Eq. (30), is realizable and thus Eq. (35), the focusing problem can be solved with conventional techniques.

V. THE APPLIED FIELD ALONE

In order to understand better the NAIBEA mechanism of acceleration, we present in this section the fully relativistic case of the interaction of a charge with a static electric field. This problem is treated in Landau and Lifshitz[7] and the results are reproduced here. The equations of motion are $(\vec{E}_{\rm app} = \vec{E}_{\rm app}\,\hat{j})$

$$\dot{p}_x = 0 \quad , \quad \dot{p}_y = e\,E_{\rm app} \quad . \tag{44}$$

Thus

$$p_x = p_x(0) \quad , \quad p_y = p_y(0) + e\,E_{\rm app}\,t \quad . \tag{45}$$

The energy is given by

$$\begin{aligned} \varepsilon &= \sqrt{m^2c^4 + c^2\,p_x^2(0) + c^2\,(p_y(0) + e\,E_{\rm app}\,t)^2} \\ &= \left[m^2c^4 + c^2\,p^2(0) + 2c^2\,e\,E_{\rm app}\,t\,p_y(0) + c^2\,(e\,E_{\rm app}\,t)^2\right]^{1/2} \quad . \end{aligned} \tag{46}$$

Thus

$$\varepsilon^2(t) \equiv \varepsilon^2(0) + 2c^2\,e\,E_{\rm app}\,t\,p_y(0) + c^2\,(e\,E_{\rm app}\,t)^2 \quad . \tag{47}$$

From the equations for the velocity components

$$v_x = \frac{p_x\,c^2}{\varepsilon} \tag{48}$$

$$v_y = \frac{p_y\,c^2}{\varepsilon} \quad . \tag{49}$$

We can integrate and find the trajectory solution

$$x(t) = \frac{p_x(0)\,c^2}{ec\,E_{\rm app}} \ln\left[\frac{2c\,e\,E_{\rm app}\,\varepsilon(t) + 2(c\,e\,E_{\rm app})^2\,t + 2c^2e\,E_{\rm app}p_y(0)}{2c\,e\,E_{\rm app}\,\varepsilon(0) + 2c^2e\,E_{\rm app}\,p_y(0)}\right] \tag{50}$$

$$\begin{aligned} y(t) &= \frac{e\,E_{\rm app}(\varepsilon(t) - \varepsilon(0))}{(e\,E_{\rm app})^2} - \left[\frac{2c^2\,e\,E_{\rm app}\,p_y(0)}{2(c\,e\,E_{\rm app})^2}\,e\,E_{\rm app} - p_y(0)\right]\frac{c}{e\,E_{\rm app}} \\ &\quad \ln\left[\frac{2c\,e\,E_{\rm app}\,\varepsilon(t) + 2(c\,e\,E_{\rm app})^2\,t + 2c^2e\,E_{\rm app}p_y(0)}{2c\,e\,E_{\rm app}\,\varepsilon(0) + 2c^2e\,E_{\rm app}\,p_y(0)}\right] \quad . \end{aligned} \tag{51}$$

The above equations represent a parabolic trajectory in the x-y plane. To see this more clearly, we take $p_y(0) = 0$. Then

$$x = \frac{p(0)\,c}{e\,E_{\rm app}}\,\sinh^{-1}\left(\frac{c\,e\,E_{\rm app}\,t}{\varepsilon(0)}\right) \tag{52}$$

and

$$y = \frac{1}{e\,E_{\rm app}}\,(\varepsilon(t) - \varepsilon(0)) \quad . \tag{53}$$

The above two equations can be combined to yield finally

$$y = \frac{\varepsilon(0)}{e\,E_{\rm app}}\left\{\left[\cosh\left(\frac{e\,E_{\rm app}}{p(0)\,c}\right)x\right] - 1\right\} \quad . \tag{54}$$

For small x, we have

$$y \simeq \frac{\varepsilon(0)(e\,E_{\rm app})}{2(p(0)\,c)^2}\,x^2 \quad . \tag{55}$$

It is clear that we have above is a free fall motion (parabolic) generalized to the relativistic regime (Eqs. (44) and (45)). We now turn to the coupled static field plus laser problem.

VI. THE INVERSE-BREMSSTRAHLUNG ELECTRON ACCELERATING PROBLEM

We consider now the case discussed by Kawata et al.[4]. An electron moving in the field of a plane electromagnetic (EM) wave which travels across a static electric field as shown in figure (1). The magnetic field (B_z) of the wave is in the x-z plane and the electric one (E_y) is in the z-y plane. The static electric field ($E_{\rm app}$) is applied in the $+y$ direction. The relativistic electron equation of motion is then given by

$$\frac{dp_z}{dt} = -e\,\beta_y\,E_y \tag{56}$$

$$\frac{dp_y}{dt} = e\,(1-\beta_z)\,E_y - e\,E_{\rm app} \tag{57}$$

where $\beta_z = \dfrac{v_z}{c}$, $\beta_y = \dfrac{v_y}{c}$ (c = velocity of light), $B_z = E_y = -E_{y_0}\sin\varphi$ and E_{y_0} and $\varphi = k(ct - x)$ are the amplitude and the phase of the EM wave. If we multiply these equations by the velocities v_x and v_y respectively and add then we obtain the energy equation

$$\frac{d}{dt}\,\varepsilon = -e\,E_y\,v_y \tag{58}$$

where $\varepsilon = mc^2\gamma + e\,E_{\rm app}\,y$ is the total mechanical energy and γ is the usual relativistic factor. From (50) and (51) we deduce

$$p_z\,c = mc^2\,\gamma + e\,E_{\rm app}\,y + K_1 \tag{59}$$

where K_1 is a constant determined by the initial conditions and from equation (51)

$$p_y = -\frac{e}{kc}\,E_{y_0}\cos\varphi - e\,E_{\rm app}\,t + K_2 \tag{60}$$

which K_2 fixed also by the initial conditions. These two first integrals of the movement may be used to reduce the system of dynamical equations to a one-dimensional problem as

we show now. First, let us change the independent variable from the time to the phase. To do this we observe that we have the relation

$$\dot\varphi = -\frac{k}{mc\gamma}\,(e\,E_{\rm app}\,y + K_1) \quad . \tag{61}$$

On the other hand, from the relativistic expression of the moment, $p_y = m\,\gamma\,v_y = m\,g\,\dot\varphi\,\dfrac{dy}{d\varphi}$, we find using the equation above

$$p_y = -\frac{k}{c}\,(e\,E_{\rm app}\,y + K_1)\,y' \tag{62}$$

where the prime denotes derivative with respect to the phase. This relation suggests that we introduce the new coordinate

$$Q = -\frac{k}{2c}\,(e\,E_{\rm app}\,y + 2\,K_1\,y) \tag{63}$$

such that $p_y = Q'$ and all the others variables may be expressed in terms of Q and p_y. Thus, using the identity $m^2c^4\gamma^2 = p^2c^2 + m^2c^4$ and Eq. (53) we have the expressions

$$mc^2\gamma = \frac{1}{2}\,\frac{m^2c^4 + p_y^2\,c^2}{\sqrt{K_1^2 - \dfrac{2c\,e\,E_{\rm app}\,Q}{k}}} + \frac{1}{2}\,\sqrt{K_1^2 - \frac{2\,c\,e\,E_{\rm app}\,Q}{k}} \quad , \tag{64}$$

$$p_z\,c = \frac{1}{2}\,\frac{m^2c^4 + p_y^2\,c^2}{\sqrt{K_1^2 - \dfrac{2c\,e\,E_{\rm app}\,Q}{k}}} - \frac{1}{2}\,\sqrt{K_1^2 - \frac{2\,c\,e\,E_{\rm app}\,Q}{k}} \quad . \tag{65}$$

From (4)

$$t = \frac{1}{e\,E_{\rm app}}\left(-p_y - \frac{e}{kc}\,E_{y_0}\cos\varphi + K_2\right) \tag{66}$$

and

$$s = \frac{1}{k}\,(kct - \varphi) \quad . \tag{67}$$

Finally, inverting (57)

$$y = -\frac{1}{e\,E_{\rm app}}\left(\sqrt{K_1^2 - \frac{2\,c\,e\,E_{\rm app}}{k}Q} + K_1\right) \quad . \tag{68}$$

Let us now derive the dynamical equation of motion for Q. Taking the derivative with respect to the phase of (54) and using (55) and (58) we get

$$Q'' = -\frac{c}{2k}\,e\,E_{\rm app}\,\frac{m^2c^2 + Q'^2}{K_1^2 - \dfrac{2\,c\,e\,E_{\rm app}}{k}Q} - \frac{e\,E_{\rm app}}{2kc} + \frac{e}{kc}E_{y_0}\sin\varphi \tag{69}$$

which is a second order non-linear inhomogeneous equation. We try now to construct a solution of it in the form of an expansion in a small parameter. Before doing this, we rewrite the equation in terms of adimensional quantities, dividing it, by the product mc and defining the variable $q = \dfrac{Q}{mc}$ we obtain

$$q'' = -\frac{1}{4}\,\frac{1+q'^2}{K_0 - q} - \frac{1}{2}BG + B\sin\varphi \tag{70}$$

$K_0 = \dfrac{k\,K_1^2}{2mc^2\,e\,E_{\rm app}}$, $G = \dfrac{E_{\rm app}}{E_{y_0}}$ and $B = \dfrac{e\,E_{y_0}}{k\,mc^2}$. Taking now B as the small parameter we substitute in the equation q for the series

$$q = q_0 + Bq_1 + B^2q_2 + \cdots \tag{71}$$

and equating in both sides of (70) the coefficients of equal powers of B we obtain a series of coupled differential equations, namely,

$$q_0'' = \frac{1}{4}\,\frac{1+{q_0'}^2}{q_0 - K_0} \quad , \tag{72}$$

$$q_1'' + \frac{q_0'}{2(K_0-q_0)}\,q_1' + \frac{1+{q_0'}^2}{4(k_0-q_0)^2}\,q_1 = -\frac{1}{2}BG + B\sin\varphi \quad , \tag{73}$$

$$q_2'' + \frac{q_0'}{2(k_0-q_0}\,q_2' + \frac{1+{q_0'}^2}{4(k_0-q_0)^2}\,q_1 = \frac{1}{4}\,\frac{{q_1'}^2}{q_0-k_0} - \frac{q_0'\,q_1\,q_1'}{2(q_0-k_0)^2} + \frac{(1+{q_0'}^2)\,q_1^2}{4(q_0-k_0)^3} \tag{74}$$

and so on.

