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COLOR GAUGE AND CONFINEMENT FOR GENTILIONIC QUARKS

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Abstract

A color gauge approach, based on the intermediate S_3 group symmetry, is proposed for gentilionic quarks. Using this gauge in the framework of Dirac's equation, we develop a dynamical model that gives quark confinement.

1. Introduction

In the last few years[1-5] we have developed, according to the postulates of quantum mechanics and the principle of indistinguishability, the concept of general statistics, first proposed by Gentile about fifty years ago.[6-9] In our theory three kinds of particles could exist in nature: bosons, fermions and gentileons. Bosons and fermions are represented by horizontal and vertical Young shapes, respectively, and gentileons would be represented by intermediate Young shapes. Bosonic and fermionic systems are described by one-dimensional totally symmetric (ψ_s) and totally anti-symmetric (ψ_a) wavefunctions, respectively. Gentilionic systems would be described by wavefunctions (Y) with mixed symmetries. Due to very peculiar properties of gentileons, like confinement and non-coalescence of systems, it seemed natural to think quarks as spin-1/2 gentileons. With this hypothesis we have shown that the baryon wavefunctions are given by[3-5] $\psi = \varphi \cdot Y(\text{color})$. The one-dimensional wavefunction $\varphi = (SU(6)xO_3)_{symmetric}$ corresponds, according to the symmetric quark model of baryons, to a totally symmetric state, and the two-dimensional state Y(color) corresponds to the intermediate representation of the symmetry group S_3. In order to preserve the intermediate S_3 symmetry, Y(color)=Y(123) ought to depend on three new quantum states, named color states, blue (|b>), red (|r>) and green (|g>). These states have been taken as the $SU(3)_{color}$ eigenstates.[4,5] We have seen[5] that the color state Y(123)=Y(brg) can be represented by $Y_+(123)$ or $Y_-(123)$, that are two equivalent irreducible representations of S_3. Thus, in what follows, the color state will be represented by $Y_+(brg)$ or $Y_-(brg)$, indicated simply by Y(brg).

2. *Rotations in the color space, color gauge and confinement*

According to the symmetric group S_3, there are six permutation operators(4,5) which leave invariant $|Y(123)|^2 = |Y(brg)|^2$. We have shown(4,5) that these transformations could be interpreted as discrete rotations by angles π and $2\pi/3$, in a three dimension space (X,Y,X), of the equilateral triangle formed by the basic triplet of the $SU(3)_{color}$. In this color space E_3, the axes X and Z correspond to the axes $\tilde{I}_3$ (color isospin) and to $\tilde{Y}$ (color hypercharge), respectively. These rotations, written in terms of the Pauli's matrices, are represented by 2x2 matrices, $\{\eta_i\}, i = 1,3,...,6$, given explicitely in our preceding papers.(4,5) It is clear from these papers the spinorial character of the color state Y(brg).

In our last paper,[5] we proposed a quantum chromodynamics for gentilionic hadrons assuming a SU(3) color gauge. With this hypothesis, the usual QCD and gentilionic QCD have the same gluons and the same Lagrangian density. In these circumstances, both theories will give identical predictions for hadronic properties.

We have called AS_3 the algebra(3) of the symmetric group S_3 spanned by the six vectors $\{\eta_i\}$, $i = 1,2,...,6$. Since the S_3 group admits two generators $a = \eta_4$ and $b = \eta_6$, we can consider AS_3 as being an associative polynomial algebra generated by a and b, $\{\eta_1,\eta_2,...,\eta_6\} = \{I,ba,ab,a,aba,b\}$. These generators, a and b, obey the commutation relation, $ab + ba = -I$. We have also seen(4,5) that this algebra has an invariant, $K^{[2;1]}_{(2,1)} = \eta_4 + \eta_5 + \eta_6 = 0$, with a zero eigenvalue. This invariant, that was named "color Casimir", has a very beautiful and simple interpretation in the color space: "the baryon color charge is an equal to zero constant of motion". This result, that automatically satisfies the Gell-Mann-Nishijima relation, can also be interpreted as a selection rule for quark confinement. Since in our scheme, color and quark confinement rules appear as a consequence of geometrical and symmetry properties defined in the color space E_3, it seems natural to expect that the dynamical confinement of quarks could be deduced from a gauge symmetry based on the E_3 gentilionic characteristics. So, with this in mind, we could write the states $|b\rangle$, $|r\rangle$ and $|g\rangle$, in the $(\tilde{I}_3,\tilde{Y})$ plane, as

$$|b\rangle = \frac{\sqrt{3}}{2}\,|+\rangle - \frac{1}{2}\,|-\rangle, \qquad |r\rangle = \frac{\sqrt{3}}{2}\,|+\rangle + \frac{1}{2}|-\rangle \quad \text{and} \quad |g\rangle = |-\rangle,$$

respectively, where $|+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ and interpret the rotations in this plane as transformations being produced by gluons exchange between quarks. Taking the properties of the hadrons as invariant by these transformations in E_3, we write the gauge field A_μ as(10)

$$A_\mu = \sum_{k=1}^{2} B_\mu^{\ k}\, T_k \tag{2.1}$$

where $B_\mu^{\ k} = \partial\theta^k(x)/\partial x_\mu$, $\theta(x)$ the rotation angles in the color plane and the generators of the internal symmetry group T_k are given by $T_1 = a$ and $T_2 = b$.

