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TWO DIMENSIONAL GRAVITY

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# On the Dirac Quantization of Two Dimensional Gravity

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## Abstract

We discuss the Dirac quantization of two dimensional gravity with bosonic matter fields. After defining the extended Hamiltonian it is possible to fix the gauge completely. The commutators can all be obtained in closed form; nevertheless, the results are not particularly simple.

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## Introduction

Two dimensional gravity has been studied in detail during the last few years<sup>1,2</sup>. Several important and clear results have been obtained for Liouville theory<sup>3</sup> as well as for the light cone gauge pure gravity<sup>1,2</sup>. Moreover correlation functions involving dressed operators have been systematically computed<sup>4</sup>. In this plethora of results, it is however disappointing that very few methods can shed some light in the canonical structure of higher dimensional gravitation theory, since two dimensional space-time has revealed strength as a theoretical laboratory for higher dimensional (specially gauge) theories<sup>5</sup>. Thus we shall follow the works [6] and [7] using the canonical method in order to study two dimensional gravity.

We shall start with a brief discussion of pure two dimensional gravity. This is a very simple case, since the number of constraints is too large, and the canonical formalism fails to provide a non trivial result. However, working in an analogous way as in ref.[6], it is possible to calculate some fundamental quantities before the gauge fixing procedure.

To begin with we consider the two dimensional gravity Lagrangian

$$L = \sqrt{-g} \left( -\frac{1}{2} \varphi \square \varphi - \frac{\alpha}{2} R \varphi + \frac{\alpha^2}{2} \beta \right) , \quad (1)$$

where  $R$  is the scalar curvature,  $\beta$  is the cosmological constant,  $\varphi$  is an scalar auxiliary field and  $\alpha$  is the renormalized matter fields central charge. We proceed calculating the Hamiltonian structure for the model using light-cone variables, and after adding a suitable surface term to (1) we end up with a Hamiltonian system of four first class constraints

$$\pi^{--} = \frac{\partial L}{\partial(\partial_- g_{--})} = 0 = \Gamma^1 , \quad (2a)$$

$$\pi^{-+} = \frac{\partial L}{\partial(\partial_- g_{-+})} = 0 = \Gamma^2 , \quad (2b)$$

$$\phi_1 = \frac{1}{2} \left[ (\partial_+ \varphi)^2 - \frac{4}{\alpha^2} (g_{++} \pi^{++})^2 - \frac{4}{\alpha} (g_{++} \pi^{++}) \pi - \frac{\alpha \partial_+ g_{++} \partial_+ \varphi}{g_{++}} + 2\alpha \partial_+^2 \varphi + \alpha^2 \beta g_{++} \right] \quad (3a)$$

$$\phi_2 = \pi \partial_+ \varphi - 2g_{++} \pi^{++} - \pi^{++} \partial_+ g_{++} . \quad (3b)$$

Equations (2) are the primary constraints while equations (3) are secondary. At this level the model reveals the well-known  $SL(2, R)$  structure<sup>6</sup> in a very clear way, when we construct the following set of variables

$$J^+ = \frac{1}{g_{++}} (\phi_2 - \phi_1) + \frac{\alpha^2 \beta}{2} , \quad (4a)$$

$$J^0 = j^0 - x^- J^+ , \quad (4b)$$

$$j^0 = \left[ g_{++} (\pi^{++} + \frac{\alpha \partial_+ \varphi}{2 g_{++}}) + \frac{\alpha}{2} (\pi - \frac{\alpha \partial_+ g_{++}}{2 g_{++}} - \partial_+ \varphi) \right] , \quad (4c)$$

$$J^- = j^- - 2x^- J^0 - (x^-)^2 J^+ , \quad (4d)$$

$$j^- = \alpha^2 (g_{++} + 1) , \quad (4e)$$

$$b = \pi - \frac{\alpha \partial_+ g_{++}}{2 g_{++}} + \partial_+ \varphi , \quad (4f)$$

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which classically satisfy the Poisson bracket  $SL(2, R)$  algebra

$$\{J^a(x), J^b(y)\} = -2\epsilon^{abc}\eta_{cd}J^d(x)\delta(x-y) + \alpha^2\eta^{ab}\partial_+\delta(x-y) . \quad (5)$$