We see that the zeroth order term satisfies a non-linear second order equation while all the others higher order terms satisfy a linear second order equation which differ only for the non-homogeneity. As the non-linear equation may be integrated the problem is therefore solved.

To find the solution of Eq. (72) we write it as

$$q_0'' + \frac{1}{4}\,\frac{q_0^2}{k_0-q_0} = -\frac{1}{4}\,\frac{1}{k_0-q_0} \tag{75}$$

where the l.h.s. may be immediately integrated giving

$$(k_0-q_0)^{\frac{1}{4}}\,\frac{d}{d\varphi}\left[\frac{q_0'}{(k_0-q_0)^{\frac{1}{4}}}\right] = -\frac{1}{4}\,\frac{1}{k_0-q_0} \quad . \tag{76}$$

Taking as independent variable q_0 instead of φ we can rewrite (76) as

$$\frac{d}{dq_0}\left[\frac{q_0'}{(k_0-q_0)^{\frac{1}{4}}}\right] = -\frac{1}{2}\,\frac{1}{(k_0-q_0)^{\frac{3}{2}}} \tag{77}$$

which may be integrated to give

$$\frac{{q_0'}^2}{(k_0-q_0)^{\frac{1}{2}}} = \frac{1}{(k_0-q_0)^{\frac{1}{2}}} + K \tag{78}$$

where K is constant fixed by the initial values of q_0 and q_0'. At this point we have to discuss how to impose the initial conditions on the terms of the expansion of q. Usually the zeroth order term of the expansion carries all the information about the beginning of the motion. Nevertheless, here the first term, q_0, is independent of the EM wave, which appears as an inhomogeneity in the equation for q_1. Thus, in the expression for the p_y component of the moment, Eq. (62), the first term must be contained in the first order of the expansion, so, we put $q_1'(0) = -1$, $q_0'(0) = K_2/mc = k_2$ and $q_2'(0) = q_3'(0) = \cdots = 0$

and finally to all orders. $q_i(0) = 0$ since $q(0) = 0$. Using these values, we obtain for the constant K, in (72) the value

$$K = \frac{1 + (K_2/m)^2}{\sqrt{k_0}} \quad . \tag{79}$$

Going on, we have from (78)

$$q_0' = \pm \sqrt{K \sqrt{k_0 - q_0} - 1} \tag{80}$$

where the $(+)$ is to be taken in the interval from $\varphi = 0$ to $\varphi = \varphi_m$, which is the value when $q_0' = 0$, i.e., when the maximum value of q_0 is attained, after that, $q_0' < 0$. It may be easily verified, using (75) and (78), that the integration of (78) gives

$$\frac{4}{3}\left(K\sqrt{k_0 - q_0} - 1\right)^{\frac{3}{2}} + 4\left(K\sqrt{k_0 - q_0} - 1\right)^{\frac{1}{2}} = \mp k^2(\varphi - \varphi_m) \tag{81}$$

where $\varphi_m = \dfrac{4}{K^2}\left(\dfrac{k_2^3}{3} + k_2\right)$, and inverting this equation we find

$$q_0 = k_0 - \left(\frac{1 + v^2}{K}\right)^2 \tag{82}$$

with

$$v = \left(-\frac{a_0}{2} + \sqrt{\frac{a_0^2}{4} + 1}\right)^{\frac{1}{3}} - \left(\frac{a_0}{2} + \sqrt{\frac{a_0^2}{4} + 1}\right)^{\frac{1}{3}} \tag{83}$$

and

$$a_0 = \pm \frac{3}{4} K^2(\varphi - \varphi_m) \quad . \tag{84}$$

These equations determines completely the zeroth order term, q_0, of the expansion.

We show now that the solutions of the linear differential for the higher order terms of expansion may be explicitly written. To do this, we observe that $q_h^{(1)} = q_0'$ is a solution of the homogeneous equation

$$q_h'' + \frac{q_0'}{2(k_0 - q_0)}\, q_h' + \frac{1 + q_0'^2}{4(k_0 - q_0)^2}\, q_h = 0 \tag{85}$$

as is readily verified. Since $\omega = \sqrt{k_0 - q_0}$ is the root of this equation, the other linearly independent solution of (85) is found to be $q_h^{(2)} = \frac{1}{3} q_0'^4 + 2q_0'^2 - 1$. The solution of (73) satisfying the imposed initial conditions will then be

$$q_1 = -\frac{q_h^{(1)}(0)\, q_h^{(2)}(\varphi) - q_h^{(1)}(\varphi)\, q_h^{(2)}(0)}{\sqrt{k_0}} - \int_0^\infty \frac{q_h^{(1)}(\varphi)\, q_h^{(2)}(\varphi') - q_h^{(1)}(\varphi')\, q_h^{(2)}(\varphi)}{\omega\varphi'}\,(G - \sin\varphi')\, d\varphi' \tag{86}$$

and the solution of (74) is

$$q_2 = +\int_0^\infty \frac{q_h^{(1)}(\varphi)\, q_h^{(2)}(\varphi') - q_h^{(1)}(\varphi')\, q_h^{(2)}(\varphi)}{\omega\varphi'} \left[q_1'^2 + 2\,\frac{q_0' q_1 q_1'}{k_0 - q_0} + \frac{\left(1 + q_0'^2\right) q^2}{(k_0 - q_0)^2}\right] \frac{d\varphi'}{4(k_0 - q_0)} \tag{87}$$

In figure (2) we show the three first terms in the B-expansion vs. φ (Eq. (71)), for the system studied by Kawata et al.[4], namely the amplitude of the laser is $E_{y_0} = 0.1\, E_0$ where $E_0 = \dfrac{mc^2}{e\lambda/32} = \dfrac{1.636 \times 10^7}{\lambda}\left[\dfrac{V}{\text{cm}}\right]$, λ is the wavelength in cm and $E_{\text{app}}/E_0 = 4.28 \times 10^{-5}$. The laser power is $3.5 \times 10^{15}\, \dfrac{W}{\text{cm}^2}$ for $\lambda = 10\, \mu$m. The initial velocity $v_0 = 0.9999\, c$. The electron is injected at an angle of $0.608°$ with respect to the laser direction (along z).

To show the convergence of the B-expansion we show in figure (3) the total mechanical energy calculated with a numerical integration of Eq. (64) and using these three first terms of the B expansion. The values of the parameters used are the same as those of Kawata et al.[4] (see above).

VII. THE NON-LINEAR AMPLIFICATION OF INVERSE BREMSSTRAHLUNG ELECTRON ACCELERATION (NAIBEA) PRINCIPLE

The previous section dealt with the acceleration principle advanced by Kawata et al.[4]. We looked in great detail at the analytical numerical aspects of the problem of a coupled charge-static electric field-laser system. Net acceleration of the particle results because the static field forces the particle to change the sign of its velocity when the laser electric field changes sign such that RHS of the energy equation (52)

$$\frac{d\varepsilon}{dt} = -e\,E_y\,v_y = |e|\,E_y\,v_y$$

is kept with the same positive sign over a cycle. After that, the electron energy oscillates around the maximally attained one. We now turn to an important generalization of the Kawata et al. model, that was developed by us and reported earlier in ref. 2). Namely we make the static electric field polarity to change at several optimally determined places, then keeping the sign of $\frac{d\varepsilon}{dt}$ positive over many cycles. The net gain in electron energy could be made as large as one wishes.

The starting point of our analysis is the energy and trajectory equations of section V

$$Q'' = -\frac{c}{2k}\,e\,E_{\rm app}\,\frac{m^2c^2 + Q'^2}{K_1^2 - \frac{2c}{k}\,e\,E_{\rm app}\,Q} - \frac{e\,E_{\rm app}}{2kc} + \frac{\epsilon}{kc}\,E_{y_0}\sin\varphi \tag{88}$$

and

$$\varepsilon' = \frac{c\,\epsilon\,E_{y_0}}{k}\,\frac{1}{\left(K_1^2 - \frac{2c\,\epsilon\,E_{\rm app}\,Q}{k}\right)^{1/2}}\,Q'\sin\varphi\ . \tag{89}$$

Equations (88) and (89) are the Inverse-Bremsstrahlung Electron Acceleration (IBEA) equations of the previous section. Equation (82) is a second order non-linear inhomogeneous one

that determines the trajectory of the particle since x and y are explicitly given in term of Q as

$$x = \frac{1}{k}\left[kc\,\frac{1}{\epsilon\,E_{\rm app}}\left(-Q' - \frac{\epsilon}{kc}\,E_{y_0}\cos\varphi + K_2\right) - \varphi\right] \tag{90}$$

$$y = -\frac{1}{\epsilon\,E_{\rm app}}\left[\left(K_1^2 - \frac{2c\,\epsilon\,E_{\rm app}\,Q}{k}\right)^{1/2} + K_1\right]\ . \tag{91}$$

The acceleration equation, Eq. (89), is easily solved once a first integration of (88) is done. The important feature to be emphasized here is that ε' is proportional to $Q'\sin\varphi$.

Before discussing the solution of Eq. (89), we analyse the expected behaviour of the energy as a function of φ (or t). Clearly whenever $\varphi = n\pi$, ε' is zero (ε is maximum or minimum). If, say, at $\varphi = \pi$, $Q' = p_y$ is also zero, then ε is at an inflection point. $\varepsilon'' = 0$. This behaviour is commonly referred to as fold catastrophe, according to the classification of Thom[8]. This catastrophe also characterizes the phenomenon of rainbow. This behaviour is shown in figure (4), where a case similar to that of Ref. 4 is considered (see paragraph after Eq. (87)). The figure exhibits $\varepsilon(\varphi)$ and $Q(\varphi)$. The inflection point alluded to above is clearly shown (indicated by the arrow). At later times ε reaches a maximum at $\varphi = 2\pi$ and then just oscillates along with the wave. If the applied field is reversed at $3\pi/2$, the original maximum in ε at $\varphi = 2\pi$ becomes an inflection point and the particle energy is then pushed up to another maximum at 3π after which the oscillation set in again. The net gain in energy after the first kick is about 300% whereas after the second kick is 600%. Thus one can double the gain by reversing the direction of the applied electric field at the appropriate time (or x). In figure (5) we show $\dot{\varepsilon}(t)$ vs. t which exhibits the stair structure of the acceleration quite clearly.

