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NON-PERTURBATIVE EFFECTS IN HEAVY MESONS

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Abstract

The existence of a non-trivial vacuum has influences on bound states. We calculate its effects on heavy pseudoscalar mesons parametrizing the non-perturbative properties by gluon condensates and using a non-relativistic approximation. We derive an effective hamiltonian taking into account the interaction with the gluonic vacuum. The background gauge formalism used preserves gauge invariance. Non-perturbative effects are shown to be more important in higher excited states.

1 Introduction

The concept of wave functions is well defined in non-relativistic quantum mechanics. The non-relativistic approximation is justified if one is interested in bound systems composed only of heavy quarks (flavors c and b), i.e. an expansion in inverse powers of the quark masses is convergent. It is probably easier to study the confinement mechanism in such systems. Also to understand the J/Ψ suppression one needs the accurate wave functions. We will obtain the eigenfunctions and eigenvalues of an effective hamiltonian derived from QCD. It includes non-perturbative effects: the interaction among quarks and vacuum. The picture we adopt for the vacuum is that of random gluon fields leading to condensation. As we will see, the vacuum interaction is very similar to the interactions of electrons with external fields. Comparisons with quantum electrodynamics are therefore very useful because in this

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case one knows how to perform a Foldy-Wouthuysen transformation on the Dirac equation in the presence of external fields and also the famous Zeeman and Stark effects. Also the one-gluon exchange for perturbative gluons in QCD is very similar to the one-photon exchange in QED which, within an instantaneous approximation, leads to the Coulomb potential and the higher order spin-dependent terms (Breit-Fermi hamiltonian).

Those similarities are no suprise once QCD was inspired in QED and constructed as a gauge invariant field theory. The differences, however, are very important. QCD is non-abelian and consequently presents asymptotic freedom. One expects also chiral symmetry and confinement as properties of the theory. The non-abelian character of the theory prevents the use of perturbative methods in the low energy limit. To describe bound states one must therefore go beyond perturbation theory. It is usually accepted that lattice calculations are the best tool we have, but so far one did not solve some fundamental problems[1]. An alternative is the use of QCD sum rules[2]. This somewhat more phenomenological method is based on the dual descriptions of correlations functions. They are calculated in terms of quark and gluon degrees of freedom using perturbation theory in the presence of a non-trivial vacuum and compared to the same correlation functions obtained with hadron degrees of freedom which, can themselves be extracted from experiments. Matrix elements of operators in the presence of the non-trivial vacuum, which otherwise in perturbation theory would vanish, are parametrized by quark and gluon condensates. Beyond the success of this approach, motivation for the existence of condensates comes from Nambu-Jona-Lasinio type models for the spontaneous breaking of chiral symmetry[3] and from the trace anomaly of QCD[4]. Sum rules, however, are suitable only for the calculation of the lowest state of each specific hadron.

Another possibility is the use of phenomenological models where gluonic degrees of freedom are eliminated. An example of these are bag like models[5] where it is assumed that in the interior of the bag there is a perturbative phase (small coupling constant) and the external phase exerts a pressure which counterbalances that of the quarks. Another class is composed by non-relativistic potential models[6]. In this case gluons are eliminated in favour of a confining potential. Spin splittings, as in bag like models, originate from perturbative one-gluon exchange. This "one-gluon exchange" (namely, the Coulomb potential) can also be corrected by the running of the coupling constant known from QCD. There are some unsolved points on the nature of the confining potential, if it is scalar or vector[7]. There are some problems with splittings[8], for instance the recently measured[9] $^1P_1$ state for $c\bar{c}$ does not fit in any potential model. Despite of these shortcomings, potential models are very successful to reproduce the most part of the experimental spectra. The agreement is particularly good for heavy mesons ($c\bar{c}$ and $b\bar{b}$) where the non-relativistic approximation is expected to be adequate. In this case, however, there are several non-equivalent forms for the potential (linear, Richardson, logarithmic, ...) which are equally good in reproducing data of mesons of radii between 0.3 and 1.0 fm.

Although our approach in this work is similar to that used by Leutwyler and Voloshin[10], we do not resort to the perturbative approach. Non-perturbative effects are parametrized by gluon condensates which are then used in the calculation of matrix elements of an effective hamiltonian. The hamiltonian is obtained from QCD separating gluons in background and quantum gluons. We neglect cubic and quartic terms in quantum (or hard) gluons and integrate them out. We use gauge invariant basis states to preserve gauge invariance while making approximations. Due to the structure of the vacuum non-usual singlet states may exist and contribute in hamiltonian matrix elements. The basis states are introduced in the next section. In section 3 we present the effective hamiltonian, first in the time-axial gauge and neglecting hard gluons. After that we use the Coulomb background gauge and take into account also hard gluons. In section 4 we present and discuss the results for heavy pseudoscalar mesons. Section 5 is devoted to final remarks where we summarize our results and conclude.

