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THE GRAVITON SELF-ENERGY IN THERMAL  
QUANTUM GRAVITY

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# The graviton self-energy in thermal quantum gravity

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## Abstract

We show generally that in thermal gravity, the one-particle irreducible 2-point function depends on the choice of the basic graviton fields. We derive the relevant properties of a physical graviton self-energy, which is independent of the parametrization of the graviton field. An explicit expression for the graviton self-energy at high-temperature is given to one-loop order.

## I. INTRODUCTION

The high-temperature properties of quantum gravity are of some interest, both in their own right as well as for their potential cosmological applications. If the temperature  $T$  is well below the Planck scale, perturbation theory can be used to calculate the  $n$ -point thermal graviton functions, with internal lines which correspond to matter and fields in thermal equilibrium. These Green functions have been studied previously [1–4], and show a leading  $T^4$  behavior.

One of the most noteworthy aspects in field theory is the freedom of choice in the fundamental fields. In the case of quantum gravity this freedom is extensive, so that one may use as basic fields the metric tensor  $g_{\mu\nu}$ , or its inverse  $g^{\mu\nu}$ , or any other function of  $g_{\mu\nu}$ . From the point of view of the S-matrix elements at zero temperature, such different choices lead to the same result [5]. In thermal field theories at high-temperatures, the leading contributions to the one-particle irreducible Green functions are gauge-independent quantities, describing

the physical properties of hot plasma [6–8]. In thermal quantum gravity, these contributions are also gauge invariant [3,4], being however dependent on the parametrization of the graviton fields. For instance, as pointed out by Kikuchi, Moriya and Tsukahara [2], the time components of the graviton two-point function  $\Pi^{00,00}$  evaluated to one-loop order at zero momenta, depend on the choice of the basic graviton fields. Nevertheless, these authors have found that in the high-temperature limit, the quantity  $\Pi^{00,00} - \Pi^{\rho,00}$  is independent of these choices.

The main purpose of this work is to generalize the above features, for all components of the IPI 2-point function  $\Pi^{\mu\nu,\alpha\beta}$  evaluated at arbitrary momenta  $k$ . As we will argue, the parametrization-dependence of  $\Pi^{\mu\nu,\alpha\beta}$  is related to the non-vanishing of the 1-point function (tadpole) in thermal quantum gravity. In section II we present the results of explicit one-loop calculations, showing that the graviton self-energy given by:

$$\bar{\Pi}^{\mu\nu,\alpha\beta}(k) \equiv \Pi^{\mu\nu,\alpha\beta}(k) - \frac{1}{4} \left( \eta^{\mu\alpha} \Pi^{\rho, \nu\beta} + \eta^{\mu\beta} \Pi^{\rho, \nu\alpha} + \eta^{\nu\alpha} \Pi^{\rho, \mu\beta} + \eta^{\nu\beta} \Pi^{\rho, \mu\alpha} \right), \quad (1.1)$$

yields in the most common cases a quantity which does not depend on the choices of the basic graviton fields. In section III, we discuss the general conditions under which the expression (1.1) represents a physical amplitude, which is independent of the graviton parametrization. These are characterized by the fact that the graviton self-energy is described by a traceless function, which is transverse with respect to the external momenta at  $k^2 = 0$ .

## II. THE GRAVITON SELF-ENERGY

In order to derive the expression of the graviton self-energy to one-loop order, we consider first the Feynman diagrams contributing to  $\Pi$ , which are shown in Fig.1. Here, the external lines denote the gravitational field and the internal lines represent hot matter and gravitational fields in thermal equilibrium.

There exists in the literature several methods for the evaluation of the leading contributions in the high temperature limit, i.e. when  $k_0, |\vec{k}| \ll T$ . In this domain all masses can be

neglected, so one can consider the particles as being effectively massless. The method we use is that of reference [9], in which the Green functions are related to the forward scattering amplitudes of the thermal gravitons, with on-shell momenta  $q$ . This method simplifies considerably the calculations in the present case. Here we will just state the main results.