The mechanism responsible for this doubling of the gain in the particle energy is governed by non-linear equations. We therefore coin it the Non-linear Amplification of Inverse-

Bremsstrahlung Electron Acceleration (NAIBEA). Clearly the NAIBEA can be repeated several times by merely alternating the sign of the applied field at the appropriate phases of the wave. A simple estimate of the net gain in energy after the elapse of $n\pi$ in φ, with accompanying changes in the sign of $E_{\rm app}$ is $\sim n(\Delta\varepsilon)$ where $\Delta\varepsilon$ is the gain after the first kick (in our case $\Delta\varepsilon$ is about 150 MeV). The way to accomplish this is by arranging an array of $E_{\rm app}$ with interchanging signs at appropriate positions along the x-direction. These locations are obtained from Eq. (90). The position x_1 at which the first inflection point in ε occurs is

$$\begin{aligned} x_1 &= \frac{E_{y_0}}{\pi E_{\rm app}}\lambda + \frac{c\,p_y(0)}{e\,E_{\rm app}} - \frac{\lambda}{2} \\ &\simeq 0.11\,{\rm m} \quad . \end{aligned} \tag{92}$$

If the applied field is reversed at $\varphi = 3\pi/2$, which corresponds to the position x_1^r,

$$\begin{aligned} x_1^r &= \frac{1}{k}\left[\frac{kc}{e\,E_{\rm app}}\left(-Q'_{3\pi/2} + p_y(0) + \frac{e}{kc}E_{y_0}\right) - \frac{3\pi}{2}\right] \\ &\simeq 0.243\,{\rm m} \quad , \end{aligned} \tag{93}$$

then the position of the second kick or inflection point is

$$\begin{aligned} x_2 &= x_1 + \frac{2|Q'_{3\pi/2}|c}{e\,E_{\rm app}} - \frac{\lambda}{2} \\ &\simeq x_1 + 0.365\,{\rm m} = 0.474\,{\rm m} \quad . \end{aligned} \tag{94}$$

It is easy to show that the position of the $n^{\rm th}$ reversing of the static field is given by

$$x_n^r = \frac{K_2^{(n-1)} + \left|Q'_{\frac{2n+1}{2}\pi}\right|}{e\,E_{\rm app}} - \frac{2n+1}{2k}\pi \tag{95}$$

where $K_2^{(n-1)}$ is given by

$$K_2^{(n-1)} = K_2^{(n-2)} + 2\,Q'_{\frac{2n+1}{2}\pi} \tag{96}$$

and corresponding following kick

$$x_{n+1} = x_n + \frac{2\,Q'_{\frac{2n+1}{2}\pi}}{2E_{\rm app}} - \frac{\lambda}{2} \quad . \tag{97}$$

The values of Q' can be calculated from the trajectory equation, Eq. (88). We mention that the trajectory of the electrons accelerated with the NAIBEA mechanism is well behaved. Figure (6) shows a typical case. The dispersion (oscillation) along the y direction is quite small. Before turning to specific examples we mention that the trajectory variable Q, can be generalized to a vector whose definition in general is[3)]

$$\frac{d\vec{Q}}{d\varphi} = -\vec{A} \tag{98}$$

where $\vec{A}$ is the total vector potential which is written as

$$\vec{A} = \vec{A}_{\rm applied} + \vec{A}_{\rm laser} \quad . \tag{99}$$

Since $\vec{A}$ is defined to within an arbitrary constant, we have here full freedom in giving the electron nonzero initial value of p_x and/or p_y.

VIII. EXAMPLES OF NAIBEA

We turn now to specific choices of the accelerator. We consider a linearly polarized pulse with $\vec{A}$ taken to be along the y direction. We first consider a constant applied electric field, $E_{\rm app}$, then

$$\vec{A} = \left[A_y^0(\varphi) - E_{\rm app}\,t - p_y(0)\right]\hat{y} \quad . \tag{100}$$

With the $\vec{A}$ above used in Eqs. (20) and (21) we recover the NAIBEA equation of Ref. 2). Inversions of $E_{\rm app}$ are made at appropriate values of z to assure the validity of Eq. (31), namely $p_y(n\pi) = 0$. This means that the applied field is inverted at φ_j's such that $\left.\frac{dp_y(\varphi)}{d\varphi}\right|_{\varphi_j=\frac{n+1}{2}\pi} = 0$. We now replace $E_{\rm app}$ by a constant magnetic field along x. Then

$$\vec{A} = \left[A_y^{(0)}(\varphi) - B_{\rm app}\, z - p_y(0)\right] \hat{y} \quad . \tag{101}$$

The resulting NAIBEA equations are almost identical to those of Ref. 2) except for a change in sign of th second term in Eq. (8) of that reference (with $E_{\rm app}$ replaced by $B_{\rm app}$). Further, Eq. (13) of Ref. 3) reduces to

$$u^2 = u_0^2 - 2\, B_{\rm app}\, Q \quad . \tag{102}$$

The y-component of the momentum is given by (Eq. (18))

$$p_y(\varphi) = p_y(0) - A_y^{(0)}(\varphi) + B_{\rm app}\, z \quad . \tag{103}$$

The continuous acceleration of the particle results if $B_{\rm app}$ is chosen so that $p_y(n\pi) = 0$. This requires changing the sign of $B_{\rm app}$ at appropriate places ($\varphi \simeq \frac{n+1}{2}\pi$).

We consider the following numerical example. The initial value of γ, $\gamma_0 = 70$, $P = \frac{3}{2}\, \nu_0^2\, 10^{15}$ W/cm^2 for $\lambda_0 = 10^{-3}$ cm. The parameter $\nu_0 = 1$ refers to the maximum value of electric field of the pulse in our units. We also take nine cycles within the pulse. The shape of the pulse is taken to be a Gaussian, $A^2 = \exp(-\varphi^2/\Delta^2)$, with $\Delta = 3\pi$ (see figure (7)). The applied field intensity is taken to be 2.34 teslas which corresponds to $\sim 5 \times 10^{-5}$ that of the laser. The electrons are injected at an angle of 0.6° with respect to the z-axis ($p_y(0) \approx \frac{0.6}{180}\pi\gamma_0$). We consider, as an example nine changes in the sign of the modulated applied magnetic field. In figure (8) we show the change of γ vs. z obtained by solving Eqs. (88) and (89), with $E_{\rm app}$ replaced by $-B_{\rm app}$. The gain in energy is a factor

of 35 over a distance (accelerator length) of seven meters! The accelerator length could be made smaller if full optimization is accomplished. It is interesting to calculate the effective field for this case. From Eqs. (1), (5) and (6) we obtain the value $e\,E_{\rm eff} \cong 150$ MeV/m.

The corresponding trajectory of the particle, confined in the (z,y)-plane is shown in figure (8). The arrows indicate the positions where the applied magnetic field is inverted. These positions (in meters) are given in table 1. Finally, in figure (10) we compare γ vs. z for a plane wave and the pulse considered above.

As a second example, we consider proton acceleration using our concept[2b]. We thus take for the initial energy of the protons, the value 0.5 TeV. This corresponds to an initial velocity of $v_0 = 0.999998$c. The laser power is taken here to be $P = 6.6 \times 10^{20}$ W/cm^2 with $\lambda_0 = 5 \times 10^{-3}$ cm. The laser electric field intensity is $e\,E_{\rm laser} = 49$ TeV/m. The initial injection angle is $\theta_0 = 0.02°$. A Gaussian laser pulse with the above maximum intensity is considered with a width of 3π. The applied magnetic field intensity was taken to be 10 teslas, which is $5.14 \times 10^{-7}\, E_{\rm laser}$. During the acceleration the sign of the applied field was changed 11 times. The result are shown in figure (11). The proton reaches an energy of 20 TeV, within a length of 3 km. The effective laser electric field intensity is found here to be $e\,E_{\rm eff} = 6.6$ GeV/m. The trajectory of the proton in the y-z plane is shown in figure (12). The arrows indicate the positions where the last seven inversion of the field are made. These positions as well as the first six of them are given in the Table 2 (in km). It is clear from Table 2 that there is an ample space along the tube to further fine tune the particle trajectory. The problem of laser + magnetic field acceleration was also considered in Refs. 9) and 10).

As a final remark we mention that our NAIBEA accelerator can be further optimized by better determining the values of the available fine tune variables: θ_0, $B_{\rm app}$, and the inversion positions. The final aim is to have a rather "small", very high energy accelerator with the lowest possible value of B_{app} for a given laser power. In the following sections, we assess the importance of a) field deformation in the space separating the capacitors for choice (100) and b) radiation damping.

IX.a. INTERACTION BETWEEN CAPACITORS

A fundamental part of the NAIBEA is the array of alternating static electric field. This can be supplied by what we generically call "capacitors". A question arises as to whether the deformation of the static field in the space between capacitors may be hazardous to the functioning of the accelerator. We analyse in this section this problem in detail, and show that for all practical purposes this problem is of minor importance.

Let us consider the two capacitor problem depicted in figure (13). The solution of the equation for the scalar potential

$$\Delta^2 \varphi = 0$$

can be obtained with the help of the usual boundary value conditions

$$\varphi(z, y) = -\frac{2V}{\pi} \int_0^\infty \frac{\sinh ky}{\sinh ka} \frac{\sin kz}{k} \, dk \quad . \tag{104}$$

Then we find for the z- and y-components of the electric field the following

$$E_z = -\frac{\partial}{\partial z} \varphi = \frac{2V}{\pi} \int_0^\infty \frac{\sin ky}{\sinh ka} \cos kz \, dk \tag{105}$$

$$E_y = -\frac{\partial}{\partial y} \varphi = \frac{2V}{\pi} \int_0^\infty \frac{\cosh ky}{\sinh ka} \sin kz \, dk \quad . \tag{106}$$

The field configurations in the y-z plane is shown in figure (14). It is clear from the figure that only in a very restricted space between the capacitors small inhomogeneity exists. At the center of the space between the capacitors the electric field is zero. Since the inflection

point that drives the NAIBEA occurs inside the capacitor, the deformation of static field in this restricted space is completely irrelevant since here the laser electric field is several orders of magnitude greater. Se the discussion in Section VI.