Using this structure it is also possible to calculate the quantum BRST charge in a gauge independent way, expanding the energy momentum tensor in terms of Virasoro modes; we define the Virasoro constraint as

$$\tau = T_m + T_S \approx 0 \quad (6a)$$

where the  $b$ -field and gravity energy momentum tensor are respectively

$$T_m = \frac{1}{4}b^2 + \frac{\alpha}{2}\partial_+b , \quad (6b)$$

$$T_S = \frac{1}{2\alpha^2}\eta_{ab}J^aJ^b - \partial_+J^0 . \quad (6c)$$

Imposing now the nilpotency ( $\hat{Q}^2 = 0$ ) for the BRST charge

$$\hat{Q} = c_0(L_0 - a) + \sum_{n \neq 0} : c_n L_{-n} : - \frac{1}{2} \sum_{-\infty}^{\infty} (m-n) : c_{-m} c_{-n} b_{m+n} : , \quad (7)$$

we obtain the usual relation for the central charges<sup>1,2</sup> in a gauge independent way

$$c_{mat} + \frac{3k}{k+2} - 6k - 28 = 0 , \quad k = \frac{\alpha^2}{8} . \quad (8)$$

As we mentioned earlier, the next step in our work was to fix completely the gauge freedom for the model using the canonical formalism. This is a difficult task because the theory is diffeomorphism invariant; as a consequence the Hamiltonian is a linear combination of constraints

$$H_c = 4 \left[ -\frac{\sqrt{-g}}{g_{++}}\phi_1 + \frac{g_{-+}}{g_{++}}\phi_2 \right] . \quad (9)$$

In order to avoid this problem we use time dependent gauge fixing constraints. Our result shows that this procedure annihilates all the physical degrees of freedom for the pure gravity theory. However if we consider the model coupled to matter fields

$$S = \frac{1}{2} \int d^2x \sqrt{-g} [-\varphi \square \varphi - X \square X - \alpha R \varphi - \gamma R X + (\gamma^2 + \alpha^2) \beta] , \quad (10)$$

It is possible to find a non-trivial reduced phase space leaving the canonical procedure as a possible technique for quantization in this case.

## Canonical Quantization of 2-D gravity coupled to matter fields

We start adding a suitable surface term to the Lagrangian (10) in order to find simpler expressions for the canonical momenta<sup>(1)</sup>; we find, after some algebra, the Lagrangian

$$\begin{aligned} L = & \frac{1}{2\sqrt{-g}} \left[ (-g_{11}\dot{\varphi}^2 + 2g_{01}\dot{\varphi}\varphi' - g_{00}\varphi'^2) + \alpha(\dot{g}_{11}\dot{\varphi} - 2g'_{01}\dot{\varphi} + g'_{00}\varphi') \right. \\ & \left. + \alpha \frac{g_{01}}{g_{11}}(\dot{g}'_{11}\dot{\varphi} - \dot{g}_{11}\varphi') - \alpha^2\beta g \right] \\ & + \frac{1}{2\sqrt{-g}} \left[ (-g_{11}\dot{X}^2 + 2g_{01}\dot{X}X' - g_{00}X'^2) + \gamma(\dot{g}_{11}\dot{X} - 2g'_{01}\dot{X} + g'_{00}X') \right. \\ & \left. + \gamma \frac{g_{01}}{g_{11}}(\dot{g}'_{11}\dot{X} - \dot{g}_{11}X') - \gamma^2\beta g \right] . \end{aligned} \quad (11)$$

The canonical momenta are derived from above, and read

$$\pi^{00} = \frac{\partial L}{\partial \dot{g}_{00}} = 0 = \Gamma_1 \quad (12)$$

$$\pi^{01} = \frac{\partial L}{\partial \dot{g}_{01}} = 0 = \Gamma_2 \quad (13)$$

$$\pi_\varphi = \frac{1}{\sqrt{-g}}(g_{01}\varphi' - g_{11}\dot{\varphi}) + \frac{\alpha}{2\sqrt{-g}} \left( \dot{g}_{11} - 2g'_{01} + \frac{g_{01}}{g_{11}}g'_{11} \right) \quad (14)$$

$$\pi^{11} = \frac{\alpha}{2\sqrt{-g}} \left( \dot{\varphi} - \frac{g_{01}}{g_{11}}\varphi' \right) + \frac{\gamma}{2\sqrt{-g}} \left( \dot{X} - \frac{g_{01}}{g_{11}}X' \right) \quad (15)$$

$$\bar{\pi} = \frac{1}{\sqrt{-g}}(g_{01}X' - g_{11}\dot{X}) + \frac{\gamma}{2\sqrt{-g}} \left( \dot{g}_{11} - 2g'_{01} + \frac{g_{01}}{g_{11}}g'_{11} \right) , \quad (16)$$