The relevant Feynman rules follow from the Einstein Lagrangian:

$$\mathcal{L}_{\text{grav}} = \frac{1}{2\kappa^2} \sqrt{-g} R, \quad (2.1)$$

where  $\kappa = \sqrt{32\pi G}$ . Since the leading thermal contributions are gauge independent [4], it is convenient for computational purposes to fix the gauge by choosing:

$$\mathcal{L}_{\text{fix}} = -\frac{1}{\kappa^2} (\partial_\mu \sqrt{-g} g^{\mu\nu})^2. \quad (2.2)$$

Then, subjecting  $\mathcal{L}_{\text{fix}}$  to an infinitesimal gauge transformation characterized by the parameter  $\xi$ , one obtains the Faddeev-Popov ghost Lagrangian:

$$\mathcal{L}_{\text{ghost}} = (\partial_\mu \bar{\chi}_\nu) \frac{\partial(\sqrt{-g} g^{\mu\nu})}{\partial \xi^\lambda} \chi_\lambda. \quad (2.3)$$

We will show that the expression for  $\Pi$  depends in general on the representation of the graviton fields. To this end, let us define the basic graviton field  $h$  by [3]:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \quad (2.4)$$

With the help of the results derived in reference [9], we can express the corresponding graviton 2-point function in the form:

$$\begin{aligned} \Pi_h^{\mu\nu,\alpha\beta}(k) = \omega \frac{\pi^2 T^4}{120} \kappa^2 \int \frac{d\Omega}{4\pi} & \left( \frac{k^\alpha Q^\mu Q^\nu Q^\beta}{k \cdot Q} + \frac{k^\beta Q^\mu Q^\nu Q^\alpha}{k \cdot Q} \right. \\ & + \frac{k^\mu Q^\nu Q^\alpha Q^\beta}{k \cdot Q} + \frac{k^\nu Q^\mu Q^\alpha Q^\beta}{k \cdot Q} - \frac{k^2 Q^\mu Q^\nu Q^\alpha Q^\beta}{(k \cdot Q)^2} \\ & \left. - \eta^{\nu\beta} Q^\mu Q^\alpha - \eta^{\nu\alpha} Q^\mu Q^\beta - \eta^{\mu\beta} Q^\nu Q^\alpha - \eta^{\mu\alpha} Q^\nu Q^\beta \right), \end{aligned} \quad (2.5)$$

where  $\omega$  is a weight factor given by the total number of degrees of freedom of the thermal particles, being 2 for a graviton field.  $Q^\alpha = q^\alpha / |\vec{q}| = (1, \hat{Q})$  and  $\int d\Omega$  denotes angular integration over the 3-dimensional unit vector  $\hat{Q}$ .

On the other hand, we may choose the basic graviton field  $\tilde{h}$  defined by [10]:  $\sqrt{-g} g^{\mu\nu} = \eta^{\mu\nu} + \kappa \tilde{h}^{\mu\nu}$ . Using Eq. (2.4), we can express  $\tilde{h}$  in terms of  $h$  as follows:

$$\tilde{h}^{\mu\nu} = -h^{\mu\nu} + \frac{1}{2} h^\lambda{}_\lambda \eta^{\mu\nu} - \frac{1}{2} h^\lambda{}_\lambda h^{\mu\nu} + h^{\mu\lambda} h^\nu{}_\lambda + \frac{1}{8} (h^\lambda{}_\lambda)^2 \eta^{\mu\nu} - \frac{1}{4} h_{\alpha\beta} h^{\alpha\beta} \eta^{\mu\nu} + \dots \quad (2.6)$$