IX.b. RADIATION LOSS

So far in our discussion of the NAIBEA accelerator, we have not considered the loss of energy due to the oscillatory motion of the electron in the y-direction. This problem is a standard one and we use, in the following, the treatment of Jackson[11]. The rate of change of the energy of the electron, when considering radiation damping, is governed by the equation

$$\frac{d\varepsilon}{dt} = e\left\langle \vec{v} \cdot \vec{E} \right\rangle - \frac{2}{3} \frac{e^2}{m^2 c^3} \gamma^2 \left\langle \left(\frac{d\vec{p}}{dt} \right)^2 - mc^2 \left(\frac{d\gamma}{dt} \right)^2 \right\rangle \quad . \tag{107}$$

where the bracket $\langle\rangle$ indicates time averaging. The loss of energy is described by the second term on the RHS of Eq. (107). For the problem of NAIBEA, which is quite similar to the inverse free electron laser principle, where a wiggler field is present, Eq. (107) becomes

$$\frac{d\varepsilon}{dt} = +e\left\langle \vec{v} \cdot \vec{E} \right\rangle - \frac{2}{3} \frac{e^2}{m^2 c^3} \gamma^2 \left\langle \left(\frac{dp_y}{dt} \right) \right\rangle^2 \quad . \tag{108}$$

From Eq. (57), we have for extremely relativistic particles

$$\frac{dp_y}{dt} = -e\, E_{app} \quad , \tag{109}$$

where E_{app} changes polarity at the times where $\frac{dp_y}{d\varphi} \simeq 0$. Substituting Eq. (109) into Eq. (108), we obtain the gain-loss energy equation

$$\frac{d\gamma}{dt} = A - B\gamma^2 \quad , \qquad \varepsilon = mc^2 \gamma \tag{110}$$

$$A \cong \frac{\epsilon}{2} \frac{v_y(0) E_{y_0}}{mc^2} \quad , \quad B = \frac{1}{3} \frac{\epsilon^2}{m^2 c^3} \epsilon^2 E_{\text{app}}^2 \quad .$$

In obtaining (110), the time average is performed, which gives an overall factor $\frac{1}{2}$. Eq. (110) is easily solved to obtain

$$\gamma = \gamma(0) + \frac{\sqrt{A}}{\sqrt{B}} \left[\frac{\exp\left(2\sqrt{AB}\,t\right) - 1}{\exp\left(2\sqrt{AB}\,t\right) + 1} \right] \quad . \tag{111}$$

We clearly see from Eq. (111), that the increase in γ with t saturates at the critical value, attained at large t.

$$\gamma_c = \gamma(0) + \frac{\sqrt{A}}{\sqrt{B}} \equiv \gamma(0) + \sqrt{\frac{3}{2} \frac{\epsilon v_y(0) E_{y_0}/mc^2}{\left(\frac{\epsilon^2}{mc^2}\right)\left(\frac{\epsilon E_{\text{app}}\, c}{mc^2}\right)^2} c} \quad . \tag{112}$$

With the parameters of Kawata et al. we find

$$\gamma_c \simeq 10^6 \quad .$$

Accordingly, very high values of γ can be obtained with our machine.

X. CONCLUSIONS

In this paper a detailed account of the Non-Linear Amplification of Inverse Bremsstrahlung Acceleration mechanism is given. Several examples of NAIBEA both for electrons and protons are worked out. The problem of capacitor-capacitor interaction of the original NAIBEA proposal of Ref. 2) is discussed. We also analyse the reradiation problem and show that the relativistic factor γ reaches a saturation at very high values.

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TABLE CAPTIONS

Table 1. The positions, in meters, of the applied field reversals for electron acceleration with the $P = 3.5 \times 10^{15}\ \mathrm{W/cm^2}$ laser.

Table 2. The positions, in kilometers, of the applied field reversals for proton acceleration with the $P = 6.6 \times 10^{20}\ \mathrm{W/cm^2}$ laser.

FIGURE CAPTIONS

Fig. 1. The coupled electron-laser-static field configuration.

Fig. 2. The contributions of the first four terms in the B-expansion of Eq. (18).

Fig. 3. Solid curve: Numerically generated solution of Eqs. (64) and (69), in the one-kick case of Ref. 4). Dashed curve: The result obtained from the analytical methods. Eqs. (71), (72), (73) and (74) (see text for details).

Fig. 4. (a) The energy ε in units of the electron rest mass mc^2 vs. the laser phase. The lower curve is the result of Ref. 4), whereas the upper one is our result (see text for details). (b) The trajectory variable in units of mc vs. the laser phase.

Fig. 5. The energy ε in units of mc^2 vs. the time in units of $1.01 \times 10^6 \lambda/32c$ (see text and Ref. 5) for details).

Fig. 6. The trajectory in the y-z plane for the case shown in figure (5).

Fig. 7. The laser pulse vs. φ used in our calculation the width $\Delta = 3\pi$.

Fig. 8. The energy of the particle in units of mc^2 vs. the traveled distance z.

Fig. 9. The trajectory of the particle in the y-z plane. Inset: scale of y in mm.

Fig. 10. The *gamma* vs. t curve for a plane wave laser and a pulse (see text for details).

Fig. 11. The relativistic factor, γ vs. z for protons. A Gaussian laser pulse with a width of $\Delta = 3\pi$ is used in the calculation (see text for details).

Fig. 12. The trajectory of the proton in the y-z plane. The arrows indicate the applied field reversal positions given in table 2.

Fig. 13. The two capacitors problem.

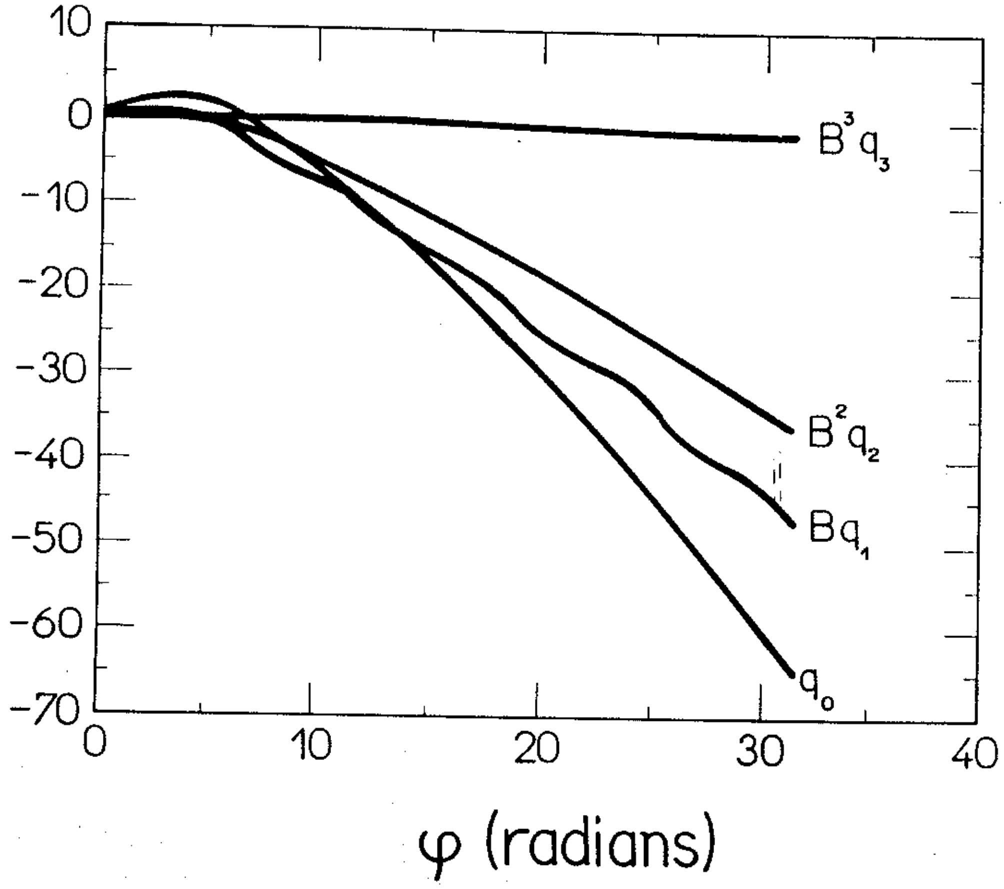
Fig. 14. The electric field intensity distribution for the two capacitor problem of figure (13). Plotted is E/v vs. $y + z$ in units of a.

Table 1

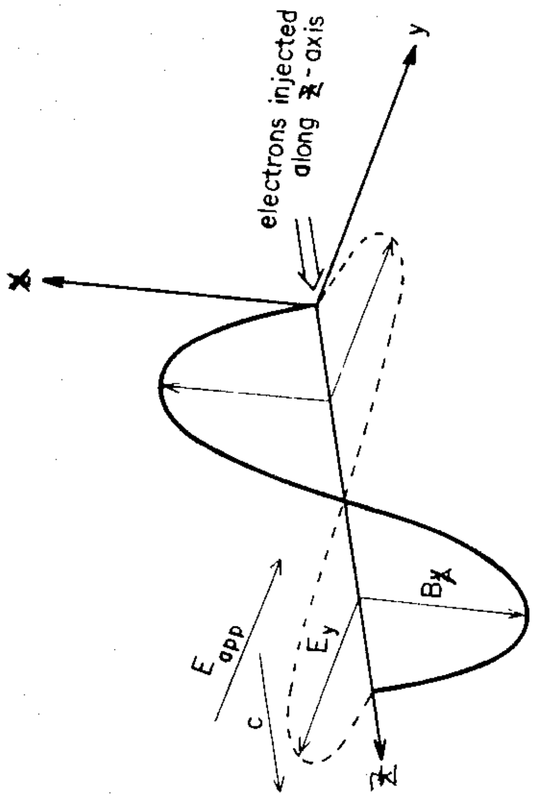
1	2	3	4	5	6	7	8	9
0.15	0.24	0.51	1.0	1.75	2.75	4.0	5.4	6.9

Table 2

1	2	3	4	5	6	7	8	9	10	11
0.031	0.05	0.085	0.18	0.375	0.665	1.045	1.515	2.052	2.63	3.23

