UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01452-990 SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-1090

OPTIMAL FILTERS FOR THE DETECTION OF CONTINUOUS GRAVITATIONAL WAVES

Nadja S. Magalhães
Instituto Nacional de Pesquisas Espaciais
C.P. 515, São José dos Campos - SP, 12201-970, Brazil

Carlos O. Escobar
Instituto de Física, Universidade de São Paulo

Novembro/1993

Optimal filters for the detection of continuous gravitational waves

Nadja S. Magalhães

Instituto Nacional de Pesquisas Espaciais

C.P. 515, São José dos Campos - SP, 12201-970, BRAZIL

Carlos O. Escobar

Instituto de Física

Universidade de São Paulo

C.P. 20516, São Paulo - SP, 01452-990, BRAZIL

Abstract

We determine the transfer functions of two kinds of filters that can be used in the detection of continuous gravitational radiation. The first one optimizes the signal-to-noise ratio, and the second reproduces the wave with minimum error. We analyse the behaviour of these filters in connection with actual detection schemes.

I. INTRODUCTION

The detection of gravitational waves (g.w.) is one of the most fascinating and challenging subjects in Physics research nowadays. Besides checking the General Relativity theory, the detection of this phenomenon will mark the beginning of a new phase in the comprehension of astrophysical phenomena by the use of gravitational wave astronomy.

Although these waves were predicted at the beginning of the century [1], the research on their detection only started around 1960, with the studies of Joseph Weber [2]. The major obstacle to this detection is the tiny amplitude the g.w. have [3]. Even though the more sensitive detector now operating [4] is capable to detect amplitudes near $h \sim 5 \times 10^{-19}$, this value must be decreased by several orders of magnitude so that impulsive waves can be detected regularly.

On the other hand, the discovery of pulsars with periods lying in the milliseconds range stimulated the investigations on the detection of gravitational waves of periodic origin. Although these waves generally have amplitudes even smaller than those emitted by impulsive sources, periodic sources are continuously emitting gravitational waves in space and they can be detected as soon as the correct sensitivity is reached. Since many of the resonant mass antennae now operating are designed to detect frequencies near 1000 Hz, the millisecond pulsars will probably be detected if these antennae ever become sensitive to amplitudes $h \leq 10^{-27}$. This value is bigger if we consider the Crab pulsar ($f \sim 60Hz$): $h \sim 10^{-24}$. There is a resonant mass detector with a torsional type antenna (CRAB IV) being developed by the Tokyo group [5] to detect gravitational waves emitted by the Crab pulsar. This group expects to reach $h \sim 10^{-22}$ soon.

The main purpose of this paper is a contribution towards the increase in sensitivity of resonant mass continuous gravitational wave detectors looking at the use of adequate filters. We study two kinds of filters, the first optimizes the signal-to-noise ratio (SNR), and is normally used in the detection of impulsive waves [6]. The second filter reproduces the wave with minimum error. Both filters apparently were not investigated in the continuous gravitational wave context yet.

II. THE FILTER THAT OPTIMIZES SNR

Linear, stationary filters obey the relation

$$\mathcal{O}(t) = \int_{-\infty}^{\infty} k(t')\mathcal{I}(t-t')dt'.$$

$k(t)$ is the impulse response function that characterizes the filter $\mathcal{K}$, $\mathcal{I}(t)$ is the input at the filter and $\mathcal{O}(t)$ is the filter output.

Generally[1] $\mathcal{I}(t)$ has a useful part, $U(t)$, and an unwanted part, $N(t)$: $\mathcal{I}(t) = U(t)+N(t)$. We have a similar relation for the filter output, given by $\mathcal{O}(t) = U'(t) + N'(t)$.