We then obtain for the corresponding graviton 2-point function the result:

$$\Pi_{\tilde{h}}^{\mu\nu,\alpha\beta}(k) = \Pi_h^{\mu\nu,\alpha\beta}(k) + \omega \frac{\pi^2 T^4}{120} \kappa^2 \int \frac{d\Omega}{4\pi} (\eta^{\nu\beta} Q^\mu Q^\alpha + \eta^{\nu\alpha} Q^\mu Q^\beta + \eta^{\mu\beta} Q^\nu Q^\alpha + \eta^{\mu\alpha} Q^\nu Q^\beta), \quad (2.7)$$

which is clearly different from the one given in (2.5). A similar conclusion is obtained by using as the basic graviton field  $h^*$ , defined by the inverse metric tensor  $g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{*\mu\nu}$ . From Eq. (2.4), we see that  $h^*$  can be expressed in a power series of  $h$  given by:

$$h^{*\mu\nu} = -h^{\mu\nu} + h^{\mu\alpha} h^\nu{}_\alpha + \dots \quad (2.8)$$

However, the graviton self-energy defined by equation (1.1) is found to be independent of these choices. In the above cases,  $\bar{\Pi}$  is given by one and the same expression, namely:

$$\begin{aligned} \bar{\Pi}^{\mu\nu,\alpha\beta}(k) = \omega \frac{\pi^2 T^4}{120} \kappa^2 \int \frac{d\Omega}{4\pi} & \left[ \frac{k^\mu Q^\nu Q^\alpha Q^\beta}{k \cdot Q} + \frac{k^\nu Q^\mu Q^\alpha Q^\beta}{k \cdot Q} \right. \\ & \frac{k^\alpha Q^\mu Q^\nu Q^\beta}{k \cdot Q} + \frac{k^\beta Q^\mu Q^\nu Q^\alpha}{k \cdot Q} - \frac{k^2 Q^\mu Q^\nu Q^\alpha Q^\beta}{(k \cdot Q)^2} \\ & \left. - \frac{1}{2} (\eta^{\nu\beta} Q^\mu Q^\alpha + \eta^{\nu\alpha} Q^\mu Q^\beta + \eta^{\mu\beta} Q^\nu Q^\alpha + \eta^{\mu\alpha} Q^\nu Q^\beta) \right] \end{aligned} \quad (2.9)$$

The angular integrations can be performed with the help of the formulas given in Appendix C of reference [9]. Then, the result for the graviton self-energy  $\bar{\Pi}$  can be expressed in terms of a basis of 14 independent tensors  $T_i^{\mu\nu,\alpha\beta}$ , built from  $\eta^{\mu\nu}$ ,  $u^\mu \equiv \delta_0^\mu$  and:

$$K^\mu \equiv \left( \frac{k_0}{|\vec{k}|}, \hat{k} \right) \equiv (r, \hat{k}). \quad (2.10)$$

Their explicit forms, presented in Tab. I, were previously given by Rebbhan in Ref [3]. In the basis of tensors listed in Tab. I, the expression for the graviton self-energy can be written as follows:

$$\bar{\Pi}^{\mu\nu,\alpha\beta}(k) = \omega \frac{\pi^2 T^4}{30} \kappa^2 \sum_{i=1}^{14} \bar{c}_i(K, r) T_i^{\mu\nu,\alpha\beta}, \quad (2.11)$$

where the 14 structure functions  $\bar{c}_i(K, r)$  are given by:

$$\bar{c}_1 = \frac{1}{12} - \frac{K^2}{24} + \frac{K^4}{8} L(r) \quad (2.12a)$$

$$\bar{c}_2 = -\frac{1}{6} + \frac{K^2}{12} - \frac{5K^4}{24} + \frac{5K^6}{8} L(r) \quad (2.12b)$$

$$\bar{c}_3 = -\frac{K^2}{3} + \frac{7K^4}{12} - \frac{35K^6}{24} + \frac{35K^8}{8} L(r) \quad (2.12c)$$

$$\bar{c}_4 = -\frac{K^2}{24} + \frac{K^4}{8} L(r) \quad (2.12d)$$

$$\bar{c}_5 = \frac{K^2}{12} - \frac{5K^4}{24} + \frac{5K^6}{8} L(r) \quad (2.12e)$$

$$\bar{c}_6 = \left( -\frac{1}{12} + \frac{5K^2}{24} - \frac{5K^4}{8} L(r) \right) r \quad (2.12f)$$

