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EVAPORATION AS A PROBE OF MATTER EVOLUTION IN RELATIVISTIC HEAVY ION COLLISIONS

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ABSTRACT

Evaporation of particles at the surface of thermalized expanding matter is studied. It is shown to have a sizable effect on the pion transverse mass spectrum so should be included in attempts to develop a complete hydrodynamical description of relativistic nuclear collisions at future accelerators. In addition, if hydrodynamical flow has indeed been established in current high energy nuclear collisions, evaporation helps to get a more consistent picture with data.

1. INTRODUCTION

At the present moment, the theoretical description of relativistic nuclear collisions is still quite controversial. On one extreme, one may try to describe heavy ion collisions as a superposition of nucleon-nucleon collisions. On the other extreme, one may apply a statistical description, assuming that a complete thermalization is attained. Technically speaking, the second description has an advantage over the first one: such poorly understood details as nucleon structure or mechanisms for nucleon-nucleon or nucleus-nucleus collisions required in the first approach can be forgotten thanks to the thermalization assumption. If this assuption is true, then one can rely on more sound methods of statistical physics.

The basic origin of this ambiguity is that we do not know the thermalization time, or time needed for the particles created early in a collision to evolve from the initial state to a thermal equilibrium state. A reliable estimate of the thermalization time can only be done by a microscopic dynamical description of the reaction, which requires knowledge of interaction cross sections, initial density reached, etc, i.e. quantities that are not well established yet. It is however thought that [1], due to the higher multiplicities and longer dense matter lifetimes available, states of thermal equilibrium should be reached (if they have not been reached yet) at the future accelerators, RHIC and LHC, that will be in use before the end of the decade. In this sense it is important to develop a complete hydrodynamical description of relativistic heavy ion collisions.

There is indeed a lot of activity in this direction. Full three-dimensional hydrodynamical codes (i.e. codes where the relativistic laws of conservation of momentum-energy and baryon number are numerically solved point by point) are becoming available [2, 3] and transverse momentum and rapidity distributions are predicted. These took over more simplified solutions [4, 5, 6, 7]. Finer details are now being studied. The effect of the freeze out criteria and initial conditions are tested using such codes or easier to handle semi-numerical approaches [8, 9, 10, 11]. The impact of resonance decays (in particular in connection with the observed low-p_t pion enhancement) is being evaluated both in static thermal models and hydrodynamical models [12, 13, 14, 15]. The aim of this paper is to study evaporation of particles at the surface of the thermalized expanding matter, this effect has not received much attention yet but has interesting qualitative features and may be quantitatively important in a hydrodynamical description.

2. THEORETICAL DESCRIPTION OF EVAPORATION

The phenomenon that we want to describe is the following. Initially, the energy density in a slice of matter perpendicular to the beam axis has some profile which, to fix ideas we take to be of a Wood-Saxon type: $\epsilon(r,t_0) \propto 1/[1+exp((r-r_0)/a)]$ (e.g. for sulphur nuclei, r_0=3.5 and a=0.5). We can distinguish three regions. First, particles at large r are so diluted that they do not interact and free stream towards the detectors. Second, the particles at small r behave as a fluid. Third, at intermediate r, we can consider that there is a mixture of fluid and free particles: particles less than a (local) mean free path away from the free particle region (or one could use, less than a mean free path from the exterior) can also escape towards the detectors without collisions if their momentum is outward oriented. This is depicted in figure 1. As time increases, the mixture region travels inward until the particles at r=0 have a mean free path big enough to be emitted whatever the direction of their momentum, at some time τ_f.