where  $\Gamma_1$  and  $\Gamma_2$  are two primary constraints. As in the pure case we find two secondary constraints ( $\phi_1, \phi_2$ ) which are also first class

$$\begin{aligned} \phi_1 = & -\frac{1}{2}(\pi^2 + \bar{\pi}^2) + \frac{(\alpha\pi + \gamma\bar{\pi} + 2\pi^{11}g_{11})^2}{2(\alpha^2 + \gamma^2)} - \frac{(\alpha^2 + \gamma^2)\beta g_{11}}{2} \\ & - \frac{1}{2}(\varphi'^2 + X'^2) + \frac{g'_{11}}{2g_{11}}(\alpha\varphi' + \gamma X') - \alpha\varphi'' - \gamma X'' \end{aligned} \quad (17a)$$

$$\phi_2 = \varphi'\pi + X'\bar{\pi} - 2g_{11}\pi^{11} - g_{11}'\pi^{11} . \quad (17b)$$

The diffeomorphism invariance of the model is verified through the constraint algebra:

$$\{\phi_1(x), \phi_1(y)\} = [\phi_2(x) + \phi_2(y)]\partial_x\delta(x-y) \quad (18a)$$

$$\{\phi_1(x), \phi_2(y)\} = [\phi_1(x) + \phi_1(y)]\partial_x\delta(x-y) \quad (18b)$$

$$\{\phi_2(x), \phi_2(y)\} = [\phi_2(x) + \phi_2(y)]\partial_x\delta(x-y) . \quad (18c)$$

<sup>(1)</sup> In this case we work with Minkowskian variables  $x^0$  representing the time variable.

This invariance leads us to impose a set of gauge fixing constraints, following the Dirac method. To fix the gauge freedom completely we need four of them:

$$\Gamma_3 = g_{11} - g_{00} + 2 = 0 \quad (19a)$$

$$\Gamma_4 = g_{11} - g_{01} + 1 = 0 \quad (19b)$$

$$\phi_3 = g_{11} - F(t, x) \quad (20a)$$

$$\phi_4 = \frac{2}{\alpha^2 + \gamma^2} (\alpha\pi + \gamma\bar{\pi}) \quad (20b)$$

which transform the whole set of constraints  $(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \phi_1, \dots, \phi_4)$  into a second class system. The calculation of the Dirac brackets, which gives us the structure of the reduced phase space, can be made in a two step procedure. In the first step we eliminate the spurious degrees of freedom corresponding to the set  $\{(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4)\}$ . While considering the second step, it is simpler to work with an extended Hamiltonian (which includes the linear combination of all constraints with arbitrary coefficients)

$$H_e = - \left( v_1 - \frac{\sqrt{-g}}{g_{11}} \right) \phi_1 + \left( v_2 + \frac{g_{01}}{g_{11}} \right) \phi_2 \quad (21)$$

in order to stop the generation of new constraints coming from the time consistency of  $\phi_3$  and  $\phi_4$ . This procedure in fact fixes the unknown coefficients  $v_1$  and  $v_2$ .

In order to construct the Dirac matrix

$$\Delta_{ij}(x, y) = \{\phi_i(x), \phi_j(y)\} \quad (22)$$

we calculate the Poisson brackets between the constraints. The non-zero ones are in this case:

$$\{\phi_1(x), \phi_3(y)\} \equiv \Delta_{13}(x, y) = -\frac{2g_{11}^2 \pi^{11}(x)}{\rho^2} \delta(x - y) \quad (23a)$$

$$\{\phi_1(x), \phi_4(y)\} \equiv \Delta_{14}(x, y) = \left( \frac{g'_{11}}{g_{11}} - \frac{2}{\rho^2} [\alpha\varphi' + \gamma X'] \right) (x) \partial_x \delta(x - y) - 2\partial_x^2 \delta(x - y) \quad (23b)$$

$$\{\phi_2(x), \phi_3(y)\} \equiv \Delta_{23}(x, y) = 2g_{11}(x) \partial_x \delta(x - y) + g'_{11}(x) \delta(x - y) \quad (23c)$$