It is well known from noise theory [7] that the filter $\mathcal{K}_o$ that optimizes SNR at its output[2],

$$SNR \equiv \frac{|U'(t_0)|^2}{< N'^2(t) >}, \tag{1}$$

must have the following transfer function:

$$K_o(\omega) = e^{-\imath\omega t_0}\frac{\tilde{U}^*(\omega)}{S_N(\omega)}, \tag{2}$$

with

$$K_o \equiv \int_{-\infty}^{\infty} e^{-\imath\omega t}k(t)dt.$$

t_0 is the instant in which the observation takes place, $\tilde{U}(\omega)$ is the Fourier transform of $U(t)$ (* denotes complex conjugation) and $S_N(\omega)$ is the noise power spectrum density:

$$S_N(\omega) \equiv \int_{-\infty}^{\infty} e^{-\imath\omega t} < N(t)N(t-\tau) > d\tau.$$

The maximum SNR at the optimal filter output is given by the expression

[1] We are only considering random, stationary processes in this work.

[2] $< f(t) >$ represents the average value of $f(t)$.

$$SNR_o = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{|\tilde{U}(\omega)|^2}{S_N(\omega)}d\omega. \tag{3}$$

From (2) and (3) we conclude that a very weak signal will leave the filter when the noise is much stronger than the useful signal at the relevant frequency range.

III. QUASI-MONOCHROMATIC SIGNALS

Equation (2) is valid as long as $\tilde{U}(\omega)$ is well behaved. For example, if $U(t)$ were a strictly monochromatic wave like

$$U(t) = h_0 \cos\omega_0 t, \tag{4}$$

it would be difficult to build this filter since $\tilde{U}(\omega) = h_0\delta(\omega-\omega_0)$.

In order to use the optimal filter (2) in continuous gravitational wave detectors we will describe these waves as *quasi-monochromatic* useful signals. It means that the waves that reach the antenna will be of the form[3]

$$h_{xx}(t) = \begin{cases} 2h_0 e^{-at}\cos(\omega_0 t + \frac{\pi}{4}) & ,t \geq 0 \\ 2h_0 e^{at}\cos(\omega_0 t + \frac{\pi}{4}) & ,t \leq 0 \end{cases}, \tag{5}$$

The constant a is related to the signal spectral density bandwidth, $\Delta\omega_h = \frac{\pi a}{2}$, and the corresponding spectral density is of the form[4]

$$S_h(\omega) = \frac{h_0^2}{2}\left[\frac{2a}{a^2+(\omega-\omega_0)^2} + \frac{2a}{a^2+(\omega+\omega_0)^2}\right].$$

The signal (5) is quasi-monochromatic whenever $a << \omega_0$, ω_0 being its central frequency. Note that when $a \to 0$ we recover (4), the monochromatic case.

The continuous gravitational waves emitted by periodic sources can be regarded as quasi-monochromatic waves. The frequency of the Crab pulsar, for example, which is centered

[3] We suppose, for simplicity, that the g.w. has only one polarization, namely "+".

[4] At this time we admit ω real and $\omega_0 \geq 0$.

near $60Hz$, has a slow down rate of $\sim 0.01Hz/year$. Besides, the orbital motion of the Earth causes a maximum variation of $\pm 0.03Hz/year$, and the spinning motion of Earth implies a maximum variation of $\pm 2 \times 10^{-5}Hz/day$ [8].

For future use in the optimal filter expression, (2), we write the Fourier transform of the quasi-monochromatic signal (5):

$$\tilde{h}(\omega) = \sqrt{2}ah_0 \left[\frac{1+\imath}{a^2+(\omega-\omega_0)^2} + \frac{1-\imath}{a^2+(\omega+\omega_0)^2} \right]. \tag{6}$$

IV. THE MATHEMATICAL MODEL OF THE DETECTOR

A resonant mass detector can be represented by the scheme of figure 1. In this model $F(t)$ represents the gravitational interaction force between the g.w. and the antenna. The two-port circuit is related to the massive antenna and the transducer[5], and it is described by its admittance matrix $y_{ij}(\omega)$, which relates the force f_1 and the velocity v_1 at the input port to the current I and the voltage V at the output port:

$$\tilde{I}(\omega) = y_{11}\tilde{f}_1(\omega) + y_{12}\tilde{v}_1(\omega) \tag{7}$$

$$\tilde{V}(\omega) = y_{21}\tilde{f}_1(\omega) + y_{22}\tilde{v}_1(\omega)$$

The transducer and the amplifier have force and velocity noise generators represented by the stochastic, stationary functions $f(t)$ and $v(t)$, respectively. $S_f(\omega)$ $[S_v(\omega)]$ is the spectral density of $f(t)$ $[v(t)]$. We will assume that these functions are not correlated, so that $S_{fv}(\omega) = 0$.