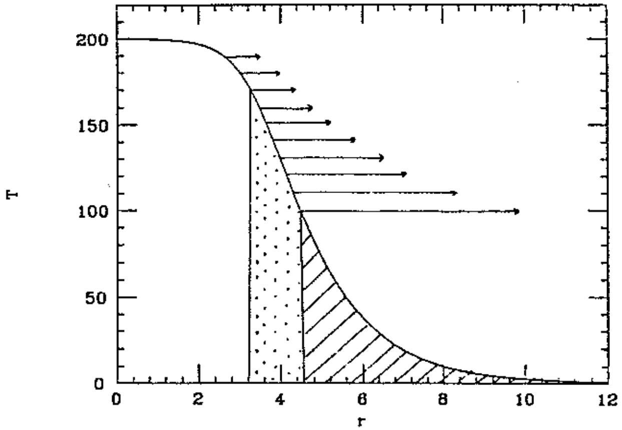

Figure 1: Initial temperature profile. The initial energy density profile is supposed to be of a Wood-Saxon type. The dashed area corresponds to the free particle region, the dotted one to the mixture region and the white one to the fluid region. Arrows indicate length of mean free path $(1/[\sigma_{\pi\pi} n_\pi(T)]$ with $\sigma_{\pi\pi} \sim \sigma_{NN}$=40 mb) at the corresponding temperature.

In this description, particles may be emited because they are in diluted matter similarly to the usual freeze our scenario. In addition they may be emitted because they are 1) traveling outward and 2) close enough to the surface to leave without collision: we say they evaporate. If condition 1) or 2) is not satisfied, these particles would not be emitted because they are not in diluted matter. Via evaporation we may have access to temperatures well above freeze out. This is in contrast to the usual scenario where particles are emitted only at freeze out and therefore the observables only keep track of low densities and low temperatures. Evaporating particles act as a probe of the thermalized matter history. This has implications for transverse mass spectra, final abundances of particles, etc.

The transverse mass distribution of the final state particles is an integration over time (from τ_0 to τ_{fo}) of the various shells of particles emitted at various temperatures from the mixture and free particle regions. To actually compute the transverse momentum spectra, we use the Cooper-Frye formula [16]

$$E\frac{d^3N}{dp^3} = \frac{dN}{dy\,d^2\vec{p_\perp}} = \frac{g}{(2\pi)^3}\int \frac{p^\mu . d\sigma_\mu}{\exp[p.u-\mu]/T \pm 1} \tag{1}$$

where the integral is over the three-dimensional surface crossed by the particles and whose normal vector is $d\sigma_\mu$. This is a hydrodynamical generalization of the formula from statistical mechanics

$$\frac{d^3N}{dp^3} = \frac{g}{(2\pi)^3}\int d^3x \frac{1}{\exp[(E-\mu)/T] \pm 1} \tag{2}$$

We need to know the time and space dependence of the temperature T, chemical potentials μ and flow velocity u^μ as well as choose the 3-surface to actually calculate quantities with equation (1). To do that, we are going to make a number of simplifying assumptions. What we want to do is calculate orders of magnitude and point out properties of evaporation. In ref. [17], we make a more detailed description.

We suppose, to fix a frame to perform simple calculations, that the longitudinal expansion can be described by the Bjorken solution [4], in particular we neglect baryonic number and do not consider transverse expansion. For simplicity we keep the radius of the dense matter region constant in time and assume that evaporation occurs on the surface defined by that radius (rather than on a shell close to the exterior). In this case, since the evaporating radius never reaches zero, particle emission stops when all matter is diluted, let us say for simplticy below some temperature T_{fo}. In reference [17], we release this assumption of emission at fixed radius. The simple example presented here maintains the gross features of the more precise solution.

Because the radius of emission is kept fixed, the transverse momentum spectrum has another component in addition to a time integrand, this new term comes from freezing out particles at $\rho < R$. This is the expected term in the usual freeze out scenario. The condition $T(t,\vec{x}) = T_{fo}$ fixes a surface $t(\rho,z)$ and the vector $d\sigma_\mu$ reads $\rho d\rho d\phi dz\,(1, -\partial t/\partial\rho\cos\phi_p, -\partial t/\partial\rho\sin\phi_p, -\partial t/\partial z)$ So we get (with $\tanh\eta \equiv z/t$)

$$\begin{aligned}\frac{dN}{dy m_\perp dm_\perp} &= \frac{g}{(2\pi)^3}\int d\phi_p \rho d\rho d\phi dz \frac{m_\perp(\cosh y - \sinh y\,\partial t/\partial z) - p_\perp \partial t/\partial\rho\cos(\phi-\phi_p)}{\exp[m_\perp ch(\eta-y)/T_{fo}] \pm 1} \\ &= \frac{gR^2\tau_f}{2\pi} m_\perp \sum (\mp)^{(n+1)} K_1(\frac{n m_\perp}{T_{fo}})\end{aligned} \tag{3}$$

This is a standard result and can be found for instance in reference [18]. The last equation pre-supposes that all particles freeze out together at some time τ_{fo} (see below).