2 Gauge Invariance and Basis States

It is usually assumed that physical states are singlet in color. For heavy mesons where the non-relativistic approximation is adequate one represents quark fields only by the two upper components and antiquark fields by the two lower components. Quark and antiquark must be connected by a color transport matrix[11] along a determined path if one demands gauge invariance also on the states. Choosing only straight paths we obtain a complete and orthogonal basis [12]. For simplicity and to illustrate the method we will restrict ourselves to pseudoscalar mesons. The simplest gauge invariant pseudoscalar state is of the form

$$\mid 21\rangle_S = \frac{1}{\sqrt{6}} \sum_{ab;\alpha} u^{\dagger a}_\alpha(\vec{x}_2) T_{ab}(\vec{x}_2, \vec{x}_1) v^b_\alpha(\vec{x}_1) \mid \Omega\rangle \qquad (1)$$

where $u^{\dagger a}(\vec{x}_2)$ creates one quark of color a in $\vec{x}_2$ with spin α. v does the same for an antiquark. We will refer to it as a singlet state since, at zero separation, the quarks are in a color singlet state.

The color transport matrix is:

$$T_{ab}(\vec{x}_2, \vec{x}_1) = Pexp(-ig \int_{\vec{x}_1,t}^{\vec{x}_2,t} dx^\mu A_\mu(x))_{ab} \qquad (2)$$

Here P denotes ordering along a straight line path from $\vec{x}_1$ to $\vec{x}_2$. These basis states are orthogonal due to the straight path chosen and the (anti-) commutation relations of the quark and antiquark creation operators.

As we mentioned before, due to the non-trivial structure of the vacuum new aspects appear in the theory. We call background fields those responsible for the gluon condensation. Quark and antiquark can couple to the background fields, i.e. they "fluctuate" with the vacuum fields to form color singlet states. In this case quark and antiquark at zero separation are in a color octet state. Following QED we define chromo-electric and -magnetic fields from the potentials. Since the background field B transforms as a pseudovector and E as a vector, the following pseudoscalar states can be formed:

$$\mid 21\rangle_B = \sum_{\alpha\beta} \frac{g}{\pi\phi} u^\dagger_\alpha(\vec{x}_2) \vec{\sigma}_{\alpha\beta} \cdot \vec{B} T(\vec{x}_2, \vec{x}_1) v_\beta(\vec{x}_1) \mid \Omega\rangle \qquad (3)$$

and

$$\mid 21\rangle_E = \sum_{\alpha} \frac{\sqrt{3}g}{\pi\phi} u^\dagger_\alpha(\vec{x}_2) \vec{E} \cdot (\vec{x}_2 - \vec{x}_1) T(\vec{x}_2, \vec{x}_1) v_\alpha(\vec{x}_1) \mid \Omega\rangle \qquad (4)$$

the summation over colour indices being implied.

As will be clear later, it is irrelevant to the order we are calculating at which point on the string the electric and magnetic background fields are inserted. The differences are expectation values of higher dimension condensates which we neglect. The above states are mutually orthogonal and normalized if we assume that

$$\langle\Omega \mid \frac{g^2}{4\pi^2}B^{ia}B^{jb} \mid \Omega\rangle = -\langle\Omega \mid \frac{g^2}{4\pi^2}E^{ia}E^{jb} \mid \Omega\rangle$$
$$= \frac{1}{96}\delta^{ij}\delta^{ab}\langle\Omega \mid \frac{\alpha}{\pi}F^{\mu\nu c}F^{c}_{\mu\nu} \mid \Omega\rangle = \frac{1}{96}\delta^{ij}\delta^{ab}\phi^2 \tag{5}$$

and $< E >=< B >= 0$. The matrix elements of the effective hamiltonian in this basis will, in lowest order, only reflect the non-trivial gluonic vacuum structure via the condensate value, denoted by the parameter ϕ. Note that Lorentz invariance of the ground state $|\Omega >$ forces one to assume that the chromoelectric background field is either antihermitian or that the "E-states" have negative norm. In effect, the two possibilities both lead to a nonhermitian hamiltonian matrix, whose eigenvalues in general are not real.

The pseudoscalar meson η is then assumed to be well represented as a linear combination of the above basis states:

$$\mid \eta\rangle = \sum_{M=S,E,B} \int_{1,2} \psi_M(2,1) \mid 2,1\rangle_M \tag{6}$$

3 Effective Hamiltonian

We start with the QCD lagrangian:

$$\mathcal{L}(x) = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \bar\psi(i\partial\!\!\!/ + gT^a V\!\!\!/_a)\psi - m\bar\psi\psi \tag{7}$$

and separate the gluonic fields in background[13] (A) and quantum (Q) fields.

$$V^a_\nu = A^a_\nu + Q^a_\nu \tag{8}$$

The background fields are assumed to satisfy:

$$D_\mu F^A_{\mu\nu} = 0 \tag{9}$$

3.1 Drastic Approximation: No quantum fields.

For the sake of simplicity we first study this case where the quantum fields are fully neglected, i.e., all gluonic fields are background fields. We choose to

work in the time-axial gauge which is defined by setting $A^l_0 = 0$. Then the QCD lagrangian becomes:

$$L = \int d^3r[\frac{1}{2}(\dot A^l_i \dot A^l_i - B^l_i B^l_i) + \psi^\dagger(i\gamma_\mu D_\mu - m)\psi] \tag{10}$$

The hamiltonian in this gauge is given by:

$$H = \int d^3r[\frac{1}{2}(\pi^l_i\pi^l_i + B^l_iB^l_i) + g\psi^\dagger\gamma_0\gamma_i T^l\psi A^l_i$$
$$+ \psi^\dagger\gamma_0(i\gamma_i\nabla_i + m)\psi] \tag{11}$$

where $T^l = \frac{\lambda^l}{2}$.

