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THEORY OF THE GLORY*

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Abstract

A new theory of the optical glory is given. The results are in good agreement with the exact Mie solution. The physical effects responsible for the glory are explained.

The optical glory, a meteorological effect first reported[1] in 1735, is a strong enhancement in the backscattering of light from clouds, often accompanied by colored rings. It arises[2] from individual water droplets, with size parameters $\beta = ka$ (k = wave number; a = droplet radius) usually[3] in the range 10^2-10^3. Exact Mie partial-wave series summed by computer in this range show[4] that the backscattering intensity, as a function of β, undergoes very complicated, rapidly-varying, quasi-periodic fluctuations, with a quasi-period $\Delta\beta \approx 0.815$ (cf. figs. 3a-3c), that have also been observed experimentally[5]. Since the number of partial waves that contribute is $\gtrsim \beta$, such results provide no answer to the question we address here: what physical effects are responsible for the glory?

We apply asymptotic techniques first developed for a scalar field[6] and later extended to electromagnetic waves[7]. For $\theta = \pi$, the scattering amplitudes $S_1(\beta,\theta)$ and $S_2(\beta,\theta)$ are given by[2]

$$S_1(\beta,\pi) = S^M(\beta) + S^E(\beta) = -S_2(\beta,\pi), \tag{1}$$

where magnetic (M) and electric (E) contributions are of the same order (cross-polarization effect). We start from the Debye multiple-reflection expansion[6]

$$S_j(\beta,\theta) = \sum_{p=0}^{\infty} S_{j,p}(\beta,\theta) \quad (j = 1,2), \tag{2}$$

where $S_{j,p}$ is associated with waves that undergo p-1 internal reflections (only external reflection for p=0). We must: (a) determine which p values yield significant contributions at a given θ; (b) find suitable asymptotic expansions for these contributions. Both problems are solved by applying the modified Watson transformation[6] to each Debye term. The following types of contributions are found: (i) geometrical-optic contributions, associated with isolated real saddle points in the λ-plane (λ= complex angular momentum); (ii) surface waves, associated with complex Regge-Debye poles; (iii) rainbow terms, associated with confluent (real or complex) saddle points[6,8]; (iv) Fock-type contributions, interpolating smoothly between the first two types[6].

For the refractive index $N \sim 1.33$ of water, geometrical-optic contributions to backscattering are far too small to account for the glory[4,6]. Van de Hulst[2] conjectured that surface waves for p=2 might be dominant. It was shown[6], however, that higher-order Debye terms must play an important role.

For $p \gg 1$, all relevant contributions arise from the domain $|\lambda-\beta| \lesssim \beta^{1/3}$, where the reflectivity is close to unity. It is found[7] that, besides surface-wave contributions, those due to higher-order rainbows formed near $\theta=\pi$ are also important. This is due to the usual rainbow enhancement[6,8] by $\beta^{1/6}$ (besides the backwards enhancement by $\beta^{1/2}$ due to the axial focusing effect), which persists at considerable deviations from the rainbow angle, because the width of the rainbow region increases with p. The backward direction falls within the dark side of the relevant rainbows, where the amplitude is exponentially damped, as it is also for the surface-wave contributions. The damping exponent is roughly proportional to $\beta(|\zeta_p|/p)^{3/2}$ for the large-p rainbow contributions, and to $\beta^{1/3}\zeta_p$ for the surface-wave contributions, where $\zeta_p \equiv \pi - p\theta_t$ (mod 2π), $-\pi \leq \zeta_p < \pi$, $\theta_t = 2\cos^{-1}(1/N)$. Positive ζ_p values represent the angle described by a surface wave before emerging in the backward direction; negative ζ_p, for the rainbow terms, represent approximately the deviation between $\theta=\pi$ and the rainbow angle. We therefore expect that the dominant Debye contributions arise from values of p satisfying $-\theta_t \leq \zeta_p < \theta_t$, and that the lowest ζ_p are dominant for surface waves, whereas the lowest $|\zeta_p|/p$ dominate for rainbow contributions.