[5]We will adopt a non-resonant transducer [9].

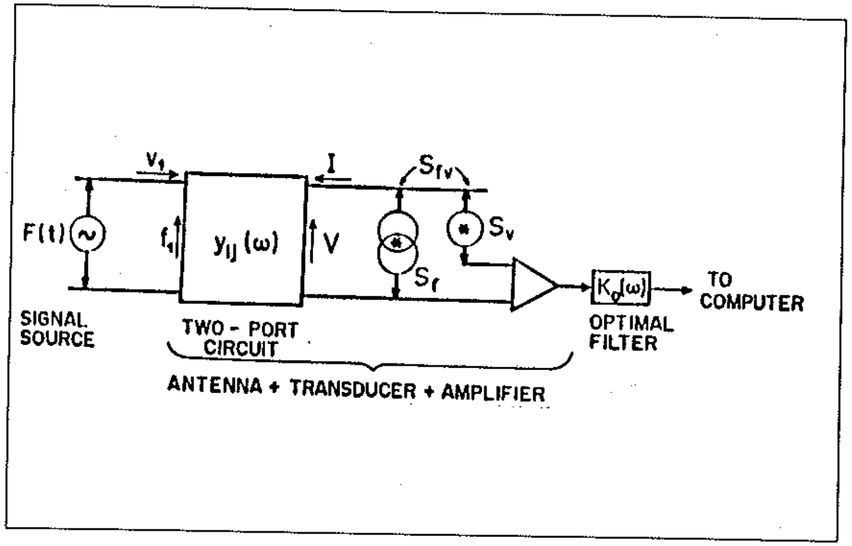

FIG. 1. Resonant mass detector scheme.

In this model the optimal filter follows the lock-in amplifier. In figure 2 the elements that precede the optimal filter in the detector are redrawn [10].

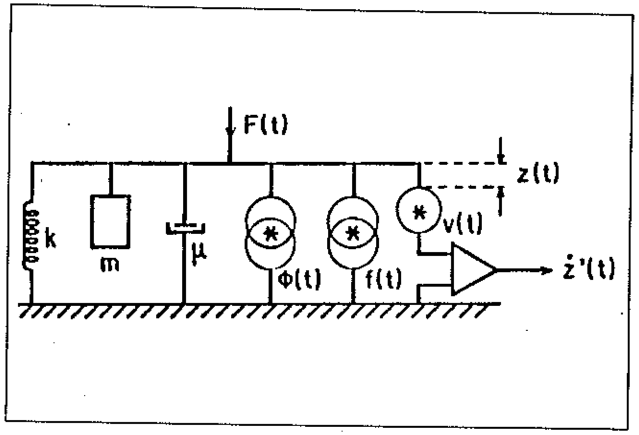

FIG. 2. Model to the antenna, the transducer and the amplifier.

In this figure m is the antenna effective mass, k its elastic constant and μ the damping constant. $\phi(t)$ represents the mechanical dissipation at the antenna, $f(t)$ is associated to he transducer back-reaction on the antenna and $v(t)$ represents the wideband, serial noise ntroduced by the amplifier.

The equation of motion of the system in given by

$$m\ddot{z}(t) + \mu\dot{z}(t) + kz(t) = -[\phi(t) + f(t)] - F(t),$$

here $z(t)$ represents the displacement. However, in the calculations we will deal with the elocity $\dot{z}(t)$; at the amplifier output we have $\dot{z}'(t) = \dot{z}(t) + v(t)$.

In the absence of $F(t)$ we can obtain the velocity noise spectral density:

$$S_N(\omega) = \left(\frac{\tau_0}{2m}\right)^2 \frac{S_\phi(\omega) + S_f(\omega)}{1 + (\omega \pm \omega_0)^2\tau_0^2} + \frac{S_s(\omega)}{1 + (\omega \pm \omega_0)^2\tau_a^2}. \qquad (8)$$

t the denominator of this expression, the minus signal is used when $\omega \geq 0$, and the plus ignal is used when $\omega \leq 0$.