$$\bar{c}_7 = \left( \frac{1}{3} - \frac{7K^2}{12} + \frac{35K^4}{24} - \frac{35K^6}{8} L(r) \right) r \quad (2.12g)$$

$$\bar{c}_8 = -\frac{1}{12} - \frac{5K^2}{24} + \frac{K^2}{2} L(r) + \frac{5K^4}{8} L(r) \quad (2.12h)$$

$$\bar{c}_9 = \frac{1}{6} - \frac{2K^2}{3} - \frac{35K^4}{24} + \frac{15K^4}{4} L(r) + \frac{35K^6}{8} L(r) \quad (2.12i)$$

$$\bar{c}_{10} = \frac{1}{6} - \frac{2K^2}{3} - \frac{35K^4}{24} + \frac{15K^4}{4} L(r) + \frac{35K^6}{8} L(r) \quad (2.12j)$$

$$\bar{c}_{11} = \left( \frac{1}{4} + \frac{35K^2}{24} - \frac{5K^2}{2} L(r) - \frac{35K^4}{8} L(r) \right) r \quad (2.12k)$$

$$\bar{c}_{12} = -\frac{13}{12} - \frac{35K^2}{24} + L(r) + 5K^2 L(r) + \frac{35K^4}{8} L(r) \quad (2.12l)$$

$$\bar{c}_{13} = -\frac{1}{12} - \frac{5K^2}{24} + \frac{K^2}{2} L(r) + \frac{5K^4}{8} L(r) \quad (2.12m)$$

$$\bar{c}_{14} = \left( -\frac{1}{12} + \frac{5K^2}{24} - \frac{5K^4}{8} L(r) \right) r \quad (2.12n)$$

and we have defined:

$$L(r) = \frac{r}{2} \ln \left( \frac{r+1}{r-1} \right) - 1. \quad (2.13)$$

We now recall [3,4], that the trace of the graviton two-point function is related in general to the graviton 1-point function  $\Gamma^{\mu\nu}$ , which is traceless. For instance, in the representation given by the  $h$  field, the trace of the function  $\Pi$  is connected to  $\Gamma$  by the Ward identity:

$$\Pi_{\rho, \mu\nu}^{\rho, \mu\nu}(k) = -\kappa \Gamma_h^{\mu\nu} = -\kappa^2 \omega \frac{\pi^2 T^4}{180} (4\eta^{\mu 0} \eta^{\nu 0} - \eta^{\mu\nu}), \quad (2.14)$$

which is a consequence of the invariance under Weyl transformations. On the other hand, it is easy to verify using (1.1) that the trace of the graviton self-energy vanishes:

$$\bar{\Pi}_{\rho, \mu\nu}^{\rho, \mu\nu}(k) = 0. \quad (2.15)$$

This property indicates that the graviton self-energy is related to the energy-momentum tensor corresponding to massless particles, which is also traceless [3,4]. Indeed, using equation (2.9), we obtain:

$$k_\alpha k_\beta \bar{\Pi}^{\mu\nu, \alpha\beta}(k) = -\frac{\kappa^2}{4} k^2 T_{(0)}^{\mu\nu}, \quad (2.16)$$

where  $T_{(0)}^{\mu\nu}$  denotes the background energy-momentum tensor:

$$T_{(0)}^{\mu\nu} = \text{diagonal}(\rho, p, p, p) \quad (2.17)$$

and  $\rho = \omega \pi^2 T^4 / 30$  is the energy density of the thermal particles, with the pressure assuming the maximal value  $\rho/3$ . It follows from these relations that, at  $k^2 = 0$ , the graviton self-energy satisfies the transversality condition,

$$k_\alpha k_\beta \bar{\Pi}^{\mu\nu, \alpha\beta}(k) \Big|_{k^2=0} = 0. \quad (2.18)$$

As a consequence of invariance under general coordinate transformations, the 1PI graviton 2-point function  $\Pi$  satisfies a Ward identity [3,4], relating its divergence to the 1-point graviton function  $\Gamma$ . At zero temperature, it turns out that this function may consistently be equated to zero [10], so that an equation like (2.18) holds directly for  $\Pi$ . But at finite temperatures, the 2-point graviton function is not transverse since the tadpole  $\Gamma$  is non-vanishing, being connected to the trace of  $\Pi$  (cf. Eq. (2.14)). In such cases, these Ward identities enforce the graviton self-energy  $\bar{\Pi}$  to obey the above physical transversality condition.

