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**INVISIBLY DECAYING HIGGS BOSON AT  $e^+ e^-$  COLLISIONS \***

**O. J. P. Éboli<sup>1</sup>**

Inst. de Física, Universidade de São Paulo,  
Caixa Postal 20516, CEP 01452-990 São Paulo, Brazil

**M. C. Gonzalez-Garcia<sup>2</sup>**

Physics Dept., University of Wisconsin, Madison, WI 53706, USA

**S. F. Novaes<sup>3</sup>**

Instituto de Física Teórica, Universidade Estadual Paulista,  
Rua Pamplona, 145 - CEP 01405-900, São Paulo, Brazil

**J. C. Romão<sup>4</sup>**

Departamento de Física, Instituto Superior Técnico  
Av. Rovisco Pais, 1 - 1096 Lisboa Codex, Portugal

**A. Lopez-Fernandez<sup>5</sup>**

PPE Division, CERN. CH-1211 Geneve 23, Switzerland

**F. de Campos<sup>6</sup> and J. W. F. Valle<sup>7</sup>**

Instituto de Física Corpuscular - C.S.I.C.  
Dept. de Física Teòrica, Universitat de València  
46100 Burjassot, València, Spain

**ABSTRACT**

In many extensions of the standard model the Higgs boson can have substantial "invisible" decay modes, for example, into light or massless weakly interacting Goldstone bosons associated to the spontaneous violation of lepton number below the weak scale. In this work, we first review the model independent limits on the Higgs boson from the analysis of the present LEP samples after including the possibility of invisible decays and study the prospects for LEP II. Next, we study the detectability prospects for such invisible Higgs boson at the Next Linear Collider.

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# 1 Introduction

Recently the LEP experiments on  $e^+e^-$  collisions around the  $Z$  peak have placed important restrictions on the mass of the standard model Higgs boson [1]

$$m_{H_{SM}} \gtrsim 60 \text{ GeV.} \quad (1)$$

There are, however, many reasons to think that there may exist additional Higgs bosons in nature. One such extension of the minimal standard model is provided by supersymmetry and the desire to tackle the hierarchy problem [2]. Another interesting motivation for an enlargement of the Higgs sector is to generate the observed baryon excess by electroweak physics [3]. This, in principle, requires  $m_{H_{SM}} \lesssim 40 \text{ GeV}$  [4] in conflict with eq. (1). This limit can be avoided in models with new Higgs bosons [5]. These could be intimately related to the question of neutrino masses [6]. In fact, one specially attractive motivation for extending the Higgs sector is the generation of neutrino masses whose existence is hinted by present data on solar and atmospheric neutrinos, as well as cosmological observations related to the large scale structure of the universe and the possible need for hot dark matter [7]. Indeed, most extensions of the minimal standard model require the presence of new Higgses to induce neutrino masses [8].

Amongst the extensions of the standard model which have been suggested to generate neutrino masses, the so-called majoron models are particularly interesting and have been widely discussed [8]. The majoron is a Goldstone boson associated with the spontaneous breaking of the lepton number. Astrophysical arguments based on stellar cooling rates constrain its couplings to the charged fermions [9], while the LEP measurements of the invisible  $Z$  width substantially restrict the majoron couplings to the gauge bosons. In particular, models where the majoron is not a singlet under the  $SU(2) \otimes U(1)$  symmetry [10] are now excluded [1].

There is, however, a wide class of models motivated by neutrino physics [11] which are characterized by the spontaneous violation of a global  $U(1)$  lepton-number symmetry by a singlet vacuum expectation value. Unlike the original model of this type [12], this new class of models may naturally explain the neutrino masses required by astrophysical and cosmological observations without introducing any very high mass scale. Another example of this type is provided by supersymmetric extensions of the standard model where R parity is spontaneously violated [13].

In any of these models with the spontaneous violation of a global  $U(1)$  symmetry around (or below) the weak scale the corresponding Goldstone boson has significant couplings to the Higgs bosons, even if its other couplings are suppressed. This implies that

the Higgs boson can decay, with a substantial branching ratio, into the invisible mode [11, 14, 15]

$$h \rightarrow J + J, \quad (2)$$

where  $J$  denotes the majoron.