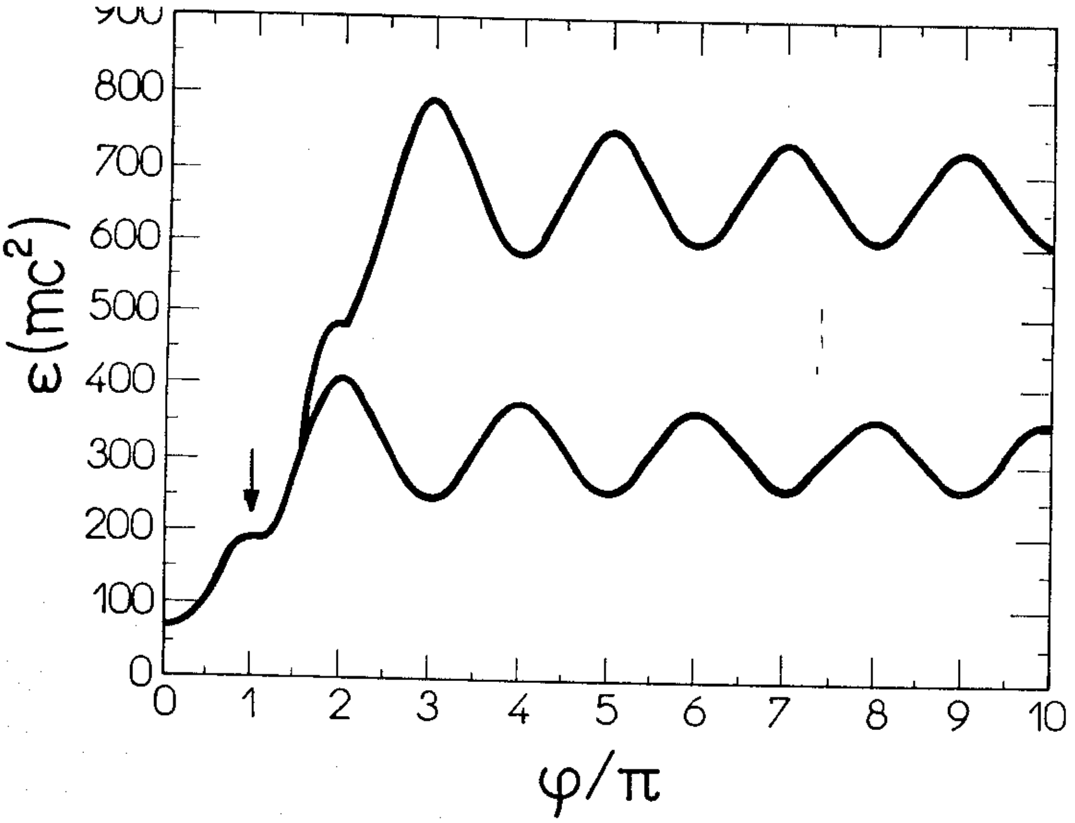


10
0
−10
−20
−30
−40
−50
−60
−70
0
10
20
30
40
φ (radians)
B^3q_3
B^2q_2
Bq_1
q_0



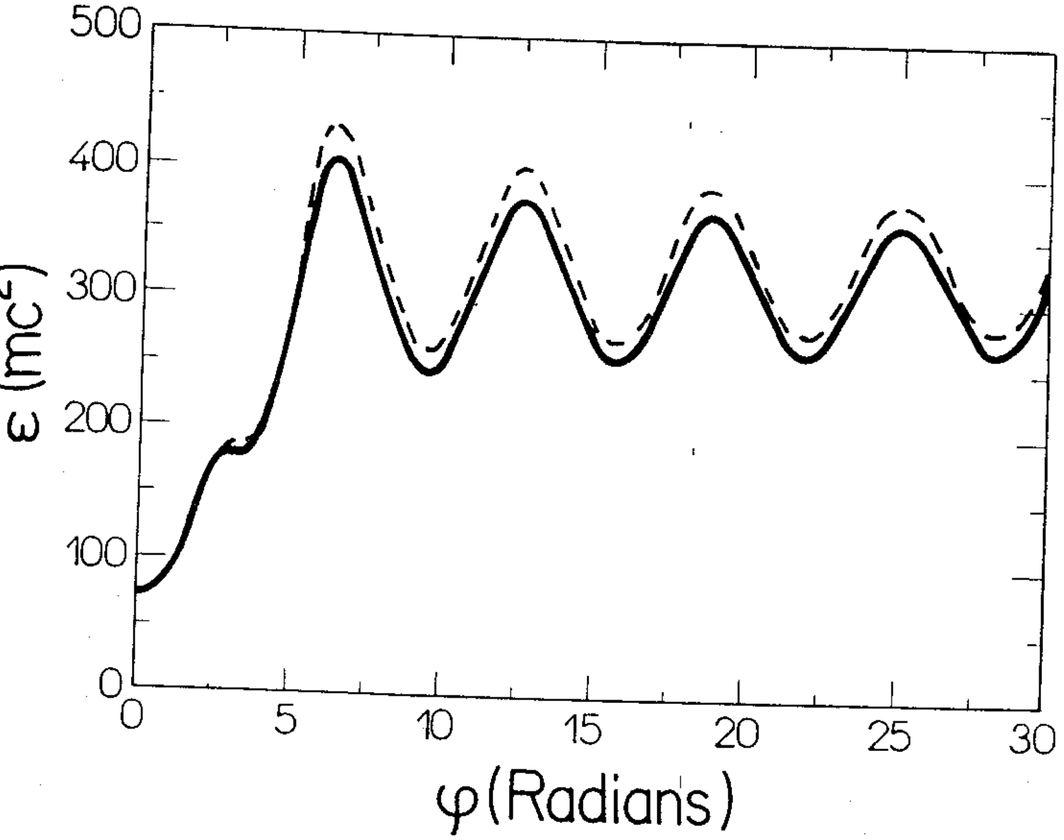
electrons injected
along z-axis
y
x
z
B_x
E_y
E_{app}
c

Fig. 1



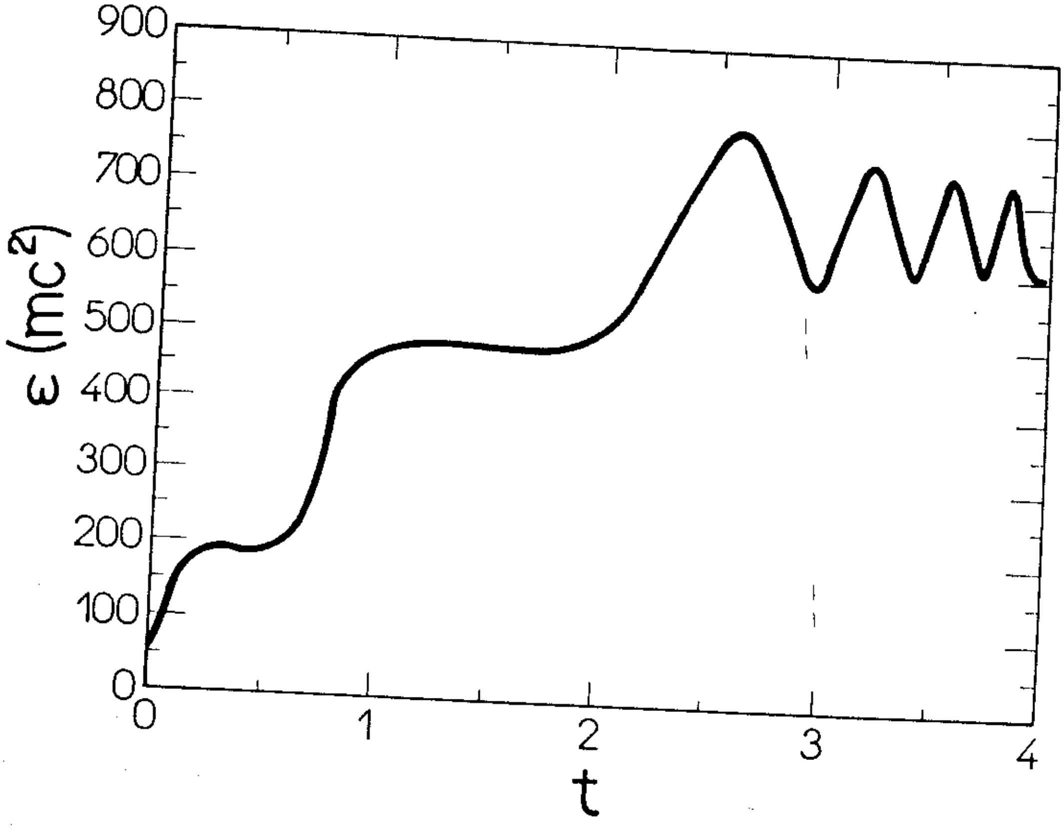
900
800
700
600
500
400
300
200
100
0
ε (mc²)
0
1
2
3
4
5
6
7
8
9
10
φ/π

Fig. 4a



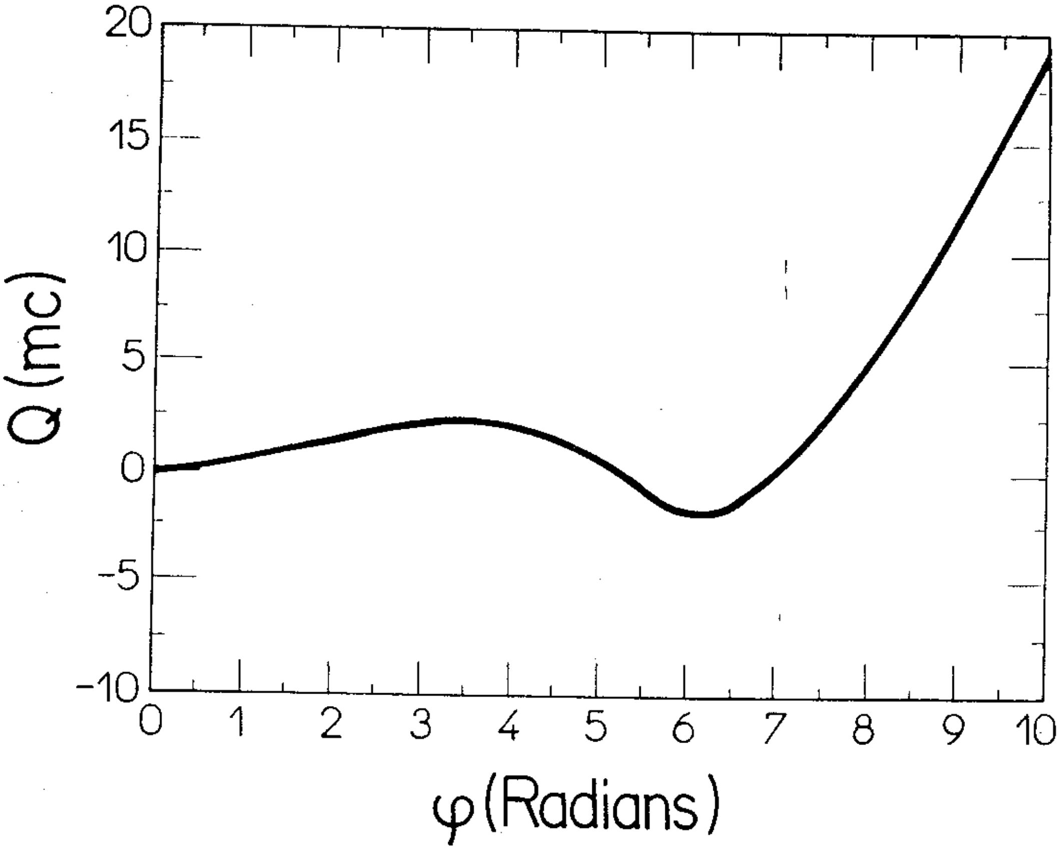
500
400
300
200
100
0
ε (mc²)
0
5
10
15
20
25
30
φ (Radians)