The properties expressed by equations (2.15) and (2.18) are important for the study of the relevant conditions which ensure the invariance of the self-energy (1.1) under reparametrizations of the graviton fields.

### III. DISCUSSION

In order to investigate this problem in a general way, we consider the effective action [4]:

$$S_{eff} = \Gamma_i^{\alpha\beta} h_{,\alpha\beta}(0) + \frac{1}{2} \int d^4k \Pi_i^{\mu\nu,\alpha\beta}(k) h_{,\mu\nu}(k) h_{,\alpha\beta}(-k) + \dots, \quad (3.1)$$

which generates the one-particle irreducible Green functions of the field theory at high temperature. The most general re-parametrization of the graviton fields, consistent with Lorentz invariance, can be written in a compact form as:

$$h_i^{\mu\nu} = a h^{\mu\nu} + b h^\lambda{}_\lambda \eta^{\mu\nu} + c h^\lambda{}_\lambda h^{\mu\nu} + d h^{\mu\lambda} h^\nu{}_\lambda + e (h^\lambda{}_\lambda)^2 \eta^{\mu\nu} + f h_{\alpha\beta} h^{\alpha\beta} \eta^{\mu\nu} + \dots, \quad (3.2)$$

where  $a, b, c, d, e$  and  $f$  denote arbitrary constants and we have chosen  $h_{,\mu\nu}$ , for definiteness, to be the field defined by Eq. (2.4). The examples given by Eqs. (2.6) and (2.8) represent particular cases of this relation. Under the above transformation, the action (3.1) can be expanded in a power series of the field  $h$ .

On the other hand, the effective action should be invariant under a re-parametrization of the graviton fields. Hence we must have that:

$$S_{eff} = \Gamma^{\alpha\beta} h_{,\alpha\beta}(0) + \frac{1}{2} \int d^4k \Pi^{\mu\nu,\alpha\beta}(k) h_{,\mu\nu}(k) h_{,\alpha\beta}(-k) + \dots, \quad (3.3)$$

where, for simplicity of notation, we drop the index  $i$  from the above Green functions. Identifying the corresponding terms given by the forms (3.1) and (3.3), we find:

$$\Gamma^{\mu\nu} = a \Gamma_i^{\mu\nu}, \quad (3.4)$$

where we used the traceless property of  $\Gamma$  [cf. Eq. (2.14)] and:

$$\begin{aligned} \Pi^{\mu\nu,\alpha\beta}(k) &= a^2 \Pi_i^{\mu\nu,\alpha\beta}(k) + a b \left( \Pi_i^{\mu\nu,\rho} \eta^{\alpha\beta} + \Pi_i^{\alpha\beta,\rho} \eta^{\mu\nu} \right) \\ &+ c \left( \Gamma_i^{\mu\nu} \eta^{\alpha\beta} + \Gamma_i^{\alpha\beta} \eta^{\mu\nu} \right) \\ &+ \frac{d}{2} \left( \Gamma_i^{\mu\alpha} \eta^{\nu\beta} + \Gamma_i^{\nu\beta} \eta^{\mu\alpha} + \Gamma_i^{\nu\alpha} \eta^{\mu\beta} + \Gamma_i^{\mu\beta} \eta^{\nu\alpha} \right) \end{aligned} \quad (3.5)$$

From the above relations we obtain, with the aid of (2.14), the Ward identity obeyed by the function  $\Pi_i$ :

$$\Pi_i^{\rho,\mu\nu} = -\frac{a+4c+2d}{a(a+4b)} \Gamma_i^{\mu\nu}, \quad (3.6)$$

where  $a \neq 0$  and  $a+4b \neq 0$ , since we assume that (3.2) can be inverted.