Such an invisible Higgs decay would lead to events with large missing energy that could be observable at LEP and affect the corresponding Higgs mass bounds. Here, we first review how one can derive, *in a model independent way*, limits on the Higgs boson from the analysis of the present LEP samples and study the prospects for LEP II. Next, we study the detectability prospects for the Higgs boson at the Next Linear Collider.

## 2 Higgs Production

In order to illustrate the main points, we consider the simplest model which contains, in addition to the standard model scalar Higgs doublet, a complex singlet  $\sigma$  carrying a nonzero vacuum expectation value  $\langle\sigma\rangle$ , which breaks a global symmetry. The scalar potential is given by [11, 14]

$$V = \mu_\phi^2 \phi^\dagger \phi + \mu_\sigma^2 \sigma^\dagger \sigma + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\sigma^\dagger \sigma)^2 + \delta (\phi^\dagger \phi) (\sigma^\dagger \sigma). \quad (3)$$

Terms like  $\sigma^2$  are omitted above in view of the imposed  $U(1)$  invariance under which we require  $\sigma$  to transform nontrivially and  $\phi$  to be trivial. Let  $\sigma \equiv \frac{w}{\sqrt{2}} + \frac{R_2 + iI_2}{\sqrt{2}}$ ,  $\phi^0 \equiv \frac{v}{\sqrt{2}} + \frac{R_1 + iI_1}{\sqrt{2}}$ , where we have set  $\langle\sigma\rangle = \frac{w}{\sqrt{2}}$  and  $\langle\phi^0\rangle = \frac{v}{\sqrt{2}}$ . The above potential leads to a physical massless Goldstone boson, namely the majoron  $J \equiv \text{Im } \sigma$ , and two massive neutral scalars  $H_i$  ( $i=1,2$ )

$$H_i = \hat{O}_{ij} R_j, \quad (4)$$

where  $\hat{O}_{ij}$  is an orthogonal mixing matrix.

In order to be able to predict the production rates of these particles in  $e^+e^-$  collisions one needs to know their couplings to the  $Z$  boson. In the simplest model only the doublet Higgs boson  $\phi$  has a coupling to the  $Z$  in the weak basis, not the  $SU(2) \otimes U(1)$  singlet field  $\sigma$ . After diagonalizing the scalar boson mass matrix, one finds that the two CP even mass eigenstates  $H_i$  ( $i=1, 2$ ) have couplings to the  $Z$  involving the mixing matrix,

$$\mathcal{L}_{HZZ} = (\sqrt{2}G_F)^{1/2} M_Z^2 Z_\mu Z^\mu \hat{O}_{i1} H_i. \quad (5)$$

Through these couplings both CP even Higgs bosons may be produced via the Bjorken process. As long as the mixing appearing in eq. (5) is  $\mathcal{O}(1)$ , all Higgs bosons can have significant production rates that are smaller than in the standard model by a factor

$\epsilon_i^2 = \hat{O}_{i1}^2$ . This is a general result which is valid for a large variety of models we are interested in.

### 3 Invisible Higgs Boson Decay

We now turn to the Higgs boson decay rates, which are sensitive to the details of the mass spectrum and to the Higgs potential. For definiteness we focus on the simplest potential, given in eq. (3). In this case, the width for the invisible  $H_i$  decay can be parametrized by

$$\Gamma(H \rightarrow JJ) = \frac{\sqrt{2}G_F}{32\pi} M_{H_i}^3 g_{H_i, JJ}^2, \quad (6)$$

where the corresponding couplings are given by

$$g_{H_i, JJ} = \tan \beta \hat{O}_{i2}. \quad (7)$$

The rate for  $H \rightarrow b\bar{b}$  also gets diluted compared to the standard model prediction, because of the mixing effects. Explicitly one has,