The canonical commutation relations for quantization are:

$$[A^l_i(\vec r,t), \pi^m_j(\vec r', t)] = i\delta_{ij}\delta^{lm}\delta^3(\vec r - \vec r')$$
$$\{\psi(\vec r,t), \psi^\dagger(\vec r', t)\} = \delta^3(\vec r - \vec r') \tag{12}$$

From the original equations of motion a constraint has still to be satisfied, which must at least be imposed on the physical states. This is the Gauss law of QCD.

$$(\nabla_i E^l_i - g\psi^\dagger T^l\psi - gf^{lmn}E^m_i A^n_i) \mid Phys\rangle = 0 \tag{13}$$

We apply the Foldy-Wouthuysen transformation to the hamiltonian (11) using the field formalism[14] where, to separate large and small components, we use the operator:

$$S = -\frac{i}{2m}\int d^3r[g\psi^\dagger(\vec r,t)\gamma_i T^l\psi(\vec r,t)A^l_i(\vec r,t) + \psi^\dagger(\vec r,t)i\gamma_i\nabla_i\psi(\vec r,t)] \tag{14}$$

A transformed state is given by:

$$\mid \psi'\rangle = e^{iS} \mid \psi\rangle \tag{15}$$

and the Schrödinger equation for the transformed state is:

$$i\frac{\partial \mid \psi'\rangle}{\partial t} = e^{iS}(H - i\frac{\partial}{\partial t})e^{-iS} \mid \psi'\rangle = H' \mid \psi'\rangle \tag{16}$$

The transformed hamiltonian can be written as:

$$H' = H + i[S,H] + \frac{i^2}{2!}[S,[S,H]] + \dots \tag{17}$$

We expand only to terms of order $1/m$. The resulting hamiltonian is:

$$\begin{aligned} H' = \int d^3r \Big\{ \mathcal{E}(\vec{r},t) + \psi^\dagger(\vec{r},t)\gamma_0 [m &- \frac{(i\vec{\nabla} + gT^l\vec{A}^l)^2}{2m} - \frac{g\vec{\sigma}\cdot\vec{B}}{2m} \\ &- \frac{g}{2m}\gamma_i T^l \pi_i^l(\vec{r},t)]\psi(\vec{r},t)\Big\} \end{aligned} \tag{18}$$

The original non-diagonal terms of order m^0 in the hamiltonian were eliminated. Now the lowest order non-diagonal term is the last one, proportional to $1/m$. Usually, following the Foldy-Wouthuysen procedure one can perform a new transformation using the odd operator in the new hamiltonian to go still one order lower[14]. This is not possible in this case. Separating the gluon fields in soft and hard gluons, the soft gluons saturate the vacuum average (5). They are responsible for the non-vanishing gluon condensate. The expectation value $< EE >$, E being the electric field, due to Lorentz invariance is proportional to $< FF >$ with a negative sign. It means, E is not an hermitian operator and it is not possible to define a unitary transformation to eliminate the non-diagonal term of order $1/m$ in the hamiltonian. It is a non-perturbative effect and can be significant in transitions where such term already at this order becomes important.

We take terms only up to order $1/m$. We can therefore neglect the small components. Particles are represented by spinors with only two upper components different from zero, and anti-particles with only two lower components. In this approximation we work with two independent two-dimensional subspaces. We wish to diagonalize the Schrödinger equation:

$$i\frac{\partial}{\partial t} \mid \eta\rangle = H \mid \eta\rangle \tag{19}$$

which is very similar to the Dirac equation in the presence of external potentials.

Although we assume $< F >= 0$ (it vanishes e.g. not to break global color and Lorentz invariance), linear terms in the background fields have to be retained because they couple octet to singlet states. The time derivative

of the meson state has two contributions: the time derivative of the wave function and the time derivative of the state itself. In the time axial gauge

$$\begin{aligned} i\frac{\partial}{\partial t}T_{ab} &= T_{ab}g\frac{\partial}{\partial t}\int_{\vec{x}_1,t}^{\vec{x}_2,t} dx^\mu A_\mu(x) \\ &= T_{ab}g\int_{\vec{x}_1}^{\vec{x}_2} dx^i \dot{A}_i(x) \\ &= -T_{ab}g\vec{E}(\vec{x}_1)\cdot(\vec{x}_2 - \vec{x}_1) \end{aligned} \tag{20}$$