We now define  $\bar{\Pi}_i$ , analogously to equation (1.1), with  $\Pi$  replaced on the right hand side by  $\Pi_i$ . Then, with the help of the Ward identity (3.6), it is straightforward to deduce the following relation:

$$\begin{aligned} \bar{\Pi}^{\mu\nu,\alpha\beta}(k) - a^2 \bar{\Pi}_i^{\mu\nu,\alpha\beta}(k) &= \left( c - \frac{a+4c+2d}{a+4b} b \right) \left( \eta^{\mu\nu} \Gamma_i^{\alpha\beta} + \eta^{\alpha\beta} \Gamma_i^{\mu\nu} \right. \\ &\quad \left. - \eta^{\mu\alpha} \Gamma_i^{\nu\beta} - \eta^{\mu\beta} \Gamma_i^{\nu\alpha} \right. \\ &\quad \left. - \eta^{\nu\alpha} \Gamma_i^{\mu\beta} - \eta^{\nu\beta} \Gamma_i^{\mu\alpha} \right) \end{aligned} \quad (3.7)$$

It is easy to see [cf. Eq. (2.15)] that the trace of the expression on the left-hand side of the above equation vanishes. This is consistent with the form appearing on its right hand side, because  $\Gamma_i$  is a traceless function.

As pointed out following Eq. (2.18), we should require the physical amplitude associated with a spin-2 massless graviton to satisfy the transversality condition:

$$k_\alpha k_\beta \bar{\Pi}_i^{\mu\nu,\alpha\beta}(k) \Big|_{k^2=0} = 0, \quad (3.8)$$

which reflects the underlying gauge invariance of the effective action. The above constraint then implies the vanishing of the expression on the right-hand side of Eq. (3.7), yielding the relation:

$$a b - a c + 2 b d = 0. \quad (3.9)$$

This is explicitly verified for all the representations of the graviton fields discussed in the previous section. Consequently, under the physical constraint expressed by (3.8), the Eq. (3.7) reduces to:

$$\bar{\Pi}^{\mu\nu,\alpha\beta}(k) - a^2 \bar{\Pi}_i^{\mu\nu,\alpha\beta}(k) = 0. \quad (3.10)$$

We next rescale the fields  $h_i^{\mu\nu}$  to:  $h_i^{\mu\nu} = a^{-1} h_i^{\mu\nu}$  [cf. Eq. (3.2)]. Making use of the same method which lead to Eqs. (3.4) and (3.5), we readily find:

$$\Gamma_{\mu\nu}^{\mu\nu} = a \Gamma_{\mu\nu}^{\mu\nu} \quad (3.11)$$

and

$$\Pi_{\mu\nu}^{\mu\nu, \alpha\beta}(k) = a^2 \Pi_{\mu\nu}^{\mu\nu, \alpha\beta}(k) \quad (3.12)$$

Then, it follows from (3.10) and (3.12) that the graviton self-energy will be invariant under general re-parametrizations, subject to the constraint (3.9), in the sense that:

$$\bar{\Pi}^{\mu\nu, \alpha\beta}(k) = \bar{\Pi}_{\mu\nu}^{\mu\nu, \alpha\beta}(k), \quad (3.13)$$

where we have defined  $\bar{\Pi}_{\mu\nu}$  similarly to  $\bar{\Pi}$ . We emphasize that this result is general, following in consequence of the invariance of the effective action under coordinate and Weyl transformations.

In conclusion, we wish to comment on the mechanism which enforces the above property of the thermal graviton self-energy. To this end, using the Ward identity (3.6) together with the relations (3.4), (3.11) and (3.12), we obtain from Eq. (3.13) that:

$$\Pi_{\mu\nu}^{\mu\nu, \alpha\beta}(k) = \Pi^{\mu\nu, \alpha\beta}(k) + \frac{\kappa}{2} \left( \frac{2b - 2c - d}{a + 4b} \right) \left( \eta^{\mu\alpha} \Gamma^{\nu\beta} + \eta^{\mu\beta} \Gamma^{\nu\alpha} + \eta^{\nu\alpha} \Gamma^{\mu\beta} + \eta^{\nu\beta} \Gamma^{\mu\alpha} \right), \quad (3.14)$$

which shows generally that the 1PI graviton 2-point function is dependent on the parametrization of the graviton fields. Clearly, this behavior occurs because of the non-vanishing of the 1-point function.