$$\Gamma(H \rightarrow b\bar{b}) = \frac{3\sqrt{2}G_F}{8\pi} M_H m_b^2 (1 - 4m_b^2/M_H^2)^{3/2} g_{H, b\bar{b}}^2, \quad (8)$$

which is smaller than the standard model prediction by the factor  $g_{H, b\bar{b}}^2$ , where

$$g_{H, b\bar{b}} = \hat{O}_{i1}. \quad (9)$$

The width of the Higgs decay to the  $JJ$  relative to the conventional  $b\bar{b}$  mode depends upon the mixing angles. For this simple model it was shown [11] that in large regions of parameter space the Higgs field decays mainly invisibly to majorons and is produced without any substantial suppression relative to the standard model predictions. The same conclusion holds for other models [15].

In summary, the invisible Higgs decay mode is expected to have quite important implications if there exists, as suggested by neutrino physics, a global symmetry that gets broken around the weak scale. From this point of view it is desirable to obtain limits on Higgs bosons that are not vitiated by detailed assumptions on its mode of decay. This can be done from the existing  $Z$  sample at LEP, as we will briefly review below following Ref. [16].

## 4 Experimental Bounds from LEP I

The production and subsequent decay of any Higgs boson, which may decay visibly or invisibly, involves three independent parameters: the Higgs boson mass  $M_H$ , its coupling strength to the  $Z$ , normalized by that of the standard model, call this factor  $\epsilon^2$ , and the invisible Higgs boson decay branching ratio.

One can use the results published by the LEP experiments on the searches for various exotic channels in order to deduce the regions in the parameter space of the model that is already ruled out. Here we briefly summarize the procedure used in Ref. [16] in order to obtain these limits. For each value of the Higgs mass, one calculates the lower bound on  $\epsilon^2$ , as a function of the branching ratio  $BR(H \rightarrow \text{visible})$ . By taking the highest of such bounds for  $BR(H \rightarrow \text{visible})$  in the range between 0 and 1, one obtains the absolute bound on  $\epsilon^2$  as a function of  $M_H$ .

For a Higgs of low mass (below 30 GeV) decaying to invisible particles one considers the process  $Z \rightarrow HZ^*$ , with  $Z^* \rightarrow e^+e^-$  or  $Z^* \rightarrow \mu^+\mu^-$  and combines the results of the LEP experiments on the search for acoplanar lepton pairs [17, 18, 19] which found no candidates in a total sample corresponding to 780.000 hadronic  $Z$  decays. The efficiencies for the detection of the signal range from 20% at very low Higgs masses to almost 50% for  $M_H = 25$  GeV.

For higher Higgs masses the rate of the process used above is too small, and one considers instead the channel  $Z \rightarrow HZ^*$ ,  $Z^* \rightarrow q\bar{q}$ . Here the results of the searches for the standard model Higgs in the channel  $Z \rightarrow Z^*H_{SM}$  with  $H_{SM} \rightarrow q\bar{q}$  and  $Z \rightarrow \nu\bar{\nu}$  can be translated, following Ref. [20]. The efficiency of these searches for an invisible Higgs increases from 25% at  $M_H = 30$  GeV to about 50% at  $M_H = 50$  GeV.

For visible decays of the Higgs boson its signature is the same as that of the standard model one, and the searches for this particle can be applied directly. For masses below 12 GeV one takes the results of a model independent analysis made by the L3 collaboration (Ref. [21]). For masses between 12 and 35 GeV the results in Ref. [17, 20, 21] can be combined; finally for masses up to 60 GeV the combined result of all the four LEP experiments given in Ref. [20] can be used. In all cases the bound on the ratio  $BR(Z \rightarrow ZH)/BR(Z \rightarrow ZH_{SM})$  was calculated from the quoted sensitivity, taking into account the background events where they existed.