Using the symmetries of the pseudoscalar states we can assume $\psi(\vec{x}_1, \vec{x}_2) = \psi(\vec{x}_1 - \vec{x}_2)$. Evaluating matrix elements and using relative coordinates we have three coupled equations:

$$\begin{aligned} \Big[(m_1+m_2) &+ \frac{\pi^2\phi^2x^2}{72}(\frac{1}{m_1}+\frac{1}{m_2}) - \frac{1}{2\mu}\vec{\nabla}^2\Big]\psi_S(x) \\ &= -\frac{\pi\phi}{3\sqrt{2}}x^2\psi_E(x) + E\psi_S(x) + \frac{\pi\phi}{2\sqrt{6}\mu}\psi_B(x) \\ \Big[(m_1+m_2)x^2 &- x_i\frac{1}{2\mu}\vec{\nabla}^2 x_i\Big]\psi_E(x) = \frac{\pi\phi}{3\sqrt{2}}x^2\psi_S(x) + Ex^2\psi_E(x) \\ \Big[(m_1+m_2) &- \frac{1}{2\mu}\vec{\nabla}^2\Big]\psi_B(x) - \frac{\pi\phi}{2\sqrt{6}\mu}\psi_S(x) = E\psi_B(x) \end{aligned} \tag{21}$$

where μ is the reduced mass. Note that while the coupling of the B states is hermitian, the E states lead to a non-hermiticity of the effective hamiltonian. We solved these equations numerically and found no bound state. We know from phenomenology that the Coulomb potential is very important in the description of heavy mesons. It comes from the hard one gluon exchange. We show in the next section how to include hard gluons in our approach.

3.2 Hamiltonian in the Coulomb Background Gauge

To take hard gluons into account we find it useful to work in the Coulomb background gauge. It is defined by:

$$D_i Q^i = 0 \tag{22}$$

where $D_\mu Q_\nu = \partial_\mu Q_\nu + gA_\mu \times Q_\nu = \partial_\mu Q_\nu + gf^{abc}A_{\mu b}Q_{\nu c}$ and

We expand the lagrangian only to second order in the quantum fields and will subsequently integrate them out in favour of an effective interaction.

For the background fields we use a modified Schwinger gauge[15]

$$A_j^b = -\frac{1}{2}F_{ji}^b x^i \quad ; \quad A_0^b = -F_{0i}^b x^i \tag{23}$$

and assume that the field-strengths corresponding to the background fields are constant (or have sufficiently low momenta that they can be regarded as essentially constant over the extent of the meson). Although ghost terms are necessary in this gauge, they do not contribute to quadratic order in the quantum fields. Our truncation of the interaction terms for the quantum fields eliminates all radiative corrections. To really have asymptotic freedom one would have to go beyond this approximation, calculating perturbative corrections before eliminating hard gluons. As a consequence of this truncation the correct perturbative behaviour at very small distances is not reproduced: the logarithmic corrections to the Coulomb potential are absent in this approximation and would have to be included "by hand" with a running coupling constant. These corrections do not seem dramatically important for describing heavy quarkonia[16] and we feel that their inclusion would only serve to unnecessarily complicate things.

After elimination of the quantum gluonic fields by their equations of motion the effective lagrangian in terms of the background and heavy quark fields becomes:[17]

$$\begin{aligned} L_{eff} &= \int d^4x \bar{\psi}(i\partial\!\!\!/ + gT^a A\!\!\!/_a)\psi - m\bar{\psi}\psi \\ &+ \frac{1}{2}\int d^4x d^4y g^2 \psi^\dagger(x)T^a\psi(x)\mathcal{D}^{ab}(x,y)\psi^\dagger(y)T^b\psi(y) \\ &- \frac{1}{2}\int d^4x d^4y g^2[j^a_{\psi_i}(x) + gj^a_{F_i}(x)]\tilde{\mathcal{D}}^{ab}_{ij}(x,y)[j^b_{\psi_j}(y) + gj^b_{F_j}(y)] \\ &+ \frac{1}{2}\int d^4x d^4y d^4z d^4z' g^2[j^d_{\psi_j}(z') + gj^d_{F_j}(z')]\tilde{\mathcal{D}}^{df}_{jl}(z',y)D_l^{fc,y} \\ &\quad \mathcal{G}^{cb}(y,z)D_k^{be,z}\tilde{\mathcal{D}}^{ea}_{ki}(z,x)[j^a_{\psi_i}(x) + gj^a_{F_i}(x)] \end{aligned} \tag{24}$$

In (24) we have omitted a part of the Lagrangian only depending on the background field A, which is regarded as a classical field. The appearance

of the propagator $\mathcal{D}$ is the result of expressing the Q_0 field in terms of its source:

$$Q_0^a(x) = \int d^4y \mathcal{D}^{ab}(x,y)j_b^0(y) \tag{25}$$

with

$$j_0^a = -g\psi^\dagger T^a\psi + 2gf^{abc}F_{bi0}Q_{ci} \tag{26}$$

and satisfies:

$$(D_jD_j)^{ab}\mathcal{D}^{bd}(x,y) = \delta^{ad}\delta^4(x-y) \tag{27}$$

Similarly the propagator $\tilde{\mathcal{D}}$ is the result of eliminating the spatial components Q_i and satisfies:

$$\int d^4z M_{ij}^{ab}\tilde{\mathcal{D}}^{bc}_{jk}(z,y) = \delta^{ac}\delta^4(x-y)\delta_{ik} \tag{28}$$

with M_{ij}^{ab} defined by:

$$\begin{aligned} M_{ij}^{ab}(x,z) &= \delta^4(x-z)[(D_\mu D_\mu)^{ab}\delta_{ij} - 2gf^{abc}F_{ij}^c]_z \\ &\quad -\frac{g^2}{\pi}f^{dea}f^{dcb}F_{i0}^c F_{j0}^e\frac{\delta(x_0-z_0)}{|\vec{x}-\vec{z}|} \end{aligned} \tag{29}$$