Since the thermal and the back-reaction noises are white noises, the force spectral denities generated by them obey the following Nyquist relations [11]: $S_\phi(\omega) = 4\frac{mk_BT_\phi}{\tau_\phi}$ and $S_f(\omega) = 4\frac{mk_BT_f}{\tau_f}$. In these expressions, τ_ϕ and τ_f are the time constants of mechanical loss t the antenna and of electrical loss at the transducer and amplifier, respectively. They re related to the energy decay time of the antenna, $\tau_0 = \frac{Q_0}{\omega_0}$, according to the expression $\frac{1}{\tau_0} = \frac{1}{\tau_\phi} + \frac{1}{\tau_f}$, where Q_0 is the antenna quality factor. T_ϕ is the antenna temperature and T_f s the back-reaction noise temperature. k_B is the Boltzmann constant.

The function $S_s(\omega)$ that appears in (8) represents a serial white noise introduced in he useful signal by the electrical network, and it has the following expression: $S_S(\omega) = \frac{1}{|y_{22}|^2}4R_tk_BT_r$, where y_{22} comes from (7). R_t is the real part of the impedance at the transucer output and T_r is the circuit noise temperature. We assume that the amplifier has a arge but limited bandwidth, as it occurs in practice , given by τ_a^{-1}. Generally

$$\tau_0 >> \tau_a. \qquad (9)$$

Introducing the antenna's *equivalent temperature*, T_e, defined by the relation $\frac{T_e}{\tau_0} = \frac{T_\phi}{\tau_\phi} + \frac{T_f}{\tau_f}$, equation (8) becomes

$$S_N(\omega) = \frac{\frac{2\tau_0k_BT_e}{m}}{1 + (\omega \pm \omega_0)^2\tau_0^2} + \frac{\frac{4R_tk_BT_r}{|y_{22}|^2}}{1 + (\omega \pm \omega_0)^2\tau_a^2}. \qquad (10)$$

This is the complete expression for the total noise spectral density at the filter input.

The velocity that the g.w. (5) generates at the antenna in the absence of the noises $\phi(t)$ and $f(t)$ has the following Fourier transform:

$$\tilde{U}(\omega) = -\frac{\tau_0}{2m}\frac{\tilde{F}(\omega)}{1 + \imath\tau_0(\omega - \omega_0)}. \qquad (11)$$

$\tilde{F}(\omega)$ is the Fourier transform of the g.w. force on the antenna, which is given by the relation [12]

$$F(t) = -\frac{1}{4}\sum_{\alpha,\beta=1}^{3} q_{\alpha\beta}\frac{d^2}{dt^2}h_{\alpha\beta}. \qquad (12)$$

$q_{\alpha\beta}$ is the dynamic part of the mass quadrupole tensor of the antenna. It is a matrix of constant elements which depend on the antenna's geometry and mass distribution. For our calculations we use $q_{xx} \sim \rho(\frac{l}{2})^4$, where l is the characteristic length and ρ is the density of the antenna.

Equation (11) is the useful signal contribution at the filter input, which corresponds to the spectral density[6]

$$S_U(\omega) = \left(\frac{\tau_0}{2m}\right)^2 \frac{S_F(\omega)}{1 + \tau_0^2(\omega - \omega_0)^2}. \qquad (13)$$

$S_F(\omega)$ is the spectral density of the force (12) and it has the form

$$S_F(\omega) = \frac{1}{4}q_{xx}^2\omega_0^4S_h(\omega). \qquad (14)$$

V. THE FILTER THAT OPTIMIZES SNR FOR THE CONTINUOUS G.W. DETECTOR

Using (2), (10) and (11) and adopting $t_0 = 0$ we obtain the transfer function of the filter that optimizes SNR for the model considered in the preceding section:

---

[6] We will simplify the expressions adopting $\omega \geq 0$ henceforth.

$$K_o(\omega) = \frac{\frac{\mathcal{G} e^{\frac{i\pi}{4}}}{[1+a^{-2}(\omega-\omega_0)^2][1-i\tau_0(\omega-\omega_0)]}}{\frac{N_p}{1+(\omega-\omega_0)^2\tau_0^2} + \frac{N_s}{1+(\omega-\omega_0)^2\tau_a^2}}. \quad (15)$$

To simplify this expression we introduced the following definitions: $\mathcal{G} \equiv \frac{Q_0 q_{zz} \omega_0 h_0}{2ma}$, $N_p \equiv 2\tau_0 k_B T_e/m$ and $N_s \equiv 4R_t k_B T_r/|y_{22}|^2$.