As an illustration, we show in Fig. 1 (from Ref. [16]) the exclusion contours in the plane  $\epsilon^2$  vs.  $BR(H \rightarrow \text{visible})$  for the particular choice for the Higgs mass  $M_H = 50$  GeV. The two curves corresponding to the searches for visible and invisible decays are combined

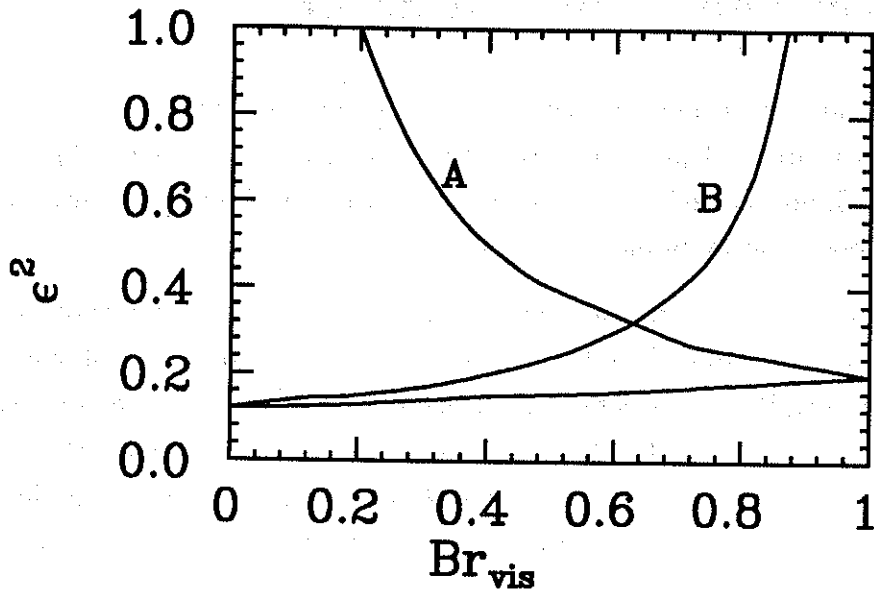


Figure 1: Exclusion contours in the plane  $\epsilon^2$  vs.  $BR(H \rightarrow \text{visible})$  for the particular choice  $m_H = 50$  GeV. The two curves corresponding to the searches for visible (curve A) and invisible (curve B) decays are combined to give the final bound, which holds irrespective of the value of  $BR(H \rightarrow \text{visible})$ .

to give the final bound; values of  $\epsilon^2$  above 0.2 are ruled out independently of the value of  $BR(H \rightarrow \text{visible})$ . The solid line in Fig. 2 shows the region in the  $\epsilon^2$  vs.  $M_H$  that can be excluded by the present LEP analyses, independent of the mode of Higgs decay, visible or invisible.

## 5 Prospects for LEP II

One can also estimate the additional range of parameters that can be covered by LEP II, assuming that the total luminosity collected will be  $500 \text{ pb}^{-1}$ , and for two possible values of the centre-of-mass energy: 175 GeV and 190 GeV.

The results on the visible decays of the Higgs are based on the study of efficiencies and backgrounds in the search for the standard model Higgs described in Ref. [22]. For the invisible decays of the Higgs one has considered only the channel  $HZ$  with  $Z \rightarrow e^+e^-$  or  $Z \rightarrow \mu^+\mu^-$ , giving a signature of two leptons plus missing transverse momentum. The requirement that the invariant mass of the two leptons must be close to the  $Z$  mass can kill most of the background from  $WW$  and  $\gamma\gamma$  events; the background from  $ZZ$  events with one of the  $Z$  decaying to neutrinos is small and the measurement of the mass recoiling