We have made numerical comparisons with the exact results[9] for $N = [\cos(11\pi/48)]^{-1} \approx 1.33007$. For this N, $\zeta_{24}=0$, so that a tangentially incident ray forms a closed path, a regular 48 - sided star-shaped polygon[10] inscribed within the spherical droplet (Fig. 1). The lowest values of p satisfying the above criterion are indicated in Fig. 1; further values are obtained by adding multiples of the period $\Delta p=48$. Fig. 2 shows the contributions from Debye terms that contribute up to $\sim 0.1\%$ to $|S^M(\beta)|^2$ and $|S^E(\beta)|^2$ for $\beta=150$, 500 and 1,500. Both the dominant p values and their relative ordering generally agree with our expectations (Fig. 1). Discrepancies are due to factors not included in the above discussion: (a) the ordering is also influenced by the damping at each internal reflection, that gives rise to another exponential, with damping exponent proportional to $p\beta^{-1/3}$; (b) for low β, lower p-values prevail, in spite of larger damping parameters, because the transition angular regions become wide[11].

Let us justify the physical interpretation proposed for the dominant terms and indicate how they are asymptotically represented. The dominant "rainbow" term in Fig. 2 is p=11; the associated 10th-order rainbow is treated by the CFU method, previously described[8] for p=2. The CFU coefficients at $\theta=\pi$ are:

$p_{01} = 1.1 \times 10^{-3}\, i$; $q_{01} = 1.5 \times 10^{-2}$; $p_{02} = 1.3 \times 10^{-4}\, i$; $q_{02} = 1.0 \times 10^{-2}$, so that the Airy theory is inadequate for both polarizations. This is due to the sharp variation of the reflection coefficients in the edge domain. A comparison between the CFU results and the exact partial-wave sum for p=11, shown in Table I, supports the validity of our physical interpretation. The dominant "surface-wave" term in Fig.2 for $\beta = 1{,}500$ is $p = 24$. Since $\zeta_{24} = 0$, the modified Watson transformation leads in this case to an asymptotic representation in terms of Fock-type functions. A comparison with the exact results, shown in Table II, again confirms our physical interpretation.

The dominant Debye terms undergo a phase change of $\simeq\pi$ for $\Delta\beta = 5\pi/(22\sqrt{N^2-1}) \simeq 0.814$, in excellent agreement with the observed quasi-period. The behavior of $|S^M(\beta)|^2$ within a quasi-period near $\beta = 150$, 500 and 1,500 is shown in Figs. 3a-3c. The main humps are seen to result from interference between the dominant "rainbow" term and the dominant "surface-wave" term. Inclusion of all terms indicated in Fig. 1 improves the agreement with the exact results, but still does not reproduce the sharp superimposed spikes. However, when one carries out the summation over the period $\Delta p = 48$, the spikes are recovered (Fig. 3c, inset). Thus, the spikes correspond to "geometrical resonances" associated with the quasi-periodic orbits with $\Delta p = 48$; they are the first to disappear[5] when a small imaginary part is added to N. They occur at different positions for $S^M(\beta)$ and $S^E(\beta)$; the total intensity $|S_1(\beta,\pi)|^2$ shows further structure due to interference between S^M and S^E.

For $\theta = \pi - \varepsilon$ and $\mu = \beta\varepsilon$ not $\gg 1$ (first few glory rings), we have

$$S_1(\beta,\theta) \approx S^M(\beta)\ J_1'(\mu) + S^E(\beta)\ J_1(\mu)/\mu \quad , \qquad (3)$$

where J_1 is Bessel's function of order one; $S_2(\beta,\theta)$ is obtained by interchanging M and E. The angular distribution and polarization of the glory rings[2,12] can be understood in terms of these results. Since rainbow contributions are predominantly magnetic (as the corresponding reflectivity is always larger) and surface-wave ones are predominantly electric, the polarization varies both with β and θ, depending on which type of contribution is dominant. Ripple effects in other directions[6] correspond to an attenuated version of the glory and can be explained in similar terms. Detailed discussions will be given elsewhere.