The Dirac brackets require the calculation of the inverse of this matrix. After a tedious calculation we find for the non zero elements

$$\Delta_{32}^{-1}(x, y) = \frac{\epsilon(x - y)}{4\sqrt{g_{11}(x)g_{11}(y)}} \quad (24a)$$

$$\Delta_{41}^{-1}(x, y) = \left\{ \int_{-\infty}^y dz \frac{\epsilon(x - z)}{4} g_{11}(z) e^{-\frac{2}{\rho^2}(\alpha\varphi(z) + \gamma X(z))} \right\} e^{\frac{2}{\rho^2}(\alpha\varphi(y) + \gamma X(y))} g_{11}(y) \quad (24b)$$

$$\Delta_{42}^{-1}(x, y) = -\frac{2}{\rho^2} \left[ \int_{-\infty}^y dz g_{11} \pi^{11}(x) \Delta_{41}^{-1}(x, z) \sqrt{g_{11}(z)} \frac{1}{\sqrt{g_{11}(y)}} \right] \quad (24c)$$

Thus we find the following Dirac brackets:

$$\begin{aligned} \{\varphi(x), \varphi(y)\}_D = & -\frac{2\alpha}{\rho^2} \left( 2g_{11} \pi^{11}(x) \frac{\alpha}{\rho^2} - \pi(x) \right) \int_{-\infty}^x dz \frac{\epsilon(z - y)}{4} C^-(z) C^+(x) \frac{\sqrt{g_{11}(z)}}{\sqrt{g_{11}(x)}} \\ & + \frac{2\alpha}{\rho^2} \left( g_{11} \pi^{11}(y) \frac{\alpha}{\rho^2} - \pi(y) \right) \int_{-\infty}^y dz \frac{\epsilon(x - z)}{4} C^-(z) C^+(y) \frac{\sqrt{g_{11}(z)}}{\sqrt{g_{11}(y)}} \\ & - \frac{2\alpha}{\rho^2} \left\{ \varphi'(y) \left[ \int_{-\infty}^y dz \frac{g_{11} \pi^{11}(z)}{\rho^2} \int_{-\infty}^x dw \frac{\epsilon(w - z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(y)}} \right] \right. \\ & \left. - \varphi'(x) \left[ \int_{-\infty}^x dz \frac{g_{11} \pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w - z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right\} \quad (25) \end{aligned}$$

where

$$C^\pm(z) = e^{\pm \frac{1}{\rho^2} (\alpha\varphi(z) + \gamma X(z))} \quad (26)$$

Further Dirac brackets can be computed in a straightforward but tedious way; we find for the purely matter field Dirac bracket the expression:

$$\begin{aligned} \{X(x), X(y)\}_D = & -\frac{2\gamma}{\rho^2} \left( 2g_{11} \pi^{11}(x) \frac{\gamma}{\rho^2} - \bar{\pi}(x) \right) \int_{-\infty}^x dz \frac{\epsilon(z - y)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(x) \frac{1}{\sqrt{g_{11}(x)}} \\ & + \frac{2\gamma}{\rho^2} \left( 2g_{11} \pi^{11}(y) \frac{\gamma}{\rho^2} - \bar{\pi}(y) \right) \int_{-\infty}^y dz \frac{\epsilon(x - z)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(y) \frac{1}{\sqrt{g_{11}(y)}} \\ & - \frac{2\gamma}{\rho^2} \left\{ X'(y) \left[ \int_{-\infty}^y dz \frac{g_{11} \pi^{11}(z)}{\rho^2} \int_{-\infty}^x dw \frac{\epsilon(w - z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(y)}} \right] \right. \\ & \left. - X'(x) \left[ \int_{-\infty}^x dz \frac{g_{11} \pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w - z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right\} \quad (27) \end{aligned}$$

The mixed gravity-matter fundamental bracket is

$$\begin{aligned} \{\varphi(x), X(y)\}_D = & -\frac{2\gamma}{\rho^2} \left( 2g_{11} \pi^{11}(x) \frac{\alpha}{\rho^2} - \pi(x) \right) \int_{-\infty}^x dz \frac{\epsilon(z - y)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(x) \frac{1}{\sqrt{g_{11}(x)}} \\ & + \frac{2\gamma}{\rho^2} \left( g_{11} \pi^{11}(y) \frac{\alpha}{\rho^2} - \pi(y) \right) \int_{-\infty}^y dz \frac{\epsilon(x - z)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(y) \frac{1}{\sqrt{g_{11}(y)}} \\ & - \frac{2\gamma}{\rho^2} \left\{ \varphi'(y) \left[ \int_{-\infty}^y dz \frac{g_{11} \pi^{11}(z)}{\rho^2} \int_{-\infty}^x dw \frac{\epsilon(w - z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(y)}} \right] \right. \\ & \left. - \varphi'(x) \left[ \int_{-\infty}^x dz \frac{g_{11} \pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w - z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right\} \quad (28) \end{aligned}$$