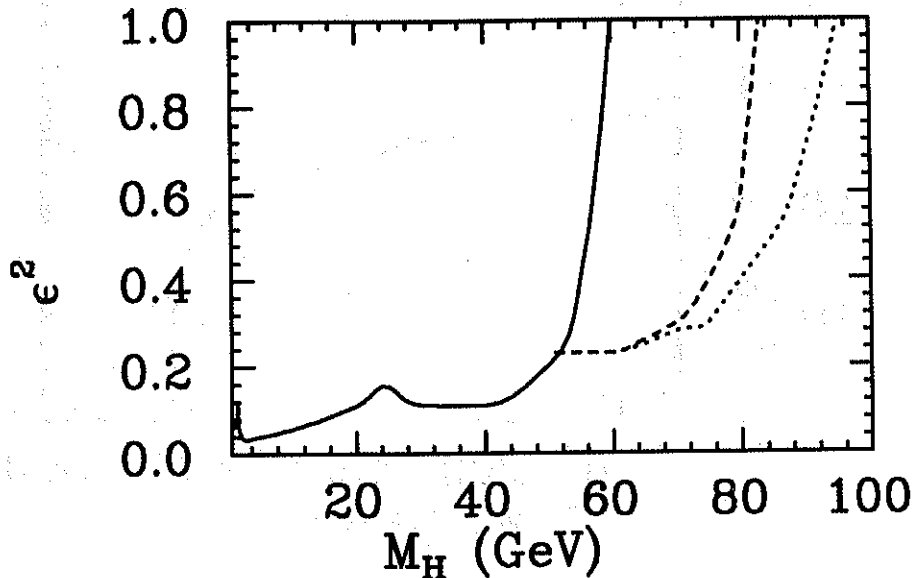


Figure 2: The solid curve shows the region in the  $\epsilon^2$  vs.  $m_H$  that can be excluded by the present LEP analyses, independent of the mode of Higgs decay, visible or invisible. The dashed and dotted curves show the that can be explored at LEP II for 175 GeV and 190 GeV centre-of-mass energy.

against the two leptons allows to further reduce it, at least for  $M_H$  not too close to  $M_Z$ . Hadronic decays of the  $Z$  were not considered in Ref. [16], since the background from  $WW$  and  $We\nu$  events is very large, and b-tagging is much less useful than in the search for  $ZH_{SM}$  with  $Z \rightarrow \nu\bar{\nu}$ , since the  $Zb\bar{b}$  branching ratio is much smaller than  $Hb\bar{b}$  in the standard model.

The dashed and dotted curves on Fig. 2 show the exclusion contours in the  $\epsilon^2$  vs.  $M_H$  plane that can be explored at LEP II, for the given centre-of-mass energies. Again, these contours are valid irrespective of whether the Higgs decays visibly, as in the standard model, or invisibly.

## 6 Invisible Higgses at the NLC

At the NLC there are two production mechanisms for Higgs particles: the Higgs bremsstrahlung off the  $Z$  boson line