The maximum SNR related to this filter is obtainable from (3), (10) and (13), and it corresponds to

$$SNR_o = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\left(\frac{\mathcal{G}}{1+a^{-2}(\omega-\omega_0)^2}\right)^2}{N_p + N_s\frac{1+(\omega-\omega_0)^2\tau_0^2}{1+(\omega-\omega_0)^2\tau_a^2}} d\omega. \quad (16)$$

Note that when $a << \tau_0^{-1} << \tau_a^{-1}$, this expression becomes

$$SNR_o \sim \frac{a}{4}\frac{\mathcal{G}^2}{N_p + N_s}.$$

Assuming, for simplicity, that $N_s = N_p$[7], we find

$$SNR_o \sim \frac{1}{64 k_B}\frac{\omega_0^3 h_0^2}{a}\frac{Q_0 \rho^2 l^8}{m T_e}. \quad (17)$$

By imposing $SNR_o \geq 1$ we obtain the following condition on the parameters of the detector:

$$\frac{Q_0 \rho^2 l^8}{m T_e} \geq 8.83 \times 10^{-22}\frac{a}{\omega_0^3 h_0^2}. \quad (18)$$

To illustrate the use of the filter (15) we will adopt two different realistic detectors. In both cases we will assume that $N_s = N_p$ and $\tau_0 = 10^2 \tau_a$ (see equation (9). The first of them, detector A, is designed to detect Crab pulsar ($\omega_0 = 376.99 Hz$, $h_0 \sim 10^{-24}$ and $a = 3.17 \times 10^{-11} Hz$. This bandwidth arises from the slow down of the pulsar after 100 milliseconds of observation); this detector has the following characteristics: $Q_0 = 5 \times 10^7$, $m = 1200 kg$ [5], $T_e = 5K$, $\rho = 2.74 g.cm^{-3}$, $l = 1.1m$.

Under these conditions, (15) shows a very narrow peak centered in ω_0. It implies a maximum SNR at its output given by $SNR_o = 1$. If the signal bandwidth is smaller (implying a smaller observation time), we have $SNR_o > 1$.

[7]We are supposing that the back-reaction noise and the serial noise give the same contribution to the total signal at the filter input.

On the other hand, if detector A has a lower equivalent temperature we attain $SNR_o = 1$ with a longer observation time. For instance, we obtain this result if $T_e \sim 0.05K$, and $a = 3.17 \times 10^{-9} Hz$. Such bandwidth corresponds to a 10 seconds observation time of the Crab pulsar's slow down .

The other detector considered, detector B, is designed to detect the millisecond pulsar PSR 1937+214 [9] ($\omega_0 = 8,066.47 Hz$, $h_0 \sim 10^{-27}$). Since we did not find any information about the bandwidth of the g.w. emitted by this pulsar, we will assume that it has the value $a = 1.2 \times 10^{-8} Hz$. This detector is characterized by $Q_0 = 5 \times 10^7$, $\rho = 2.74 g.cm^{-3}$, $l = 2m$, $m = 2300 kg$ and $T_e \sim 0.1K$; these are typical values of several ultracryogenic cylindrical antennae.

For detector B, (15) also shows a very narrow peak centered in ω_0, implying $SNR_o = 1$. Like detector A, SNR_o is greater if the signal bandwidth is smaller.

VI. THE FILTER THAT REPRODUCES THE USEFUL SIGNAL WITH MINIMUM ERROR

After the g.w. is detected we have to determine its shape with minimum error. This can be accomplished with the help of an adequate linear filter, $\mathcal{K}_r$, designed to reproduce the useful signal with the greatest possible accuracy (depending on the noise and the useful signal present at its input). This accuracy is characterized by the mean square error, $< \epsilon^2(t) >$, which is obtained from the instantaneous reproduction error, $\epsilon(t)$, defined by

$$\epsilon(t) = \mathcal{O}(t) - \eta(t). \quad (19)$$

$\eta(t)$ is the desired signal at the filter output. In a simple filtering process, as the one we are considering in this work, $\eta(t)$ must be equal to the useful signal at the filter input, $U(t)$.