Now we turn to the evaporation term. The condition $\rho = R$ defines a surface of normal vector $d\sigma_\mu = Rd\phi dtdz(0, \cos\phi, \sin\phi, 0)$. So we get

$$\begin{aligned} \frac{dN}{dym_\perp dm_\perp} &= \frac{gR}{(2\pi)^3} \int d\phi_p dtdzd\phi \frac{p_\perp \cos(\phi - \phi_p)}{exp[m_\perp \cosh(\eta - y)/T_\tau] \pm 1} \\ &= \frac{gR}{\pi^2} \int_{\tau_0}^{\tau_{fo}} \tau d\tau p_\perp \sum (\mp)^{(n+1)} K_0(\frac{nm_\perp}{T(\tau)}) \end{aligned} \tag{4}$$

This (new) result shows that at various times, particles that evaporates contribute to the transverse mass distribution with various temperatures. The final distribution is the sum of equations (3) and (4).

To actually complete our calculations of equations (3) and (4), we need to know the time dependence of the evaporating particle temperature. The change of the temperature is due to the hydrodynamical expansion and the evaporation. To compute this, we write the conservation of energy in a thin dense matter slice of radius R, perpendicular to the collision axis, extending from -Δ to +Δ around $z = 0$ (i.e. a slice at rest in the center of mass). For simplicity we suppose that the temperature is initially ρ independent inside the disk and stays ρ independent at later times. This amounts to say that the loss of energy at the surface due to evaporation is imediately transmited inside the whole slice, i.e. that there is an instantanous re-thermalization at every instant, in the whole slice. Also, all particles at $\rho < R$ freeze out together. In ref. [17], a more realistic calculation is done with energy being removed via evaporation close to the exterior only. Again the main features of the more precise solution are described by the simplified solution presented here. Conservation of energy then reads

$$\partial_t E = -\int dS_z j^z - \int dS_\perp j^\perp \tag{5}$$

In the last equation, the amount of energy inside the disk (left hand side term) changes with time because due to longitunal expansion, some fluid goes out along the collision axis Oz (first term in the right hand side), transporting energy and executing work to push neighboring volumes, and because close to surface, particles can also escape (second term in the right and side). We describe matter $\rho < R$ as a perfect fluid (with equation of state $p = c_s^2 \epsilon$) and evaporating matter as free particles. We get

$$\partial_t E \mid z = 0 \quad = \quad \partial_t \int \epsilon \rho d\rho d\phi dz = \partial_t \epsilon \times \pi R^2 2\Delta \tag{6}$$

$$\begin{aligned} \int j^z . dS_z &= \int \partial_z T_{fl}^{0z} = (1 + c_s^2)\epsilon/t \times \pi R^2 2\Delta \\ \int j^\perp . dS_\perp &= \frac{3}{(2\pi)^3} \int d^3p \frac{p_\perp cos(\phi - \phi_p)}{e^{E/T} - 1} Rd\phi dz = \frac{3}{(2\pi)^3} \int d^3p \frac{p_\perp}{e^{E/T} - 1} \times 2R2\Delta \end{aligned}$$

It is easy to integrate this and get

$$\epsilon(t) = \epsilon_0 (\frac{t_0}{t})^{1+c_s^2} - \frac{3}{(2\pi)^3} \frac{2}{\pi R} \int d^3p \frac{p_\perp}{t^{1+c_s^2}} \int_{t_0}^{t} dt' \frac{t'^{1+c_s^2}}{e^{E/T(t')} - 1} \tag{7}$$

The first term in $\epsilon(t)$ is the usual one in the Bjorken solution [4] showing how the energy density decreases due to the longitudinal expansion of the volume occupied by matter. The second term indicates that, due to evaporation, the energy density in fact decreases more quickly.