The other currents appearing in the effective lagrangian (24) are:

$$j^a_{\psi_j} = \psi^\dagger(x)T^a\gamma_j\psi(x) \tag{30}$$

and

$$j^a_{F_j} = -\frac{1}{2\pi}f^{abc}F_{j0}^c\int d^4z\frac{\delta(x_0-z_0)}{|\vec{x}-\vec{z}|}\psi^\dagger(z)T^b\psi(z) \tag{31}$$

Finally, the propagator $\mathcal{G}$ enters when the lagrange multiplier of the gauge fixing condition for the quantum fields (22) is eliminated in turn. Its equation of motion is:

$$\int d^4x D_k^{dc,z}\tilde{\mathcal{D}}^{ca}_{ki}(z,x)D_i^{ae,x}\mathcal{G}^{eb}(x,y) = \delta^{bd}\delta(z-y) \tag{32}$$

These rather formidable integro-differential equations for the Green functions can be formally solved order by order in the background field A. Since we will only retain terms of the hamiltonian matrix proportional to the lowest dimensional condensate $< g^2FF >$, we only keep terms up to second order in the background fields in this gauge (23).

In order to obtain a tractable hamiltonian, however, further approximations are necessary. We first make a nonrelativistic reduction keeping only terms up to $\mathcal{O}(1/m)$. We will also neglect all retardation effects in the effective interaction. Because the interaction is then no longer time dependent, this additional approximation makes the hamiltonian time independent, even in the presence of the background fields (which are themselves time independent in this gauge (23)), and greatly simplifies the interpretation of results. The resulting hamiltonian is:

$$
\begin{aligned}
H \;=\; & \int d^3x\{mu^\dagger u + u^\dagger \frac{(i\vec{\nabla}+g\vec{A})^2}{2m}u - u^\dagger \frac{g\vec{\sigma}\cdot\vec{B}}{2m}u - gu^\dagger A^0 u \\
& \quad + mvv^\dagger + v\frac{(-i\vec{\nabla}+g\vec{A})^2}{2m}v^\dagger - v\frac{g\vec{\sigma}\cdot\vec{B}}{2m}v^\dagger - gv\tilde{A}^0 v^\dagger\} \\
& -\frac{g^2}{2}\int d^3x d^3y u_x^\dagger T^a u_x v_y \tilde{T}^a v_y^\dagger \frac{d^3q}{(2\pi)^3} e^{i\vec{q}(\vec{x}-\vec{y})}[\frac{1}{\vec{q}^2} - \frac{g^2(B^2/8-E^2)}{\vec{q}^6}] \\
& -\frac{g^3}{16\pi}\int d^3x d^3y \frac{f^{abc}F^b_{ij}y^i x^j u_x^\dagger T^a u_x v_y \tilde{T}^c v_y^\dagger}{|\vec{x}-\vec{y}|} \qquad (33) \\
& -\frac{g^4 B^2}{512\pi}\int d^3x d^3y \frac{(\vec{x}\times\vec{y})^2}{|\vec{x}-\vec{y}|} u_x^\dagger T^a u_x v_y \tilde{T}^a v_y^\dagger \\
& -\frac{g^3}{2\pi m}\int d^3x d^3y u_x^\dagger T^a (\vec{\sigma}\times\frac{(\vec{x}\times\vec{y})}{|\vec{x}-\vec{y}|})_k u_x f^{abc}F^c_{k0} v_y \tilde{T}^b v_y^\dagger \\
& -\frac{g^3}{2\pi m}\int d^3x d^3y v_x \tilde{T}^a (\vec{\sigma}\times\frac{(\vec{x}\times\vec{y})}{|\vec{x}-\vec{y}|})_k v_x^\dagger f^{abc}F^c_{k0} u_y^\dagger \tilde{T}^b u_y
\end{aligned}
$$

The correction to the Coulomb potential from the $1/q^6$-terms in (33) is infrared divergent. Our short distance expansion in powers of the backround field clearly cannot be correct at small momentum transfers. Note however that since these terms are already of order $< FF >$ we may, in the order we are working in, replace the background fields there by their vacuum expectation values. The modification of the Coulomb potential is then seen to be just the first term in the expansion of an effective hard gluon propagator of the form

$$\mathcal{D}(q^2) \propto \frac{1}{q^2(1+\frac{\nu^4}{q^4})} = \frac{1}{q^2}(1-\frac{\nu^4}{q^4}+\cdots) \qquad (34)$$

where ν^4 is a dimensionfull constant essentially proportional to the gluon condensate (apart from logarithmic corrections). This modified form of the effective gluon propagator has recently been found as a nonperturbative solution of the Dyson Schwinger equations[18] of pure QCD and this expansion of the gluon polarization is also suggested by the operator product expansion[19]. In this case we therefore replace the expansion in the background fields by the "whole" effective propagator (34) and thus obtain an effective Coulomb potential, the fourier-transform of (34), which is:

$$V^{coul}_{eff}(r) = \frac{1}{4\pi r}e^{-\nu r}\cos\nu r \qquad (35)$$

In a short distance expansion this potential differs from the coulombic one by a constant and a mass independent harmonic force.