$$e^+e^- \rightarrow Z^* \rightarrow ZH \quad (10)$$

and the fusion process

$$e^+e^- \rightarrow \nu\bar{\nu}W^*W^* \rightarrow \nu\bar{\nu}H \quad (11)$$

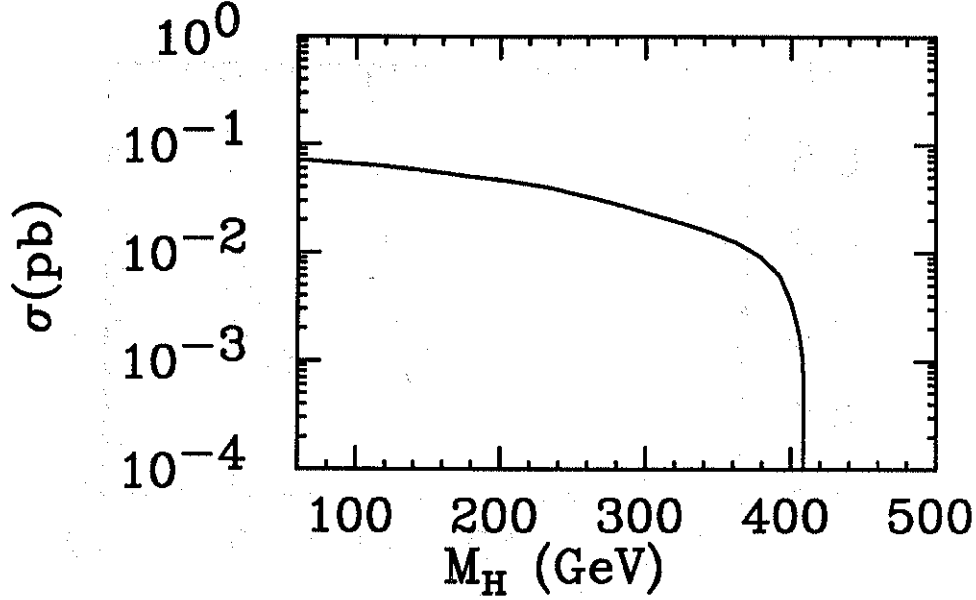


Figure 3: Total cross section for the Higgs bremsstrahlung process at 500 GeV.

For Higgses decaying invisibly, this second mechanism becomes irrelevant since it would lead to no visible signature. In Fig. 3 we plot the cross section for the Higgs bremsstrahlung process as a function of the Higgs mass at  $\sqrt{s} = 500$  GeV.

The main sources of background for the invisible decays of the Higgs are the processes

$$\begin{aligned}
 e^+e^- &\rightarrow \nu\bar{\nu}Z & (\sigma = 0.48 \text{ pb}) & \quad (A) \\
 e^+e^- &\rightarrow WW & (\sigma = 7.8 \text{ pb}) & \rightarrow (q\bar{q}') [l] \nu \quad (B) \\
 & & & \rightarrow (l\bar{l}) \nu \quad (C) \\
 e^+e^- &\rightarrow e\nu_e W & (\sigma = 5.9 \text{ pb}) & \rightarrow (q\bar{q}') [e] \nu_e \quad (D) \\
 & & & \rightarrow (e^+e^-) \nu_e \quad (E)
 \end{aligned} \tag{12}$$

where the particles in square brackets escape undetected and the fermion pairs have an invariant mass close to the  $Z$  mass. The large values of the last two total cross sections makes the  $WW$  and  $e\nu_e W$  backgrounds very large for the hadronic decays of the  $Z$  even after imposing the  $Z$  invariant mass reconstruction. For this reason we will consider in our study only the leptonic decay modes of the  $Z$ ,  $Z \rightarrow e^+e^-$  or  $Z \rightarrow \mu^+\mu^-$ . The signature will be therefore two leptons with invariant mass compatible with the  $Z$  mass plus missing transverse momentum. The requirement of missing transverse momentum eliminates the  $\gamma\gamma$  and  $ll(\gamma)$  events as well. In this case the process  $e\nu_e W$  ( $E$ ) becomes irrelevant. The most dangerous background we are left with is the process  $A$  [23]. To