Neglecting the pion mass, equation (7) gives

$$T(t) \sim T_0 \left(\frac{t_0}{t}\right)^{1/3} e^{[-15 \times I/(\pi^5 R)]\,(t - t_0)} \tag{8}$$

where $I = \int dxdyx^2/[exp(\sqrt{x^2 + y^2}) + 1] \sim 5.1$. Again, the first factor in equation (8) is the Bjorken solution [4] and will dominate for small times. The second factor in (8) is exponential because in that case, evaporation is proportional simply to what is contained in the disk.

3. NUMERICAL RESULTS

The time behavior of temperature is depicted below. We show explicitly the expansion and evaporation contributions to the cooling. We see that $T(t) \simeq T_{Bjorken}(t)$ at least for values of t below freeze out time. Since the effect of evaporation on the temperature is much smaller than that of expansion at each instant, we expect that there will be no breakdown of hydrodynamics.

We now show the transverse mass distribution for light and heavy particles. For pions, at small $m_\perp$, freeze out and evaporation are comparable and the slope reflects the freeze out temperature. For high transverse mass, evaporation dominates and the slope reflects the initial temperature. This gives rise to a sizable curvature of the spectrum. Note that we have not included resonance decays though in principle, at high temperature (e.g. T_0), there should be a copious number of them decaying into low $p_\perp$ pions. This will tend

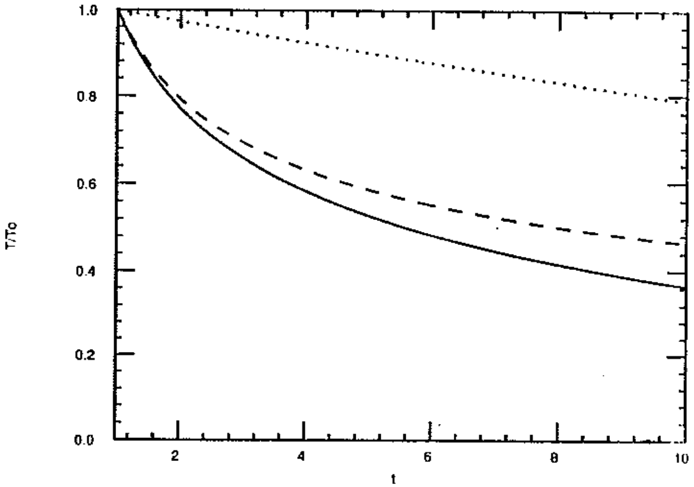

Figure 2: Temperature as a function of time. Dashed line is the Bjorken part, dotted line is the evaporation contribution and solid line is Bjorken plus evaporation. Equation (8) is evaluated with t_0=1 fm and R=3.7 fm.

to increase the curvature. It is important to realize that though evaporation contributes strongly to the distribution, there is no breakdown of hydrodynamcis at each instant. The strong influence of evaporation comes from 1) the fact that at high T, there is a lot of particles, so even if a small percentage escapes at early time it will still be sizable compared to the freeze out contribution 2) the evaporation contribution is an integral over time.

For nucleons, evaporation dominates and the slope reflects the initial temperature all along. Note that this result concerns thermal nucleons and comes from the fact that heavy particles are normally very suppressed when the temperature decreases. To take into account baryon number, a term $exp[\mu(t)/T(t)]$ should be included in the integrand of the evaporation term in equation (4) and a term $exp(\mu_{fo}/T_{fo})$ in the freeze out term in equation (3). If μ/T stays constant, then taking into account baryon number only affects the overall scale of the spectrum. Otherwise, the time evolution of μ/T will affect the precise shape of the spectrum.