In the above approach we would have only two gluon fields, associated with the two generators of the rotations, a and b. It is not our intention to develop here a quantum field theory based in these new gluon fields or to present a rigorous proof for the quark confinement. We intend only to propose a phenomelogical dynamical model that gives quark confinement based on the symmetry properties(4,5) defined in the $(\tilde{I}_3,\tilde{Y})$ plane. This model will be elaborated within the framework of Dirac's equation assuming that the quark is submitted to an external field A_μ given by Eq. (2.1). So, taking, in a first approach, that $B_\mu^{\ 1} = B_\mu^{\ 2} = B_\mu$ and averaging A_μ over the color states, the statefunction $\psi(x)$ of a quark inside a hadron would be described by the Dirac's equation

$$\left[\gamma^{\mu}(p_{\mu} - ig\, B_{\mu}) - imc\right] \psi(x) = 0, \qquad (2.2)$$

where g is the coupling constant for the strong color interaction.

Now, we adopt a simple model for the quark interaction with B_μ: taking the hadron radius as r_o, we assume that the quark moves freely in the region with $r < r_o$ and that there is an interaction between the quark and the field only when it reaches the frontier $r = r_o$. In this interaction the quark color is changed. We also assume that B_μ is a vector field, that is, $B_\mu = (0,\vec{B})$, where $\vec{B} = \nabla\theta(x)$, which corresponds to a Coulomb gauge. Analyzing this interaction in terms of rotations in the $(\bar{I}_3,\bar{Y})$ plane, we see that one color state is effectively transformed into another only when a rotation by angles of π or $2\pi/3$ is accomplished. Thus, we could imagine $\theta(x)$ as a step function that, at the point $r = r_o$, varies from zero up to π or $2\pi/3$ due to the color change in the interaction. This would imply that $\vec{B} = \nabla\theta(x) = \delta(r-r_o)\,\vec{n}$, where $\vec{n}$ is the unit vector in the radial direction. In these conditions Eq. (2.2) becomes

$$\left[i\gamma^{o}\frac{\partial}{\partial t} + i\,\vec{\gamma}.\nabla - i\,g\,\vec{\gamma}.\vec{n}\;\;\delta(r-r_o) - m\right]\psi(x) = 0. \qquad (2.3)$$

In order to solve Eq. (2.3) we use polar co-ordinates and write[11]

$$\psi(x) = \exp(-iEt)\begin{pmatrix} f(r)\;\Omega_{j\ell m} \\ (-1)^{(1+\ell-\ell')/2}\;\; g(r)\;\Omega_{j\ell' m}\end{pmatrix}, \qquad (2.4)$$

where $\Omega_{j\ell m}$ are the spinor spherical harmonics, $\ell = j \pm 1/2$ and $\ell' = 2j - \ell$.

Taking into accont Eq. (2.4) and using the property[11] $\Omega_{j\ell' m} = i^{\ell-\ell'}(\vec{\sigma}.\vec{n})\,\Omega_{j\ell m}$ we get from Eq. (2.3):

$$\begin{aligned}\frac{d}{dr}f(r) + (1+K)\,f(r)/r + g\,\delta(r-r_o)\,f(r) - (E+m)\,g(r) &= 0,\\ \frac{d}{dr}g(r) + (1-K)\,g(r)/r + g\,\delta(r-r_o)\,g(r) + (E-m)\,f(r) &= 0,\end{aligned} \qquad (2.5)$$

where $K = -(\ell+1)$ when $j = \ell + 1/2$ and $K = \ell$ when $j = \ell - 1/2$.

Our Eqs. (2.5) are similar to Eqs. (3.13) obtained by Villani[12] analysing the freedom and confinement of quarks in a classical field-theoretic context. From Eqs. (2.5) we deduce, in agreement with Villani, that $\bar{\psi}\,\vec{\gamma}\,\psi\,.\,\vec{n} = 0$ at $r = r_o$. This implies that there is no flow of quarks through the surface of the hadron. This result can be interpreted as the manifestation, in the Lorentz space, of the confinement rule predicted by the color Casimir.

Thus, in our gentilionic dynamical model, quarks behave as free particles at short distances, but at the same time are confined, in agreement with the successful "bag model".[13]

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