Fig. 3



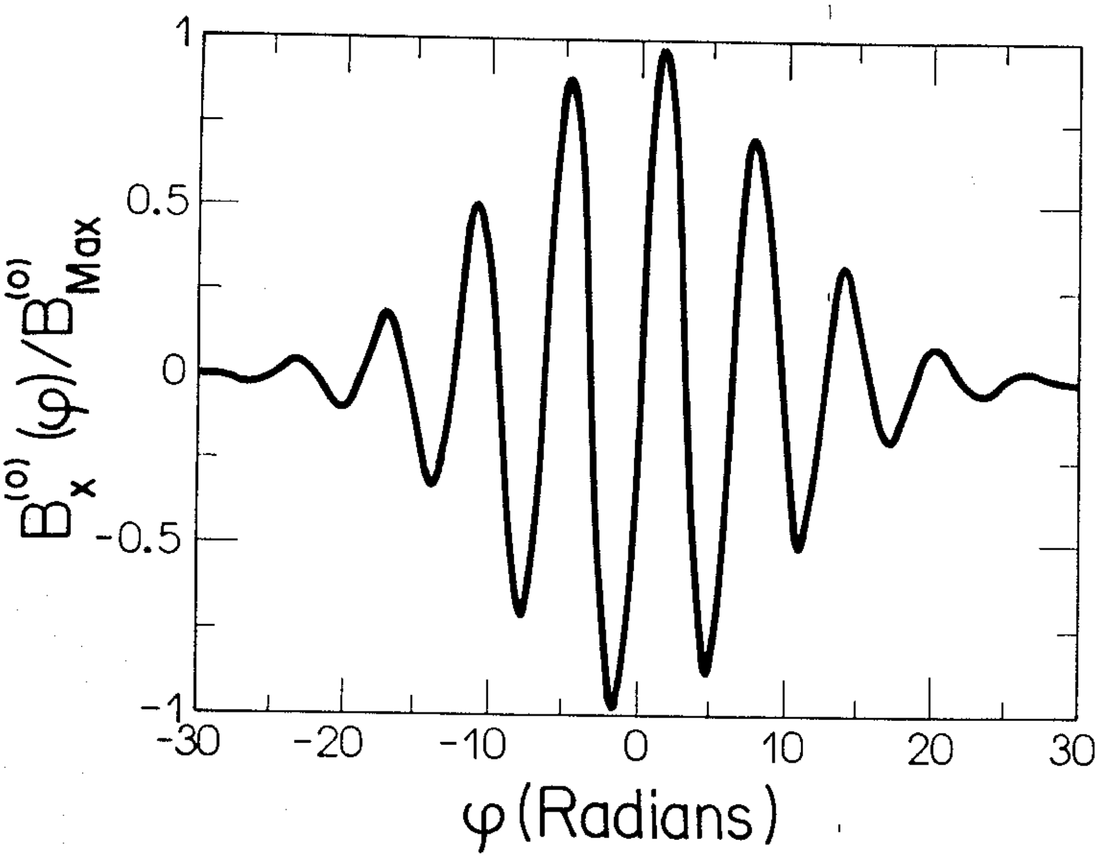
900
800
700
600
500
400
300
200
100
0
ε (mc²)
0
1
2
3
4
t

Fig. 5



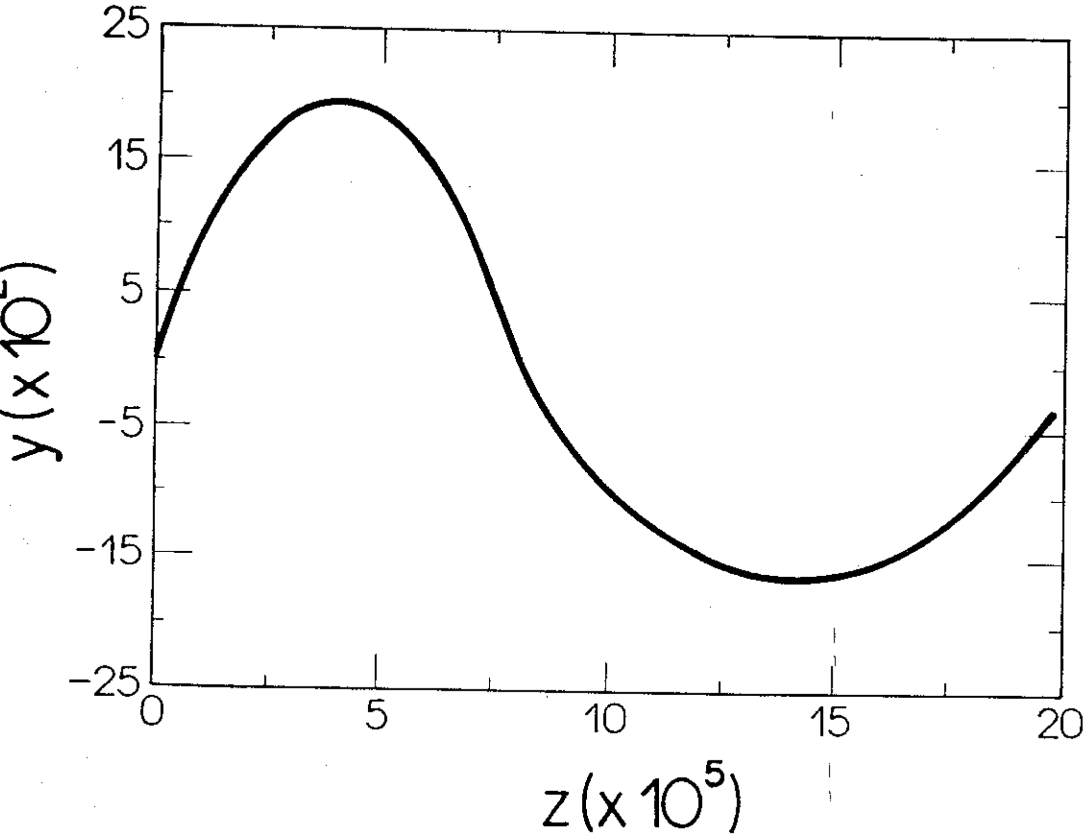
20
15
10
5
0
-5
-10
Q (mc)
0
1
2
3
4
5
6
7
8
9
10
φ (Radians)

Fig. 4b



1
0.5
0
-0.5
-1
-30
-20
-10
0
10
20
30
$B_x^{(o)}(\varphi)/B_{Max}^{(o)}$
φ (Radians)

Fig. ~~7~~



25
15
5
-5
-15
-25
0
5
10
15
20
$y\,(\times 10^2)$
$z\,(\times 10^5)$

Fig. 6

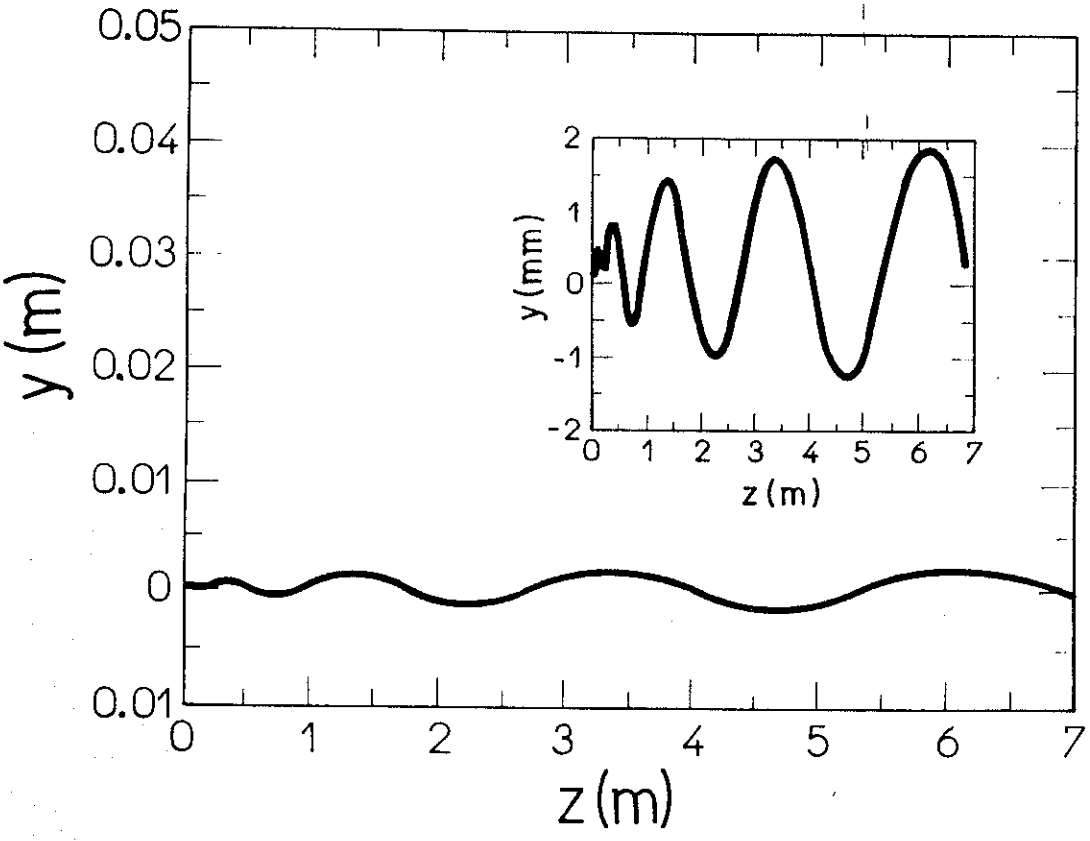

0.05
0.04
0.03
0.02
0.01
0
0.01
y (m)
0 1 2 3 4 5 6 7
z (m)
2
1
0
-1
-2
y (mm)
0 1 2 3 4 5 6 7
z (m)