These conclusions can be contrasted with those for dense matter with longitudinal and transverse expansion. There, the high $p_\perp$ part of the transverse mass spectra has a slope that reflects the freeze-out temperature blue-shifted by the fluid transverse expansion speed. In addition, all the particle types that decouple at the same temperature

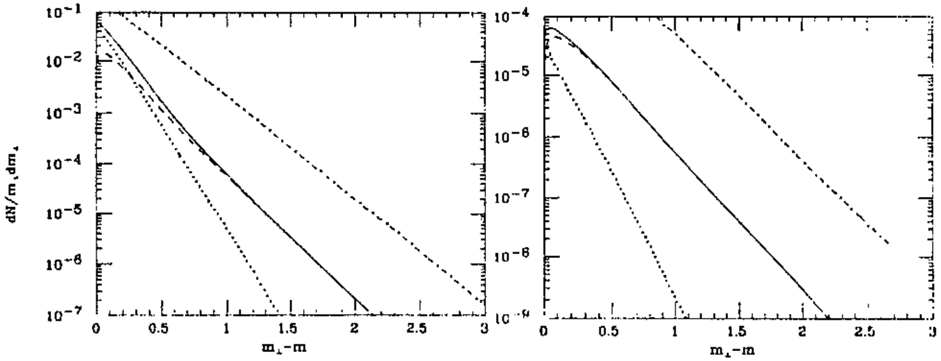

Figure 3: The various contributions to the observed transverse momentum distribution (equations (3) plus (4) a) for pion b) for nucleon. Dashed line is the evaporation contribution, dotted line is the freeze out contribution and solid line is the combination of both. For comparison, a thermal distribution (same formula as for freeze out) is also shown, with temperature T_0, as a dash-dotted line. We took t_0=1 fm, T_0=200 MeV and T_{fo}=100 MeV.

acquire the same expansion speed (in addition to thermal speed) but different momentum. Thus, the transverse mass spectrum of heavier particles extends to higher value of $p_\perp$ giving a higher value of inverse slope. So for increasing particle mass, we expect increasing apparent temperature (at large $p_\perp$), i.e. an ordering of the temperatures [19] (see also [9]). However at really high value of $p_\perp$ ($p_\perp \gg m$) the theoretically predicted curves look parallel whatever the mass.

4. COMPARISON WITH EXPERIMENTAL DATA

Since we have worked with a simplified model, it would be unwise to use it to fit data. Instead we want to see if its features go qualitatively in the right direction to reproduce data and if there are quantitatively sizable.

Data on transverse mass spectra have been obtained by most experiments. At Cern, NA34, NA35, WA80, EMU05 seem to agree to say that the pion spectrum has a curvature [20] and NA35, NA36, WA85 find within error bars similar slopes for heavy particles [21]. This is in agreement with what our simple model predicts and below, we superpose its predictions for $T_0 = 200 MeV$ and $T_{fo} = 100 MeV$ with NA35 S+S data. Taking into account resonance decays would permit a higher value of T_{fo}. Other hydrodynamical

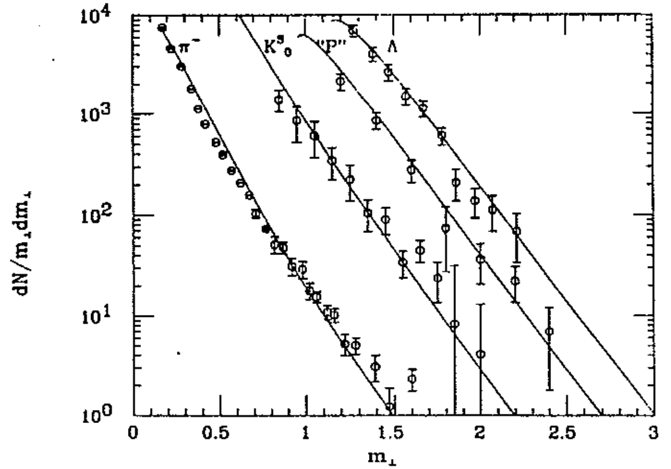

Figure 4: Transverse mass spectra. The points are NA35 data (see reference [21]) and curves are from equation (4) plus (5) implemented with (10) and for T_0=200 MeV and T_{fo}=100 MeV. This is not a least square fit.

models *without evaporation* are able to reproduce NA35 data see e.g. Lee et al. [22] for a model with spherical expansion (the authors noted that the first points in the pion spectrum were not well predicted and suggested that this might be due to resonance decays not included) and Ornik & Weiner [15] for a model with longitudinal and transverse expansion and resonance decays.