4 Masses and Wave Functions of Pseudoscalar Mesons

We will now diagonalize the hamiltonian (33) in the basis defined in section 2, suited for pseudoscalar mesons and invariant under a change of gauge for the background fields A. A numerical diagonalization is feasible since only a few octet states are relevant. As in the time-axial case, after commuting the color transport matrices through the derivative operators of the hamiltonian , one is left with an effective hamiltonian in the meson sector, which to order $A^2 \propto F^2$ has an interaction that only depends on the relative coordinate $\vec{x}_1 - \vec{x}_2$

$$
\begin{aligned}
H_{12} = M - \frac{\nabla^2}{2\mu} + \frac{ig[\vec{A}(\vec{x}_1)-\vec{A}(\vec{x}_2)]\cdot\vec{\nabla}}{\mu} + \frac{g^2[\vec{A}(\vec{x}_1)-\vec{A}(\vec{x}_2)]^2}{2\mu} - \frac{g\vec{\sigma}\cdot\vec{B}}{2\mu} \qquad (36) \\
-g\vec{E}\cdot(\vec{r}) - \frac{g^3}{2\mu}(\vec{\sigma}\times\frac{\vec{r}}{r})\cdot\vec{E} + \begin{cases} -\frac{4}{3}\frac{\alpha_s}{r}e^{-\nu r}\cos\nu r & \text{for singlet states} \\ +\frac{1}{6}\frac{\alpha_s}{r} & \text{for octet states} \end{cases}
\end{aligned}
$$

whose matrix elements can be evaluated in the above basis, setting the color transport matrix $T = 1$.

To obtain the masses of the pseudoscalar mesons one has to diagonalize the hamiltonian. Evaluating the matrix elements one arrives at the following set of coupled equations:

$$
\begin{aligned}
\left[-\mathcal{E} + r^2 - \frac{\partial^2}{\partial r^2} + \frac{\alpha'_{sing}}{r}\right] S(r) &= \sqrt{6}B(r) - 4\mu\sqrt{\frac{3}{\pi\phi}}\, r E(r) \\
\left[-\mathcal{E} - \left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right) + \frac{\alpha'_{oct}}{r}\right] E(r) &= 4\mu\sqrt{\frac{3}{\pi\phi}}\, r S(r) \\
\left[-\mathcal{E} - \frac{\partial^2}{\partial r^2} + \frac{\alpha'_{oct}}{r}\right] B(r) &= \sqrt{6}S(r)
\end{aligned}
\tag{37}
$$

which was brought into this dimensionless form using the definitions:

$$
r = \sqrt{\frac{\pi\phi}{6}}x; \quad \mathcal{E} = \frac{12\mu(E-M)}{\pi\phi}; \quad \alpha'_{sing} = \sqrt{\frac{6}{\pi\phi}}2\mu\alpha_s; \quad \alpha'_{oct} = -\frac{\alpha'_{sing}}{8}
\tag{38}
$$

M is the sum of the heavy quark masses, μ is the reduced mass, α_s is the strong coupling constant and the wave functions were redefined by:

$$
\psi_S(x) = \frac{1}{x}S(x) \quad ; \quad \psi_E(x) = \frac{1}{x^2}E(x) \quad ; \quad \psi_B(x) = \frac{1}{x}B(x)
\tag{39}
$$

One has to resort to numerical methods to solve the set of coupled equations obtained. An expansion of the wave functions in a given basis, for instance the harmonic oscillator basis, was found not very appropriate in this case. The solutions don't converge with increasing the oscillator basis.

We solved the coupled equations for bound states only, i.e. states of finite norm whose wave functions vanish rapidly enough at large distances. The electric states have negative norm. Equivalently we can use positively normalized states and a non-hermitian hamiltonian. Non-hermitian hamiltonians need not have a complete set of eigenstates. As we will show, higher excited states have a larger contribution from octet states. The norm of a state will be negative if the electric component dominates and such state would have to be considered. However we did not find any such states - they are probably unbound.