Using the above relation and Eq. (1.1), we can write the expression (3.13) in the form:

$$\bar{\Pi}^{\mu\nu, \alpha\beta}(k) = \Pi_{\mu\nu}^{\mu\nu, \alpha\beta}(k) + \Pi_{\mu\nu}^{\mu\nu, \alpha\beta}, \quad (3.15)$$

with  $\Pi_{\mu\nu}^{\mu\nu, \alpha\beta}$  given by:

$$\Pi_{\mu\nu}^{\mu\nu, \alpha\beta} \equiv \frac{\kappa}{4} \left( \frac{a + 4c + 2d}{a + 4b} \right) \left( \eta^{\mu\alpha} \Gamma^{\nu\beta} + \eta^{\mu\beta} \Gamma^{\nu\alpha} + \eta^{\nu\alpha} \Gamma^{\mu\beta} + \eta^{\nu\beta} \Gamma^{\mu\alpha} \right). \quad (3.16)$$

We may represent  $\Pi_{\mu\nu}^{\mu\nu, \alpha\beta}$  diagrammatically as shown in Fig. 2. It is important to distinguish these graphs from the usual tadpole contributions to the graviton self-energy, which are very ambiguous, being actually infinite. In our case, the contributions given by Eq.

(3.16) are well defined and finite. Furthermore, these are also gauge independent, being the same for internal scalars, quarks or gravitons, up to numerical factors which just count degrees of freedom. We see that  $\Pi_{\mu\nu}^{\mu\nu, \alpha\beta}$  depends on the choice of the basic graviton fields in a way that ensures  $\bar{\Pi}$  to be independent of this parametrization. Hence, in order to obtain a physical self-energy, one must consider in addition to the 1PI 2-point function, also the corresponding tadpole contributions. It is interesting to note that, when  $a + 4c + 2d = 0$ , the graviton self-energy can be identified with the proper 2-point function, since in this case  $\Pi_{\mu\nu}^{\mu\nu, \alpha\beta}$  vanishes.

Finally, we mention that tadpole-free graviton fields might describe the metric perturbations at high temperatures, in a given background spacetime. Such fields arise naturally in the context of a radiation dominated Robertson-Walker universe [3,11].

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## TABLES

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$$T_1^{\mu\nu,\alpha\beta} = \eta^{\alpha\nu} \eta^{\beta\mu} + \eta^{\alpha\mu} \eta^{\beta\nu}$$

$$T_2^{\mu\nu,\alpha\beta} = u^\mu (u^\beta \eta^{\alpha\nu} + u^\alpha \eta^{\beta\nu}) + u^\nu (u^\beta \eta^{\alpha\mu} + u^\alpha \eta^{\beta\mu})$$

$$T_3^{\mu\nu,\alpha\beta} = u^\alpha u^\beta u^\mu u^\nu$$

$$T_4^{\mu\nu,\alpha\beta} = \eta^{\alpha\beta} \eta^{\mu\nu}$$

$$T_5^{\mu\nu,\alpha\beta} = u^\mu u^\nu \eta^{\alpha\beta} + u^\alpha u^\beta \eta^{\mu\nu}$$

$$T_6^{\mu\nu,\alpha\beta} = u^\beta (K^\nu \eta^{\alpha\mu} + K^\mu \eta^{\alpha\nu}) + K^\beta (u^\nu \eta^{\alpha\mu} + u^\mu \eta^{\alpha\nu})$$

$$+ u^\alpha (K^\nu \eta^{\beta\mu} + K^\mu \eta^{\beta\nu}) + K^\alpha (u^\nu \eta^{\beta\mu} + u^\mu \eta^{\beta\nu})$$