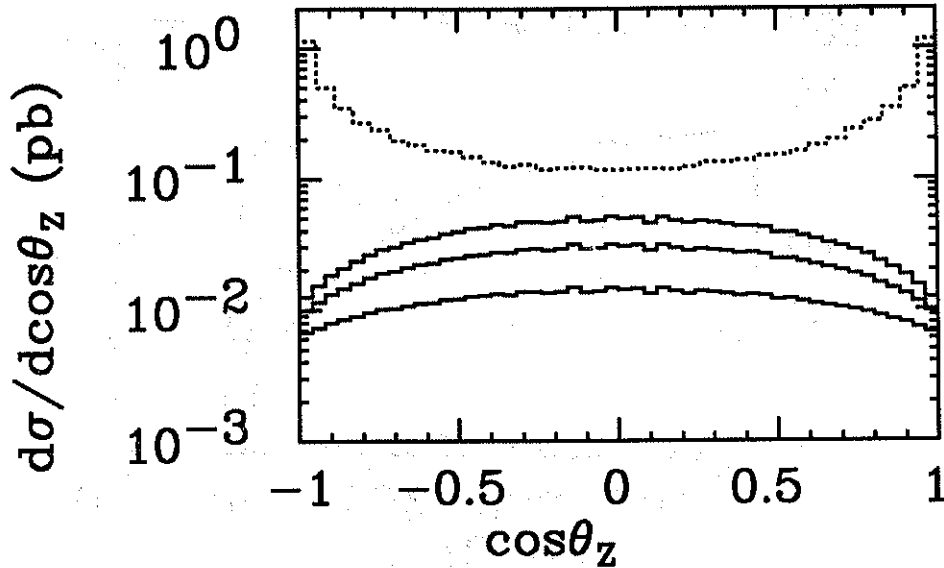


Figure 4: Angular distribution of  $Z$  in the background process  $e^+e^- \rightarrow \nu\bar{\nu}Z$  (dotted line) and in the signal (solid lines). The signal distributions are shown for  $m_H = 20, 200, 300$  GeV from upper to lower.

suppress it we impose the reconstruction of the  $Z$  energy

$$E_Z(m_H) = (s + m_Z^2 - m_H^2)/(2\sqrt{s}) \pm \Delta E \quad (13)$$

We assume an energy resolution  $\Delta E = 10$  GeV. Further suppression can be obtained from the fact that the  $Z$ 's in the signal are produced to larger polar angles than in the background (see Fig. 4). We impose an angular cut  $|\cos\theta_Z| < 0.7$ . After imposing these cuts the  $WW$  background ( $C$ ) becomes very small (see Fig. 5.). In Fig. 5 we show the number of events we are left with for the signal and backgrounds  $A$  and  $C$  for a luminosity  $\mathcal{L} = 10^4 \text{ pb}^{-1}$  and for  $\epsilon^2 \times Br_{invis} = 1$ .

In Fig. 6 we show the exclusion contours (at 95% CL) in the  $\epsilon^2 \times Br_{invis}$  vs.  $M_H$  plane that can be explored at the NLC. Invisible Higgses with masses below 200 GeV can be detected if their coupling to the  $Z$  is higher than 1/3 of the "standard" Higgs coupling; fully coupled Higgs bosons can be detected up to masses of almost 300 GeV.

Finally, we point out that at the NLC it will be possible to transform an electron beam into a photon one through the laser backscattering mechanism. This kind of process will allow the NLC to operate also in the  $e\gamma$ , and  $\gamma\gamma$  and will provide us with new mechanisms for production and detection of an invisibly decaying Higgs particle [24].

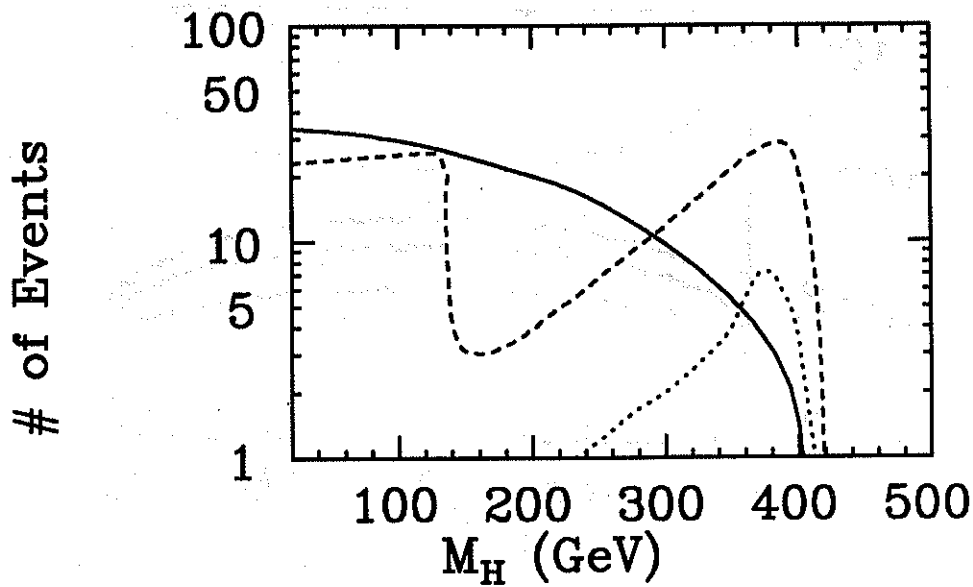


Figure 5: Final number of events for the signal for  $\epsilon^2 \times Br_{invis} = 1$  (solid) and backgrounds  $e^+e^- \rightarrow \nu\bar{\nu}Z$  (dashed) and  $WW$  background (dotted)