The parameters in this approach are the heavy quark masses m_c and m_b, the gluon condensate value and the strong coupling constant. It is interesting to note, however, that these parameters are not exactly the same as used in other approaches. Beyond renormalization effects due to the non-relativistic reduction[20] there are other finite renormalizations. The "physical" mass

in the works of Leutwyler and Voloshin[10] (or other models based only on singlet states) is already defined with the self-interaction induced by the soft gluons. In our case, where perturbation theory is not used and therefore octet states are not only intermediate states, this interaction enters explicitly among basis states and should not be included in the definition of the quark mass. However this mass renormalization is finite, since it only comes from soft gluons with restricted momenta. Within our formalism we cannot calculate this shift in the mass of the quarks explicitly and therefore have to live with the fact that the mass-parameter in the model cannot be directly compared with that of other non-relativistic calculations. Due to the interplay between perturbative and non-perturbative effects the one gluon propagator is modified. This leads to finite vertices renormalizations and therefore to a different coupling constant. In second order perturbation theory, the low momentum component of the one gluon exchange renormalizes the definition of the gluon condénsate in first order in α_s. In fact, the use of intermediate Coulomb octet wave functions already implies renormalization to all orders in the strong coupling constant. This is a point missed in the Leutwyler-Voloshin model - comparisons to the condensate value used in QCD sum rules should be done with this renormalized condensate and not with the "bare" condensate used in the model[17].

The wave functions and masses for several values of the parameters are shown in figures 1-6. Note that the states η_c' and η_B' appear as bound states only if the strong coupling constant is larger than some critical value. As mentioned before, renormalization effects give margin for speculation on the parameters and changing them to unusual values causes the appearence of a second bound state. This second bound state is predicted by all model calculations, but not surely observed in experiments. For bottom quarks, even the pseudoscalar ground state has not been observed and we can not determine the parameters. The wave function of the ground state is practically a pure singlet state, in agreement with usual suppositions. This is not the case of the first radial excitation. Although small, octet components are not negligible. The calculation of the experimentally more accessible 1^- states is in progress. In this case the inclusion of octet states enlarges the basis far more than for pseudoscalars and the numerical solution is more involved.

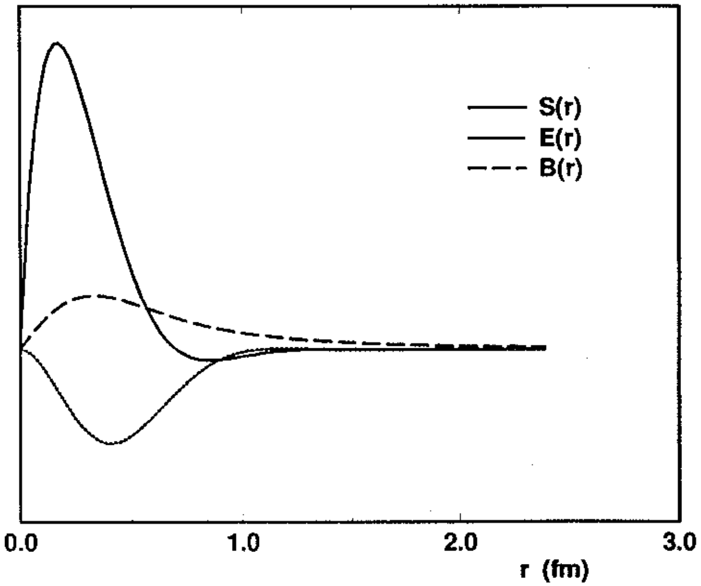

Figure 1: Wave functions of the ground state of the pseudoscalar η_b ($b\bar{b}$). $S(r), E(r), B(r)$ correspond respectively to the singlet, electric and magnetic wave functions redefined in equation (39). The parameters used are: $m_b = 4780$, $\alpha_s = 0.32$ and $\phi = (360MeV)^2$. With such parameters the mass of this state is $M = 9460MeV$ and there is no bound excited state.

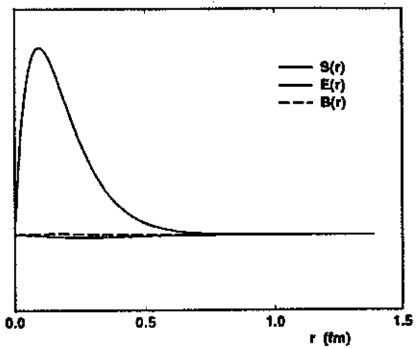

Figure 2: The same as figure 1 but with parameters: $m_b = 5090$, $\alpha_s = 0.62$, $\phi = (191MeV)^2$. M obtained is the same. The use of these unusual values for the parameters is discussed in the text.

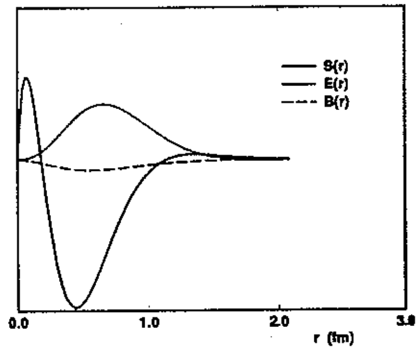

Figure 3: Wave functions of the first excited state of η_b. The parameters are the same as in figure 2. $M = 10360MeV$