Furthermore the brackets for the fields  $\varphi$ ,  $X$  and their momenta

$$\begin{aligned} \{\pi(x), \varphi(y)\}_D = & -\delta(x-y) + \frac{2\alpha}{\rho^2} \left[ \partial_y \left[ \left( \int_{-\infty}^x dz \frac{\epsilon(z-y)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(x) \frac{1}{\sqrt{g_{11}(x)}} \right) \right. \right. \\ & \times \left. \left. \left( \frac{\alpha g'_{11}(z)}{2g_{11}(z)} - \varphi'(z) \right) \right] \right] + \frac{2\alpha^2}{\rho^2} \partial_y^2 \left[ \int_{-\infty}^x dz \frac{\epsilon(z-y)}{4} \frac{\sqrt{g_{11}(z)}}{\sqrt{g_{11}(x)}} C^-(z) C^+(x) \right] \\ & + \frac{2\alpha}{\rho^2} \partial_y \left[ \pi(y) \left[ \int_{-\infty}^x dz \frac{g_{11}\pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w-z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right], \end{aligned} \quad (29)$$

$$\begin{aligned} \{\bar{\pi}(x), X(y)\}_D = & -\delta(x-y) + \frac{2\gamma}{\rho^2} \left[ \partial_y \left[ \left( \int_{-\infty}^x dz \frac{\epsilon(z-y)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(x) \frac{1}{\sqrt{g_{11}(x)}} \right) \right. \right. \\ & \times \left. \left. \left( \frac{\gamma g'_{11}(z)}{2g_{11}(z)} - X'(z) \right) \right] \right] + \frac{2\gamma^2}{\rho^2} \partial_y^2 \left[ \int_{-\infty}^x dz \frac{\epsilon(z-y)}{4} \frac{\sqrt{g_{11}(z)}}{\sqrt{g_{11}(x)}} C^-(z) C^+(x) \right] \\ & + \frac{2\gamma}{\rho^2} \partial_y \left[ \bar{\pi}(y) \left[ \int_{-\infty}^x dz \frac{g_{11}\pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w-z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right]; \end{aligned} \quad (30)$$

we have also

$$\begin{aligned} \{\pi(x), X(y)\}_D = & \frac{2\gamma}{\rho^2} \left[ \partial_y \left[ \left( \int_{-\infty}^x dz \frac{\epsilon(z-y)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(x) \frac{1}{\sqrt{g_{11}(x)}} \right) \right. \right. \\ & \times \left. \left. \left( \frac{\alpha g'_{11}(z)}{2g_{11}(z)} - \varphi'(z) \right) \right] \right] + \frac{2\gamma}{\rho^2} \alpha \partial_y^2 \left[ \int_{-\infty}^x dz \frac{\epsilon(z-y)}{4} \frac{\sqrt{g_{11}(z)}}{\sqrt{g_{11}(x)}} C^-(z) C^+(x) \right] \\ & + \frac{2\gamma}{\rho^2} \partial_y \left[ \pi(y) \left[ \int_{-\infty}^x dz \frac{g_{11}\pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w-z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right], \end{aligned} \quad (31)$$

and

$$\begin{aligned} \{\bar{\pi}(x), \varphi(y)\}_D = & \frac{2\alpha}{\rho^2} \left[ \partial_y \left[ \left( \int_{-\infty}^x dz \frac{\epsilon(z-y)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(x) \frac{1}{\sqrt{g_{11}(x)}} \right) \right. \right. \\ & \times \left. \left. \left( \frac{\gamma g'_{11}(z)}{2g_{11}(z)} - X'(z) \right) \right] \right] + \frac{2\alpha}{\rho^2} \gamma \partial_y^2 \left[ \int_{-\infty}^x dz \frac{\epsilon(z-y)}{4} \frac{\sqrt{g_{11}(z)}}{\sqrt{g_{11}(x)}} C^-(z) C^+(x) \frac{1}{\sqrt{g_{11}(x)}} \right] \\ & + \frac{2\alpha}{\rho^2} \partial_y \left[ \bar{\pi}(y) \left[ \int_{-\infty}^x dz \frac{g_{11}\pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w-z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right]. \end{aligned} \quad (32)$$