We obtain the transfer function $K_r(\omega)$ of the filter $\mathcal{K}_r$ by imposing $< \epsilon^2(t) >=< \epsilon^2(t) >_{min}$. This condition implies [7]

$$K_r(\omega) = \frac{S_U(\omega)}{S_U(\omega) + S_N(\omega)}, \quad (20)$$

supposing there is no cross-correlation between $U(t)$ and $N(t)$. The corresponding mean square error is

$$\epsilon_r \equiv < \epsilon^2(t) >_{min} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{S_U(\omega)S_N(\omega)}{S_U(\omega)+S_N(\omega)} d\omega. \quad (21)$$

From this equation it is evident that the error becomes smaller if so becomes the noise. On the other hand, if the noise is too strong ($S_N(\omega) \to \infty$) it results $K_r(\omega) \to 0$ and no signal leaves the filter.

If we define the *total* power of a signal $Z(t)$ by $< Z^2(t) > = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_Z(\omega)d\omega$, the SNR at the filter input will be[8]

$$SNR_{input} \equiv \frac{< U^2(t) >}{< N^2(t) >}. \quad (22)$$

At the filter output the signal $Z(t)$ will have the following total power:

$$< Z'^2(t) > = \frac{1}{2\pi}\int_{-\infty}^{\infty} |K_r(\omega)|^2 S_Z(\omega)d\omega. \quad (23)$$

Using this relation we can find the SNR at the filter output,

$$SNR_{output} = \frac{< U'^2(t) >}{< N'^2(t) >}. \quad (24)$$

VII. SIMPLE FILTERING OF THE QUASI-MONOCHROMATIC SIGNAL

We now consider the particular case of noise spectral density given by (10) and useful signal spectral density given by (11). In this case the filter that reproduces $U(t)$ with minimum error has the following transfer function:

$$K_r(\omega) = \left(1 + \frac{\frac{N_p}{1+(\omega-\omega_0)^2\tau_0^2} + \frac{N_s}{1+(\omega-\omega_0)^2\tau_a^2}}{\frac{\mathcal{G}^2 a}{[1+a^{-2}(\omega-\omega_0)^2][1+\tau_0^2(\omega-\omega_0)^2]}}\right)^{-1}. \quad (25)$$

[8]Note that (22) is different from (1). This happens because we are now interested on the total spectrum of the useful signal, while in the analysis of the first kind of filter we were interested only on the maximum amplitude of this signal.

The minimum error introduced by this filter is obtained from (21) using (10) and (13):

$$\epsilon_r = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{[1+a^{-2}(\omega-\omega_0)^2][1+\tau_0^2(\omega-\omega_0)^2]}{\mathcal{G}^2 a} + \frac{1}{\frac{N_p}{1+(\omega-\omega_0)^2\tau_0^2} + \frac{N_s}{1+(\omega-\omega_0)^2\tau_a^2}}\right)^{-1} d\omega. \quad (26)$$

In detector A the total noise power at the filter input is $< N^2(t) > = 5.8 \times 10^{-24} m^2/s^2$. If we use filter (25) in this detector we obtain $\epsilon_r = 3.42 \times 10^{-31} m^2/s^2$, which is $\sim 6 \times 10^{-8}$ times smaller than $< N^2(t) >$. Comparing (22) and (24) for this case, we find $SNR_{input} \sim 8.34 \times 10^{-8}$ and $SNR_{output} \sim 0.68$, so that $SNR_{output} \sim 8.11 \times 10^6 SNR_{input}$.

On the other hand, using filter (25) in detector B we obtain an output error of $\epsilon_r = 6.24 \times 10^{-32} m^2/s^2$, which is almost 10^{-6} times smaller then the input noise, $< N^2(t) > = 6.06 \times 10^{-26} m^2/s^2$. In this case, $SNR_{input} \sim 1.46 \times 10^{-6}$ and $SNR_{output} \sim 0.67$, which imply $SNR_{output} \sim 4.67 \times 10^5 SNR_{input}$.

