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MEASURING THE EARTH'S MAGNETIC FIELD IN SÃO PAULO CITY, BRAZIL

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Abstract

It is presented a system, assembled with a home made oscilloscope, to measure the Earth's magnetic field magnitude as well as its inclination, with reasonable quantitative results for teaching purposes. We got $B=(262\pm4)\times10^2$nT for the magnitude and a magnetic inclination of $I=(30\pm1^\circ)$S, that are in good accordance with the values obtained with more sensible equipment, such as the proton-precession magnetometer.

Typeset using REVTEX

I. INTRODUCTION

As the Earth's magnetic field is present in everyone's quotidian, the measurement of the geomagnetic field can be a motivating task for undergraduate students to study a charged particle movement in a weak magnetic field.

There are many sources of the Earth's steady magnetic field or geomagnetic field [1], as the geomagnetic dynamo; as well as many sources of time dependent disturbances, as the secular variation and the solar wind. However we are interested only in the steady magnetic field. The geomagnetic field close to the Earth's surface can be approximately represented by a field produced by a magnetic dipole, (with a magnetic moment $\mu \approx 8 \times 10^{22}\ A/m^2$), whose axis is inclined about 11^o from the Earth's geographical axis. Therefore the magnitude and the direction of the geomagnetic field vary from point to point of the observation. The difference between the real geomagnetic field and the field given by the geomagnetic dipole is the residual magnetic field, called geomagnetic regional anomaly.

At each point on the Earth's surface the magnetic field is characterized by its intensity (B); by its inclination (I) which is the angle between the geomagnetic vector and the horizontal plane; and by is declination (D) which is the angle between the geographical meridian and the magnetic meridian, which is the plane defined by the geomagnetic vector and the vertical direction.

The local geomagnetic field can suffer artificial distortions from electrical transmission lines, from magnetic materials as the iron and steel found in fences and building. To minimize these effects, we collected the data outdoor, about 20m far from the nearest building and about 100m far from the nearest transmission line. The presence of magnetic minerals in the ground that could affect our data was ignored.

II. THEORY

Choosing 3 referential systems, as shown in Fig. 1, the first one is the Earth's system, with the z-axis in the vertical direction, whose plane yz is contained on the magnetic meridian plane; the primed referential so is got rotating the unprimed referential by $\theta(= 90^o - I)$ around the x-axis, the z'-axis is along the geomagnetic field vector. The double primed referential is attached to the oscilloscope, (with the z''-axis along the electron beam and the plane $x''y''$ is the oscilloscope screen plane), and it is got rotating the primed referential by an angle α. The measurement of the electron deviation under the Lorentz's force was done in this later referential.

The coordinates in the different referential are related by the rotating matrix $\mathbf{A}(\theta)$:

$$A(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} \qquad (1)$$

The electrons leave the cathode with despicable speed and are accelerated by the anode potential V until they get a speed given by $\frac{1}{2}mv_o^2 = eV$. Supposing the electrons leaving the anode with a velocity parallel to the z"-axis and after all suffer only the Lorentz's force, we can get their trajectories in the different referential using the adequate rotating matrix.

Therefore in the oscilloscope referential the initial velocity is:

$$\overrightarrow{v}_o = (0, 0, v_o) \; and \; v_o = \sqrt{\frac{2eV}{m}} \qquad (2)$$

and in the magnetic referential the initial velocity is given by:

$$\overrightarrow{v_o}' = \mathbf{A}^{-1}(\alpha)\overrightarrow{v_o}'' = (0, v_o\sin(\alpha), v_o\cos(\alpha)) \qquad (3)$$

The electron trajectory under the Lorentz's force in the magnetic referential is shown in Fig. 2. The electrons perform a spiral line until they reach the oscilloscope screen at the point C ($z'' = L$ =the distance between the anode region and the screen). The projection

f this trajectory on the plane $x'y'$ is a circumference of radius R given by the centripetal rce:

$$mv_o^2\sin^2(\alpha)/R = ev_o\sin(\alpha)B \qquad (4)$$

here B is the magnitude of the Earth's magnetic field, e and m are the charge and the ass of the electron, respectively, therefore:

$$R = v_o\sin(\alpha)/\omega \; and \; \omega = eB/m \qquad (5)$$

The equations of the movement in the magnetic field referential are:

$$\begin{aligned} x' &= R(\cos(\omega t) - 1) = v_o\sin(\alpha)(\cos(\omega t) - 1)/\omega \\ y' &= R\sin(\omega t) = v_o\sin(\alpha)\sin(\omega t)/\omega \\ z' &= tv_o\cos(\alpha) \end{aligned} \qquad (6)$$

nd rotating them to the oscilloscope referential we get:

$$\begin{aligned} x'' &= x' = v_o\sin(\alpha)(\cos(\omega t) - 1)/\omega \\ y'' &= v_o\sin(\alpha)\cos(\alpha)(\sin(\omega t)/\omega - t) \\ z'' &= v_o\sin^2(\alpha)\sin(\omega t) + t\cos^2(\alpha) \end{aligned} \qquad (7)$$

Each electron spends a time t_c to go from the focusing region to the oscilloscope screen, as $\omega t_c \ll 1$ eq. (7) can be expanded until second order in ωt_c:

$$\begin{aligned} x'' &\simeq -v_o\sin(\alpha)\omega t_c^2/2 \\ y'' &\approx 0 \\ z'' &\simeq v_o t_c \simeq L \end{aligned} \qquad (8)$$

therefore from eqs. (2), (5) and (8) we get for the electrons dislocations a sine function whose amplitude depends on the acceleration voltage value:

$$x'' = -L^2\sqrt{e/(8mV)}B\sin(\alpha) \qquad (9)$$

We can rewrite this equation in a linear form defining $\xi = x''\sqrt{V}$ and $\eta = \sin(\alpha)$, thus

$$\xi = -L^2\sqrt{e/(8m)}B\eta \qquad (10)$$

whose angular coefficient is proportional to the Earth's magnetic field magnitude.

III. EXPERIMENTAL APPARATUS

It consists of a home made oscilloscope built for teaching purposes, with all the deflection plates grounded, so any deviation of the electrons from a linear trajectory is due to external forces. It was used a Toshiba 150BTB31 cathode ray tube (CRT). Since the description of a CRT and of a power supply, similar to that one we have been used, can be found in reference [2] we will not describe the oscilloscope in details.

The apparatus was assembled to allow horizontal and vertical rotations of the oscilloscope. The angle α between the electron beam and the geomagnetic field was got by measuring the angle γ $(= I - \alpha)$ with a plastic protractor and a plummet line, as shown schematically in Fig.1.

IV. RESULTS AND DISCUSSION

São Paulo City is situated at $733m$ of altitude, latitude $23°33'34"S$ and longitude $46°44'06"W$, the local magnetic declination is $D = 18°39'20"W$ [3].

First of all we aligned the oscilloscope along the horizontal component of the geomagnetic field with the help of a needle compass, next we rotated the oscilloscope around the horizontal axis until we got the spot close to the screen's center. In this position we measured the angle θ. Reversing the sense of the electron beam, rotating the oscilloscope of 180 degrees around the horizontal axis, we checked the spot position and the θ value. Fine adjustments, in both directions, were done by little rotations around the vertical and horizontal axis, respectively, until we got the same value for the angle θ for the same position of the spot, close to the screen's center. In that way we got $I = (30 \pm 1^0)S$ for the magnetic inclination, in a good accordance with the mean value of $I = 31^0 15'50"S$, predict by 1990.0's Geomagnetic Field Model for Brazil [4].

The angle α was varied from 0 up to 360 degrees for five different values of the anode voltage , $V = 300, 400, 500, 600$ and $700V$. As predict by the eq. (8), it was always observed $y'' \approx 0$, with fluctuations much smaller than our precision of $0.0005m$.

In Fig. 3 it is shown ξ *vs.* α, for the five different values of V, as well as the linear form ξ *vs.* η. The continuous lines were got from the fitting of a straight line to the linearized experimental data. The angular coefficient obtained from the fitting is $(252.73 \pm 0.40) \times 10^{-3} m\sqrt{V}$, as $L = 0.255 \pm 0.002m$ then $B = (262 \pm 4) \times 10^2 nT$, also in a good accordance with $B = 26568 \pm 12nT$, got with a proton-precession magnetometer (sensitivity $= 1nT$) at the same place.