For brackets involving  $\pi^{11}$  we arrive at

$$\begin{aligned} \{\varphi(x), \pi^{11}(y)\}_D = & -\frac{\varphi(x)\epsilon(x-y)}{4\sqrt{g_{11}(x)g_{11}(y)}} \\ & - \frac{2\alpha}{\rho^2} \left( \int_{-\infty}^y dz \frac{\epsilon(x-z)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(y) \frac{1}{\sqrt{g_{11}(y)}} \left( \frac{2g_{11}\pi^{11^2}(y)}{\rho^2} \right. \right. \\ & \left. \left. - \frac{\rho^2\beta}{2} - \frac{(\alpha\varphi' + \gamma X') g'_{11}(y)}{2g_{11}^2(y)} \right) \right) \\ & - \frac{2\alpha}{\rho^2} \partial_y \left[ \int_{-\infty}^y dz \frac{\epsilon(x-z)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(y) \frac{1}{\sqrt{g_{11}(y)}} \frac{(\alpha\varphi' + \gamma X')(y)}{g_{11}(y)} \right] \\ & + \frac{4\alpha}{\rho^2} \pi^{11'}(y) \left[ \int_{-\infty}^x dz \frac{g_{11}\pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w-z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \\ & - \frac{2\alpha}{\rho^2} \partial_y \left[ \pi^{11}(y) \left[ \int_{-\infty}^x dz \frac{g_{11}\pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w-z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right] \end{aligned} \quad (33) \text{ and}$$

$$\begin{aligned} \{X(x), \pi^{11}(y)\}_D = & -\frac{X(x)\epsilon(x-y)}{4\sqrt{g_{11}(x)g_{11}(y)}} \\ & - \frac{2\gamma}{\rho^2} \left( \int_{-\infty}^y dz \frac{\epsilon(x-z)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(y) \frac{1}{\sqrt{g_{11}(y)}} \left( \frac{2g_{11}\pi^{11^2}(y)}{\rho^2} \right. \right. \\ & \left. \left. - \frac{\rho^2\beta}{2} - \frac{(\alpha\varphi' + \gamma X') g'_{11}(y)}{2g_{11}^2(y)} \right) \right) \\ & - \frac{2\gamma}{\rho^2} \partial_y \left[ \int_{-\infty}^y dz \frac{\epsilon(x-z)}{4} \sqrt{g_{11}(z)} C^-(z) C^+(y) \frac{1}{\sqrt{g_{11}(y)}} \frac{(\alpha\varphi' + \gamma X')(y)}{g_{11}(y)} \right] \\ & + \frac{4\gamma}{\rho^2} \pi^{11'}(y) \left[ \int_{-\infty}^x dz \frac{g_{11}\pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w-z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \\ & - \frac{2\gamma}{\rho^2} \partial_y \left[ \pi^{11}(y) \left[ \int_{-\infty}^x dz \frac{g_{11}\pi^{11}(z)}{\rho^2} \int_{-\infty}^y dw \frac{\epsilon(w-z)}{4} C^-(w) C^+(z) \frac{\sqrt{g_{11}(w)}}{\sqrt{g_{11}(x)}} \right] \right]. \end{aligned} \quad (34)$$

## Conclusions

We find a closed Dirac algebra for the set of physical fields of the theory. There is always a non-local tail in the commutator function, in such way that the theory does not simplify in some limit. For  $\alpha \rightarrow -\infty$  (semi-classical limit) the exponential functions simplify to one; the commutators go to zero, with a  $\mathcal{O}(1/\alpha)$  non-local correction of the type

$$\int_{-\infty}^z dx \epsilon(z-y) g_{11}(z) g_{11}(x) \quad . \quad (35)$$

In this example, there is a feature that can not be forgotten, concerning well known non-perturbative results obtained for several correlation functions. Simple results concerning correlators for vertex functions, cannot be obtained in this case. If we delete the matter field  $X$ , the canonical quantization turns out to imply a trivial result, whereas the exact solution is far from trivial<sup>1,3</sup>. This single result suggests that canonical quantization of higher dimensional gravity may be missing an entire sector of the theory. A second example concerning the impossibility of describing non-perturbative effects by means of canonical quantization concerns 3 -  $D$  topological Yang Mills theory, where canonical quantization leads to results far simpler than the expected ones.

Finally, the non-local tail can be traced back to the smearing of the fundamental fields as implied by the gravity "fog", and should play an important role in problems tied with the horizon in quantum gravity.

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