Fig. 9

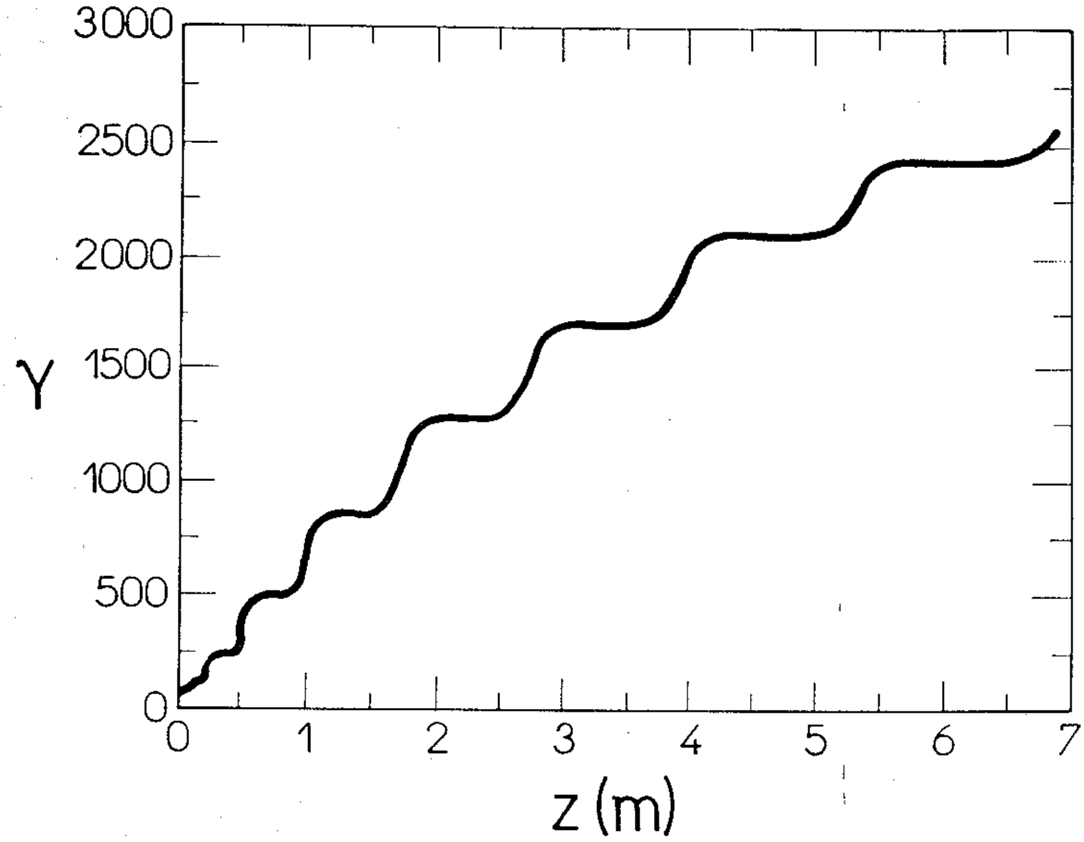

3000
2500
2000
1500
1000
500
0
Y
0 1 2 3 4 5 6 7
z (m)

Fig. 8

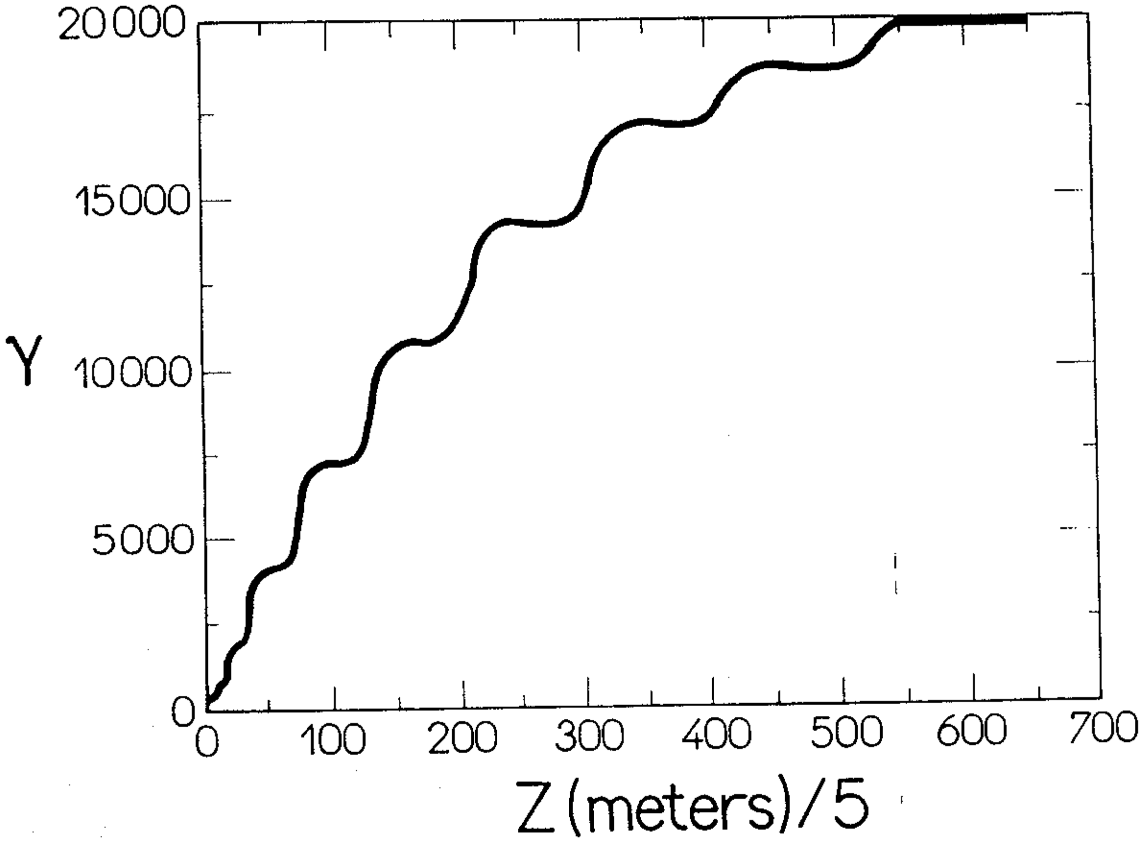

20000
15000
10000
5000
0
Y
0
100
200
300
400
500
600
700
Z (meters)/5

Fig. 11

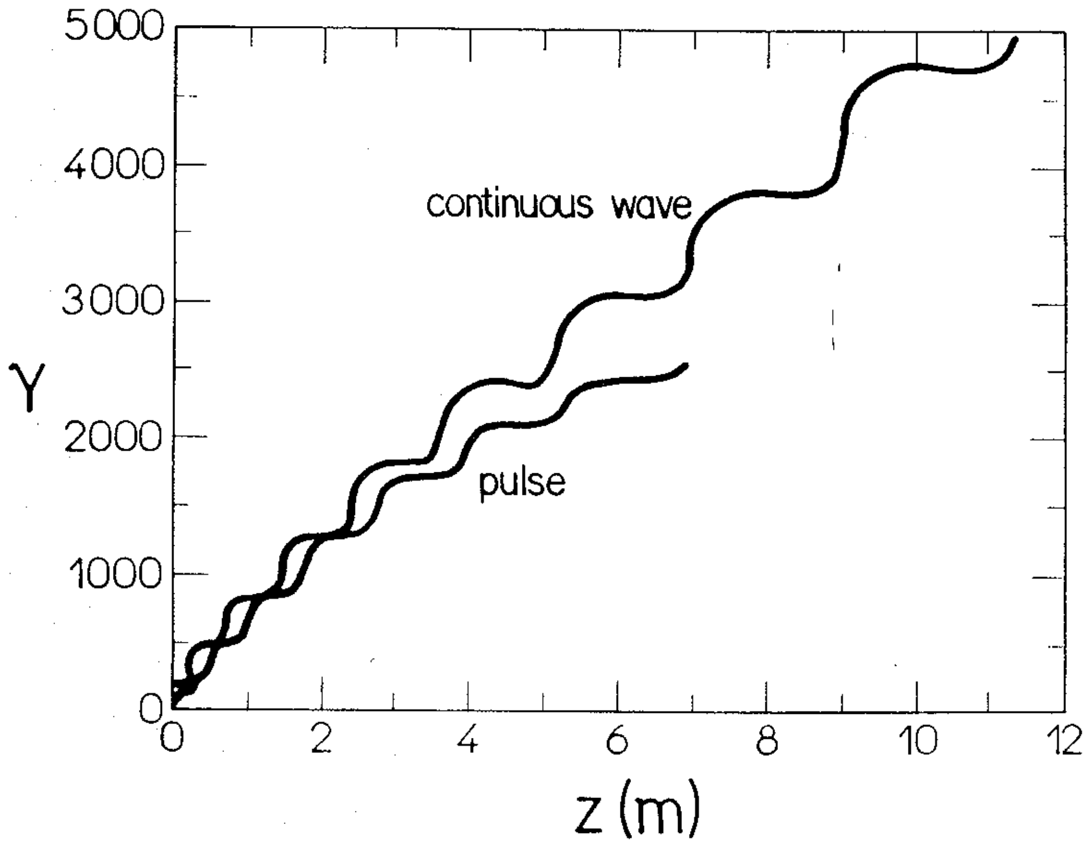

5000
4000
3000
2000
1000
0
Y
continuous wave
pulse
0
2
4
6
8
10
12
z (m)