V. CONCLUSION

The experiment is a useful exercise to illustrate the movement of electrons in a weak magnetic field and to get the magnitude of the Earth's magnetic field as well as its inclination, with reasonable quantitative results for teaching purposes.

ACKNOWLEDGMENTS

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FIGURES

Fig.1 Referential definitions relative to the local meridian; and schematically the oscilloscope with the system to measure the angle α $(= I - \gamma)$.

Fig.2 Electrons trajectory under the Lorentz's force observed in the geomagnetic referential. C is the collision point of the electrons with the oscilloscope screen. R is radius of the projected trajectory in the plane $x'y'$.

Fig.3 ξ vs. α for five different values of the anode voltage. The closed circles correspond to the linear function ξ vs. η taking all data. The continuous lines are the best fit to the experimental data.

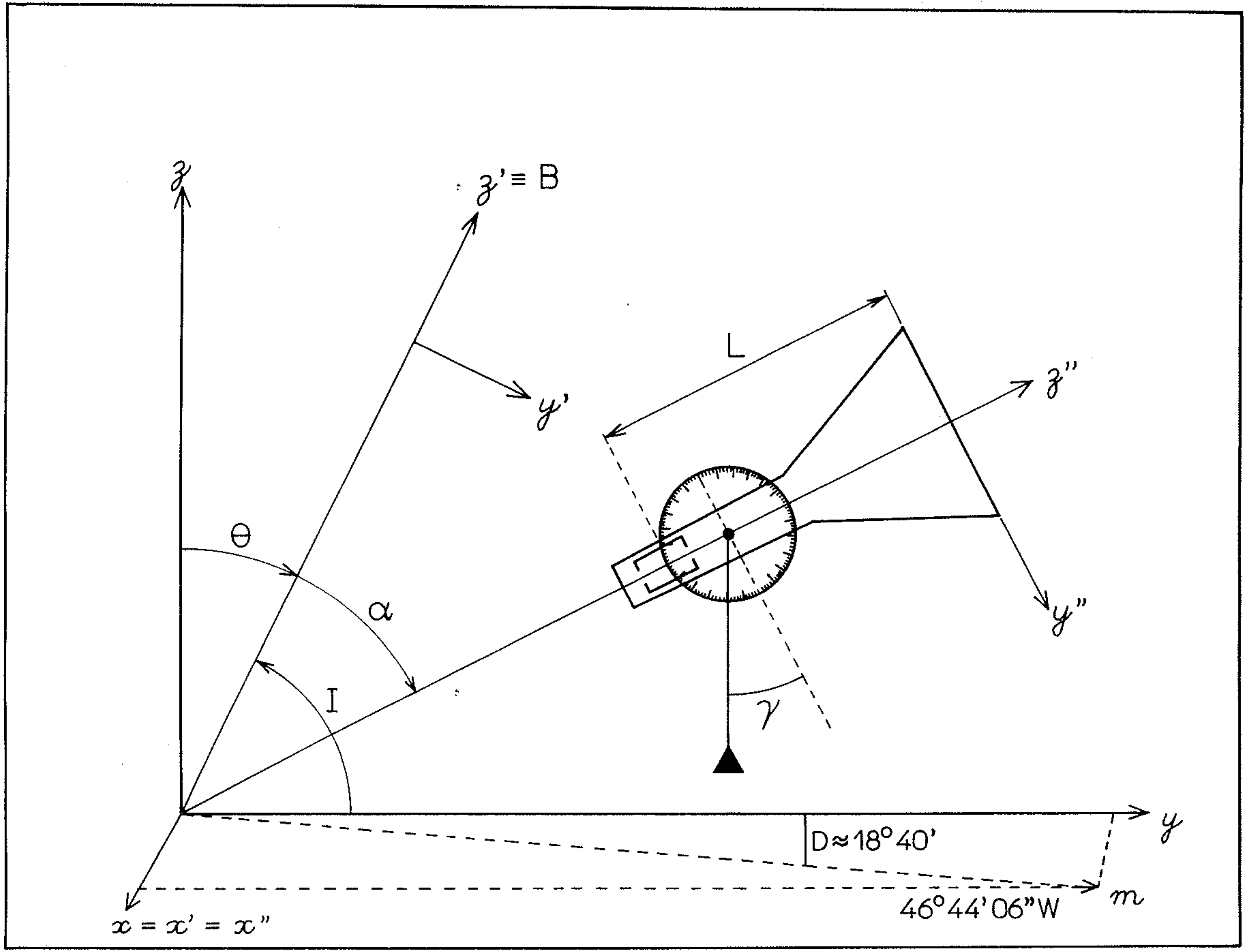

Fig. 1

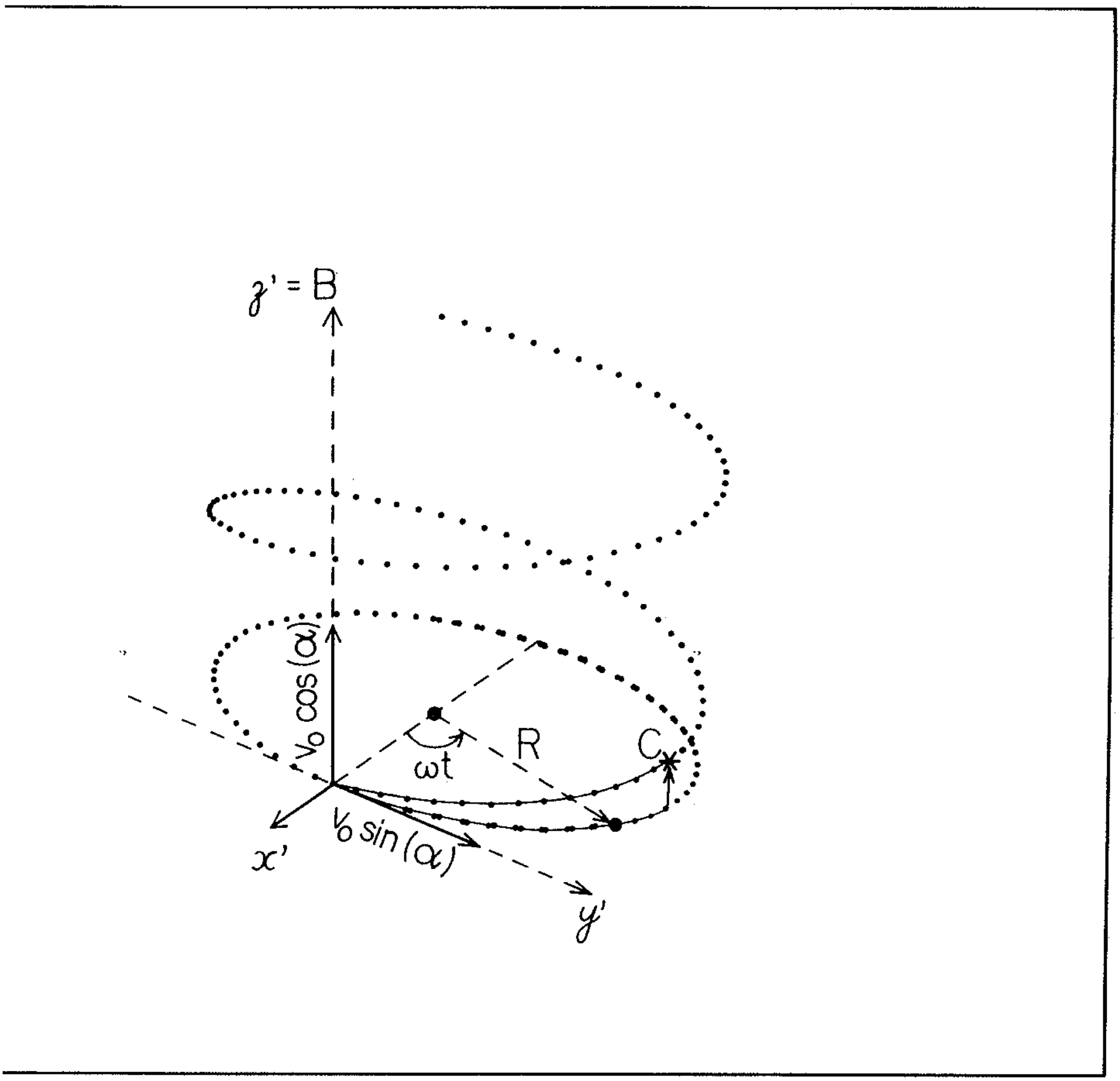

Fig. 2

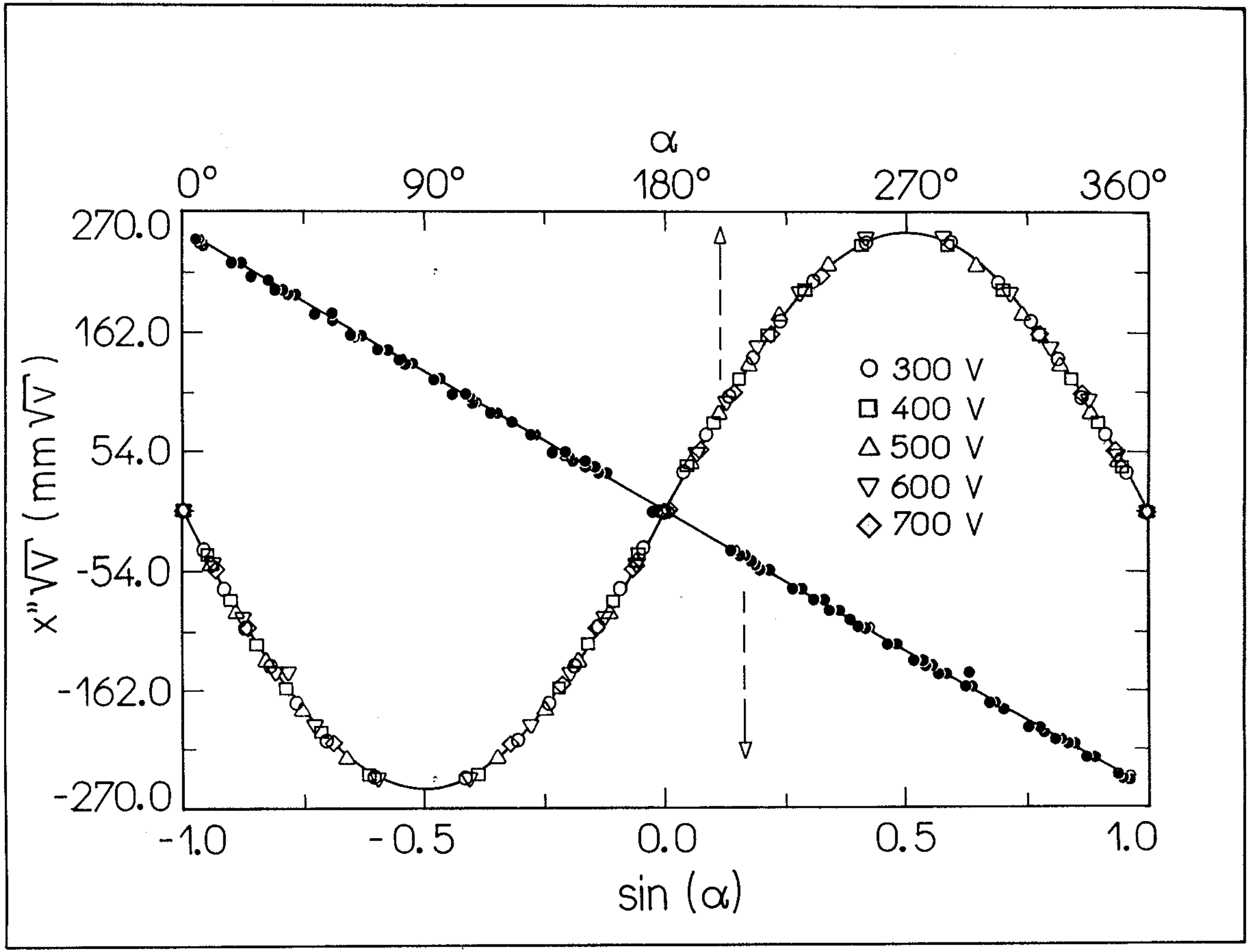
α
0°
90°
180°
270°
360°
270.0
162.0
54.0
-54.0
-162.0
-270.0
x"√V (mm √V)
-1.0
-0.5
0.0
0.5
1.0
sin (α)
○ 300 V
□ 400 V
△ 500 V
▽ 600 V
◇ 700 V

Fig. 3