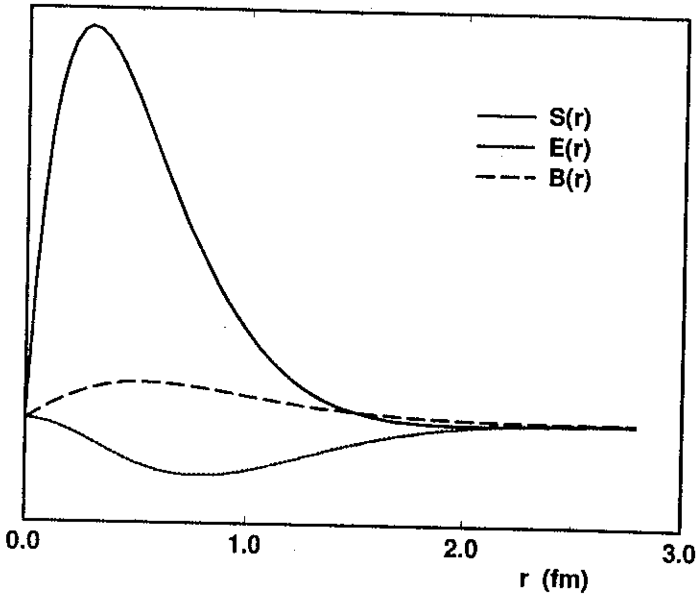

Figure 4: Wave functions of the ground state of the pseudoscalar η_c $(c\bar{c})$. $S(r), E(r), B(r)$ correspond respectively to the singlet, electric and magnetic wave functions redefined in equation (39). The parameters used are: $m_c = 1550$, $\alpha_s = 0.64$ and $\phi = (191MeV)^2$. With such parameters the mass of this state is $M = 2980MeV$ and there is no bound excited state.

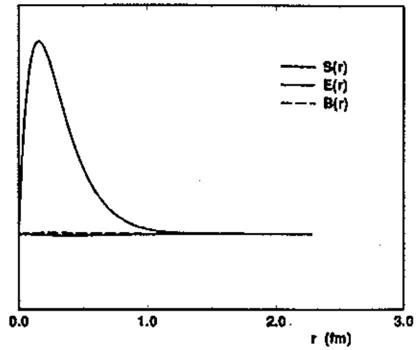

Figure 5: The same as figure 4 but with parameters: $m_c = 1850$, $\alpha_s = 1.05$, $\phi = (120MeV)^2$. The mass M obtained is the same. The use of these unsual values for the parameters is discussed in the text.

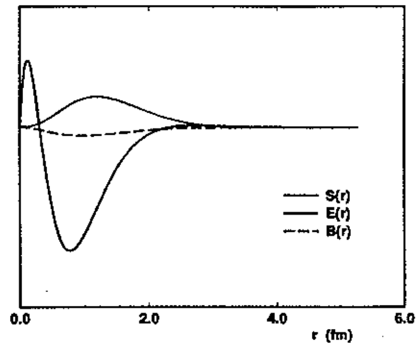

Figure 6: Wave functions of the first excited state of η_c. The parameters are the same as in figure 5. $M = 3680MeV$

5 Conclusions

We have presented an approach especially well suited to heavy quarkonia. It is based on a gluonic ground state composed of random background fields leading to gluon condensation, a consequence of the trace anomaly of QCD. It is expected that in such a medium color-charged objects cannot propagate. This eliminates the need for explicitly introducing a confining potential. We take account of this ground state by splitting the interaction into a well known perturbative part and a soft background part.

An effective hamiltonian was derived in a short distance and 1/m expansion, considering only the dimension four gluon condensate $< F^2 >$ and terms to order 1/m. We showed that the perturbative one-gluon exchange is already modified by the background fields in this order, with additional spin dependent terms emerging. Gauge invariance is a fundamental ingredient of any effective theory aspiring to be a good approximation to QCD. We have explicitly shown how to derive a gauge invariant effective interaction from the background fields. It was essential to construct a gauge invariant basis for colour singlet states. Completeness forces us to include states where quarks in octet configuration are coupled with the background fields to color singlet states. Recently, in a different context such octet states were also evocated[21].

We saw that the interaction with chromo-electric background fields is large for heavy quarkonia. We therefore did not resort to perturbation theory for diagonalizing the effective hamiltonian. The coupling to octet states leads to a non-local flavor and energy dependent interaction in the singlet channel. Its description by a local potential is therefore questionable.

One has previously attempted to include the effects from the background fields in heavy quarkonia as a perturbation to the Coulomb interaction. To justify this treatment it was necessary to strongly diminish the contribution from the background fields by introducing finite lenght correlations[22], which effectively can also be thought of as including higher dimensional condensates. In this work we showed that most of these effects are absorbed in a renormalization of the parameters of the model.

We examined the 0^- spectra of heavy quarkonia. We found that the inclusion of background fields influences the low lying states by at most $\sim 100MeV$. This modification of the Coulomb spectrum is in the right direction and of the same order as that of the usual linear confining potential.

However, we only obtained one or two bound states in our approach, what is not in disagreement with experimental observation. Extrapolating these results one could say that what confining potentials simulate in phenomenological potential models are vacuum effects. Sure one must yet show that colored states do not propagate in the non-trivial vacuum, but to do it one can not simply parametrize the vacuum by condensates. One must resort to models of the vacuum. Before one can say definitively whether a phenomenological potential is still necessary one will however have to calculate heavy mesons with other spin-parities and resonances. To reproduce the experimental spectra it will probably be necessary to go to the next order in the approximations, taking $(1/m^2)$ corrections and correlations (or higher order condensates) into account. Work on this line is in progress.

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