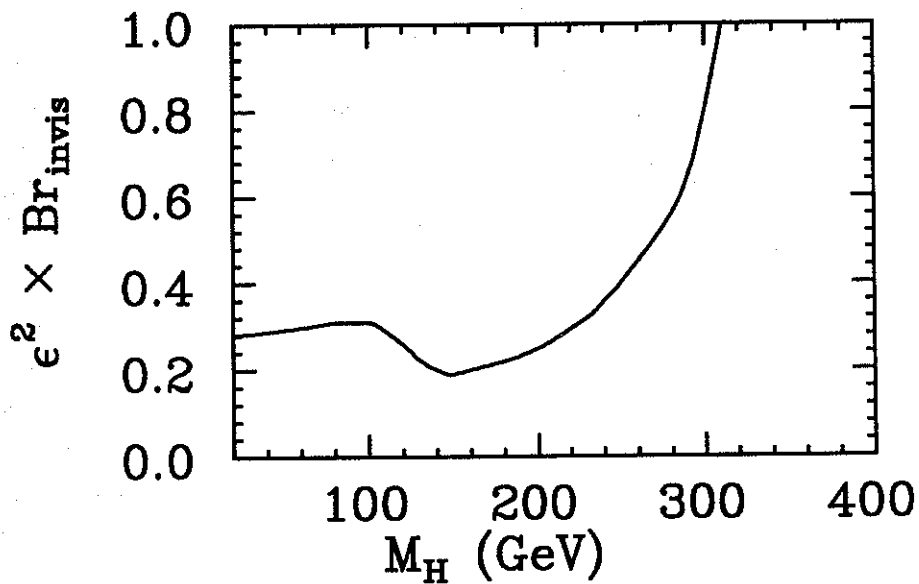


Figure 6: Accessible region at the NLC in the plane  $\epsilon^2 \times Br_{invis}$ ,  $M_H$  at 95 % CL.

## 7 Discussion

The Higgs boson can decay to invisible Goldstone bosons in a wide class of models in which a global symmetry, such as lepton number, is broken spontaneously around or below the weak scale. These models are attractive from the point of view of neutrino physics and suggest the need to search for the Higgs boson in the invisible mode.

We have reviewed the model-independent limits on the Higgs boson mass and Z coupling strength that can be deduced from the present LEP samples. The limits are summarized in Fig. 1 and 2 and do not depend on the mode of Higgs boson decay. They are probably conservative and could be somewhat improved with more data and/or more refined analysis.

Moreover we have investigated the reach of a high energy linear  $e^+e^-$  collider to discover a Higgs boson in the invisible mode. In Fig. 6 we show the exclusion contours (at 95% CL) in the  $\epsilon^2 \times Br_{invis}$  vs.  $M_H$  plane that can be explored at the NLC. Invisible Higgses with masses below 200 GeV can be detected if their coupling to the Z is higher than 1/3 of the “standard” Higgs coupling; fully coupled Higgs bosons can be detected up to masses of almost 300 GeV.

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<sup>1</sup> Internet EBOLI@USPIF.IF.USP.BR – DecNet USPIF::EBOLI

<sup>2</sup> Bitnet CONCHA@WISCPHEN – DecNet 47397::CONCHA

<sup>3</sup> DecNet UEIFT1::NOVAES

<sup>4</sup> Bitnet ROMAO@PTIFM

<sup>5</sup> Bitnet ALFON@CERNVM

<sup>6</sup> Bitnet CAMPOSC@EVALUN11 – Decnet 16444::CAMPOSC

<sup>7</sup> Bitnet VALLE@EVALUN11 – Decnet 16444::VALLE

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