$$T_7^{\mu\nu,\alpha\beta} = K^\nu u^\alpha u^\beta u^\mu + K^\mu u^\alpha u^\beta u^\nu + K^\beta u^\alpha u^\mu u^\nu + K^\alpha u^\beta u^\mu u^\nu$$

$$T_8^{\mu\nu,\alpha\beta} = K^\beta K^\nu \eta^{\alpha\mu} + K^\beta K^\mu \eta^{\alpha\nu} + K^\alpha K^\nu \eta^{\beta\mu} + K^\alpha K^\mu \eta^{\beta\nu}$$

$$T_9^{\mu\nu,\alpha\beta} = K^\mu K^\nu u^\alpha u^\beta + K^\alpha K^\beta u^\mu u^\nu$$

$$T_{10}^{\mu\nu,\alpha\beta} = (K^\beta u^\alpha + K^\alpha u^\beta) (K^\nu u^\mu + K^\mu u^\nu)$$

$$T_{11}^{\mu\nu,\alpha\beta} = K^\beta K^\mu K^\nu u^\alpha + K^\alpha K^\mu K^\nu u^\beta + K^\alpha K^\beta K^\nu u^\mu + K^\alpha K^\beta K^\mu u^\nu$$

$$T_{12}^{\mu\nu,\alpha\beta} = K^\alpha K^\beta K^\mu K^\nu$$

$$T_{13}^{\mu\nu,\alpha\beta} = K^\mu K^\nu \eta^{\alpha\beta} + K^\alpha K^\beta \eta^{\mu\nu}$$

$$T_{14}^{\mu\nu,\alpha\beta} = (K^\nu u^\mu + K^\mu u^\nu) \eta^{\alpha\beta} + (K^\beta u^\alpha + K^\alpha u^\beta) \eta^{\mu\nu}$$


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TABLE I. A basis of 14 independent tensors

FIGURES

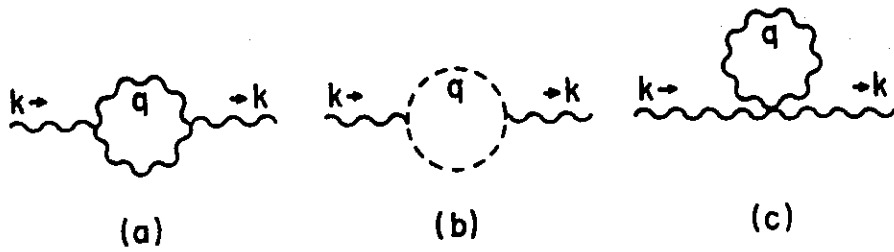


FIG. 1. Lowest order contributions to the thermal 1PI graviton 2-point function. Wavy lines denote gravitons and dashed lines represent ghost particles

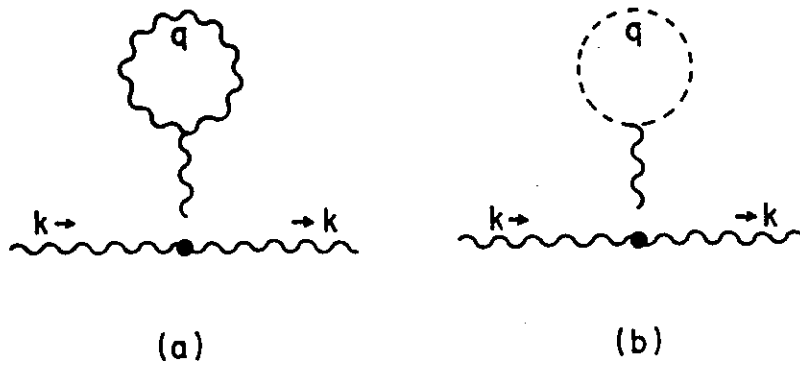


FIG. 2. One-loop tadpole contributions of graviton and ghost fields to  $\Pi_{// tad}$ . The black dot stands for terms proportional to the  $\eta$  functions in Eq. (3.16)