Fig. 10

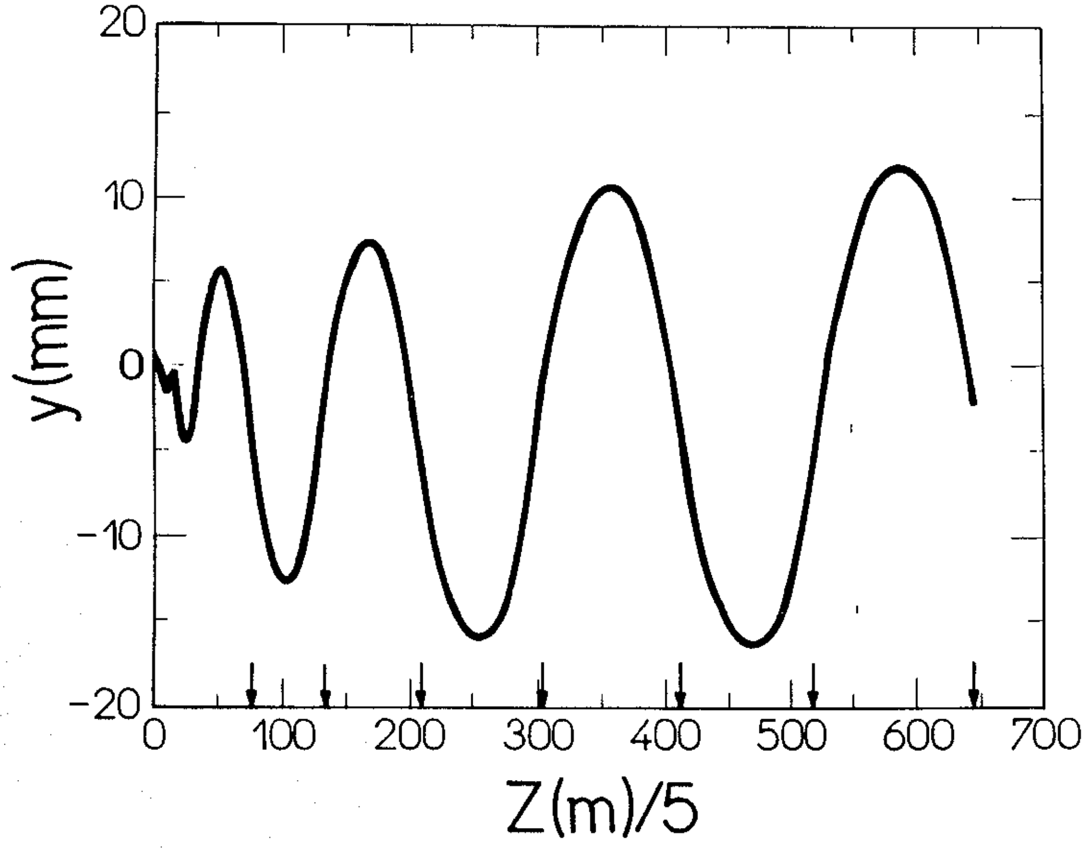


Fig. 12b

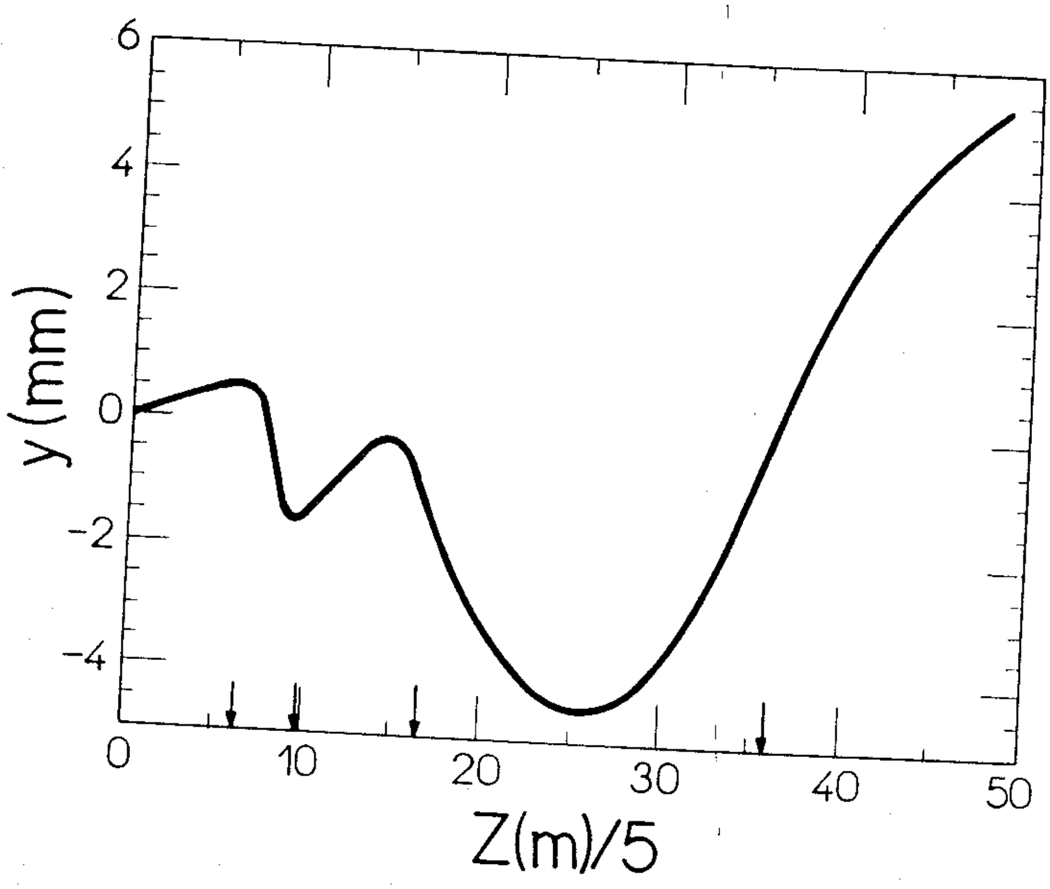
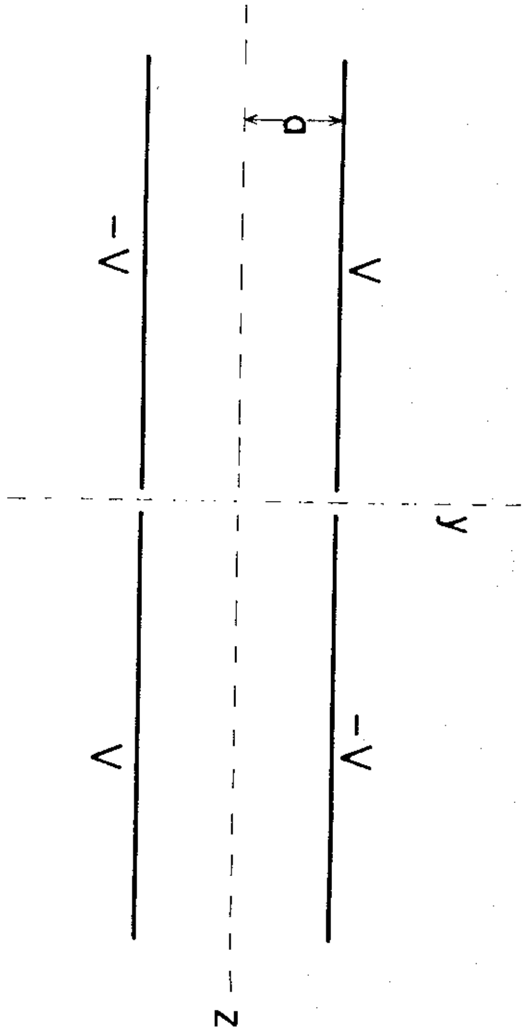


Fig. 12a



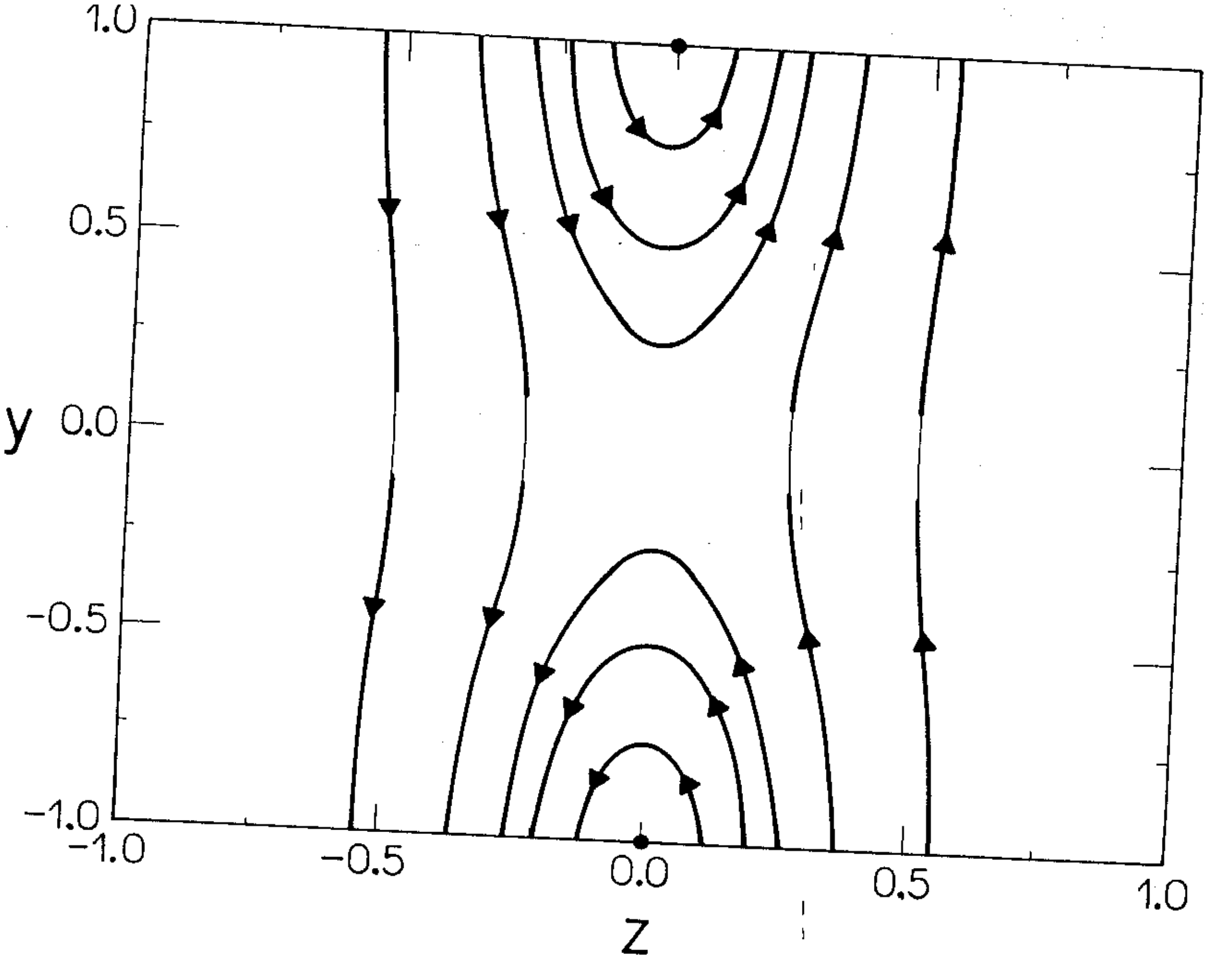


Fig. 14