There exist however some data where *evaporation might be crucial*, namely particle ratios such as those obtained at Cern by NA35, NA36 and WA85. Various groups (see e.g. references [23, 24]) have shown that to reproduce with a non-expanding thermal gas the WA85 ratios $\overline{\Lambda}/\Lambda$, $\overline{\Xi}^-/\Xi^-$, Ξ^-/Λ and $\overline{\Xi}^-/\overline{\Lambda}$ and NA35 ratios $\overline{\Lambda}/\Lambda$ and K^0_S/Λ, temperatures of order 200 MeV are needed. Such temperatures are hard to understand in term of freeze out and transverse expansion that only "mimicks" high temperatures should not help. For instance, in the numerical hydrodynamical description of ref. [25], the $\overline{\Lambda}$ abundance is not reproduced. In our description, since these experimental ratios concern heavy particles, their spectra should be dominated by evaporation i.e. exhibit naturally a high temperature. Ratios involving pions and heavy particles should not correspond simply to a single temperature (pions and heavy particles may be emitted with different

temperatures). We have not yet performed more precise calculations because this requires the knowledge of $exp(\mu(t)/T(t))$.

5. CONCLUSION

The idea that high $p_\perp$ particles could come from evaporation (at the surface of expanding matter) and low $p_\perp$ ones from a different mechanism (e.g. freeze out) was suggested in '76 by Gorenstein et al. [26], as a possible explaination for the change of curvature in the transverse momentum spectrum of secondaries in p-p collisions at the ISR. As no attempt was made there to compute the possible magnitude of evaporation with regard to the "other" mechanism, it was not clear whether the transverse mass spectrum could indeed be fitted or not. Today it also seems controversial to use hydrodynamics to describe ISR data (particularly at the lowest energies). Evaporation was studied as well, though more as a secondary idea, by Glendening and Karant a few years later [27]. These authors, in an attempt to find signals sensitive to the hadronic spectrum, described high energy (10 GeV/nucleon in the center of mass) nuclear collisions via the formation and disassembly due to evaporation of thermal (non-hydrodynamical) fireballs. Recently Lee et al. [9] commented briefly that (concave) experimental pion spectra could be reproduced by evaporation, but only using irrealistic weights in the time integral of equations analogous to Eq.(5). Basically, evaporation has not drawn much attention.

The model that we used is simplified. Even in ref.[17], transverse expansion of the fluid and energy transport due to the temperature gradient are not taken into account. Chemical potentials are not included. Neither are resonance decays. The aim of the present work is to assess whether it is worth investing more time to incorporate surface effects in attempts (particularly numerical ones) to develop a complete hydrodynamical of relativistic nuclear collisions at future accelerators. We think the answer is yes because evaporation can have a sizable effect on the pion transverse mass spectrum and affect the heavy particle spectra as well. In addition, if hydrodynamical flow has indeed been established in high energy nuclear collisions, a scenario with expansion and evaporation may lead to a more consistent explaination of experimental data: it may reproduce not just the shape of spectra but also the ratios of the integrated spectra.

In fact, evaporation and freeze out are different aspects of the same dynamics of the dense gas. The exact particle emission criterion is that a given particle does not interact anymore. This criterion is not easy to manipulate so approximate ones, corresponding to specific properties of particles, are used. The usual freeze out conditions $T \sim T_{fo}$ or $R \sim 1/[\sigma n(T(t))]$ are independent of the initial energy. A better criterion is $R(t) \sim 1/[\sigma n(T(t))]$ [10] or $1/\sum_j \langle \sigma_{ij} v_{ij} \rho_j \rangle \sim min(R(t)/\langle v_i \rangle, -\rho/\dot{\rho})$ [11, 8, 9]. This includes more of the dynamics but still neglects the position of the particle (close or not to the surface) and the direction of its momentum (inward or outward). Here we have kept track of the particle location and direction to compute their contribution to the transverse mass spectrum. Our criterium is closer to the exact one and can be viewed as an improvement over previous ones. However it permits something new, namely the emission of particles from the early hot matter. As a result, the transverse mass spectrum does not carry information only on the freeze out time but on the whole hydrodynamical evolution of the collision. Similarly, final abundances will not reflect the conditions that prevailed at freeze out only. In summary, evaporation probes the whole thermal history of a collision.

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