If the Crab bandwidth were $a \sim 1.85 \times 10^{-11} Hz$ we would obtain $SNR_{output} \sim 1.05$; this bandwidth also implies $\epsilon_r \sim 2.94 \times 10^{-32} m^2/s^2$. We would find the same SNR_{output} if the bandwidth of PSR1937+214 were $a \sim 6.9 \times 10^{-9} Hz$, corresponding to $\epsilon_r \sim 5.36 \times 10^{-32} m^2/s^2$.

VIII. CONCLUSIONS

We have derived expressions for the transfer functions of two kinds of filters, both designed to detect continuous monochromatic waves. The first filter, $\mathcal{K}_o$, optimizes SNR at its output (equation (1)) and is important for a first detection of the wave. The second filter, $\mathcal{K}_r$, reproduces the wave with minimum error and should be used when we intend to know the complete shape of the wave.

In the study of $\mathcal{K}_o$ we have first analysed the detection of Crab pulsar. Supposing $N_p = N_s$ (see section V) we concluded that this pulsar could be detected if its signal were as monochromatic as $\sim 3 \times 10^{-11} Hz$; it means a sampling time of the order of 100 msec. We have also analysed a possible detection of PSR 1937+214 and suggested a maximum limit for its bandwidth, allowing its detection by third generation resonant mass detectors.

The use of $\mathcal{K}_r$ is effective in the detection of both pulsars if their signals are still more monochromatic. We then obtain $SNR_{output} > 1$ with the form of the signal as preserved as possible.

With the continuous optimization of present detectors the condition of monochromaticity of the signal becomes weaker. For example, if $N_p > N_s$, the bandwidths of the signal can be larger than those we obtained in this paper. Besides, the inequality (18) can be used as a reference to optimize resonant mass continuous gravitational detectors. Note that continuous sources with high frequency, small bandwidth and high amplitude are the most favourable for detection with less improvement of the detector. On the other hand, the detector should have a small equivalent temperature and materials with high quality factor and density should be preferred for the antenna body, which should be as large as possible.

ACKNOWLEDGMENTS

N.S.M. thanks CNPq (Brasília-DF, Brazil) and FAPESP (São Paulo-SP, Brazil) for financial support, and C.O.E. thanks CNPq for partial support. We are grateful to O.D.Aguiar and C.Frajuca for fruitful discussions.

REFERENCES

[1] A.Einstein: Ann.Phys.,Lpz., **49** (1916) 769.

[2] J.Weber: Phys.Rev. **117** (1960) 306.

[3] K.S.Thorne, in *300 years of Gravitation*, ed. S.Hawking and W.Israel, Cambridge Univ. Press (1987), chap. 9.

[4] N.Solomonson et al., in *Proceedings of the 6th Marcel Grossmann Meeting on General Relativity*, eds. H.Sato and T.Nakamura, World Sci. (1992) 1517.

[5] K.Morimoto et al., in *Proceedings of the 5th Marcel Grossmann Meeting on General Relativity*, eds. D.G.Blair and M.J.Buckingham, World Sci. (1989) 1719.

[6] J.C.Price: Phys.Rev. **D36** (1987) 3555.

[7] L.A.Wainstein and V.D.Zubakov, *Extraction of signals from noise*, Dover Publ. Inc. (1970) section 16.

[8] K.Tsubono, in *The detection of gravitational waves*, ed. D.G.Blair, Cambridge Univ. Press (1991) 243.

[9] G.V.Pallottino and G.Pizzella: Il Nuovo Cim. **7C** (1984) 155.

[10] R.P.Giffard: Phys.Rev. **D14** (1976) 2478.

[11] D.K.C.MacDonald, *Noise and fluctuations: an introduction*, John Wiley (1962) 51-55.

[12] H.Hirakawa, S.Owa and K.Iso: J.Phys.Soc.Jpn. **54** (1985) 1270.

[13] M.-K.Fujimoto and H.Hirakawa: J.Phys.Soc.Jpn. **46** (1979) 703.