We conclude that the glory arises from the following physical effects conjointly: (i) incidence in the edge domain (near the top of the centrifugal barrier); (ii) the consequent almost-total internal reflection, leading to many Debye contributions (an effect related with "orbiting"); (iii) enhancement by axial focusing; (iv) the cross-polarization effect; (v) surface-wave contributions from Regge-Debye poles; (vi) imaginary rays in the shadow of higher-order rainbows formed near the backward direction; (vii) geometrical resonances associated with closed or nearly-closed quasi-periodic orbits; (viii) the competition among various kinds of damping: radiation damping of surface waves, the damping of imaginary rays in the shadow of a caustic (rainbow), and that due to internal reflection. The glory represents a new domain in optics, where complex orbits give rise to dominant effects, suggesting that complex extremals of Feynman path integrals may play an important role under more general conditions.

TABLE I. $S_{11}^{M}(\beta,\pi)$ exact compared with "rainbow" term

β	Exact	Asymptotic
1,500.1	-333 - 794i	-304 - 823i
1,500.2	860 - 43i	878 - 8i
1,500.3	-249 + 824i	-289 + 829i
1,500.4	-690 - 514i	-681 - 553i
1,500.5	719 - 474i	752 - 454i

TABLE II. $S_{24}^{M}(\beta,\pi)$ exact compared with "surface-wave" term[a]

β	Exact	Asymptotic
1,500.1	183 - 231i	214 - 193i
1,500.2	- 250 - 102i	- 245 - 111i
1,500.3	52 + 244i	38 + 255i
1,500.4	205 - 187i	229 - 178i
1,500.5	- 254 - 117i	- 236 - 131i

[a]Both exact and asymptotic results include summation over $\Delta p = 48$.

Footnotes and References

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[2]H.C. Van de Hulst, J. Opt. Soc. Am. 37, 16 (1947); Light Scattering by Small Particles (Wiley, New York, 1957).

[3]The average β for observed glories is ~160 (ref.2, p. 258).

[4]H.C. Bryant and A.J. Cox, J. Opt. Soc. Am. 56, 1529 (1966).

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[6]H. M. Nussenzveig, J. Math. Phys. 10, 82, 125 (1969).

[7]V. Khare, Ph. D. thesis, University of Rochester (1975).

[8]V. Khare and H.M. Nussenzveig, Phys. Rev. Lett. 33, 976 (1974).

[9]In all comparisons, the direct-reflection term ($p=0$) has been omitted. This does not affect the conclusions.

[10]The rapid variability with N of the glory pattern is related with the closeness of approach to such periodic closed paths.

[11]E.g., for $p=7$, $\theta = \pi$ is within the main rainbow

peak for $\beta=150$, but lies deep within the shadow for $\beta=1{,}500$.

[12] J.V. Dave, Appl. Opt. **8**, 155 (1969).

Figure Captions

FIG. 1. Path of a tangentially incident ray for $N = [\cos(11\pi/48)]^{-1} \approx 1.33007$. The values of p for the dominant Debye terms are indicated next to the arrows (—— rainbow terms; --- surface-wave terms). The directions of the arrows determine the corresponding values of ζ_p; e.g., $\zeta_{11} = -7.5^\circ$. The ordering by increasing ζ_p (surface waves) or $|\zeta_p|/p$ (rainbow terms) is indicated by the lengths of the arrows.

FIG. 2. Relative contributions of various order Debye terms to $|S^M(\beta)|^2$ and to $|S^E(\beta)|^2$: —— rainbow terms; ---- surface-wave terms. For $\beta=150$, there appear some terms not present in Fig. 1 (shown by -·-·-).

FIG.3. Behavior of $|S^M(\beta)|^2$ within a quasi-period: —— exact; -·-·- contribution from the two leading Debye terms in Fig.2 ; ---- contribution from all Debye terms in Fig. 2 (without summation over $\Delta p=48$): (a) Near $\beta=150$; (b) Near $\beta=500$; (c) Near $\beta=1{,}500$. The inset shows the effect of summation over $\Delta p=48$ over the approach to the spike marked A: —— exact; --- contribution from all Debye terms in Fig.2 summed over $\Delta p=48$.

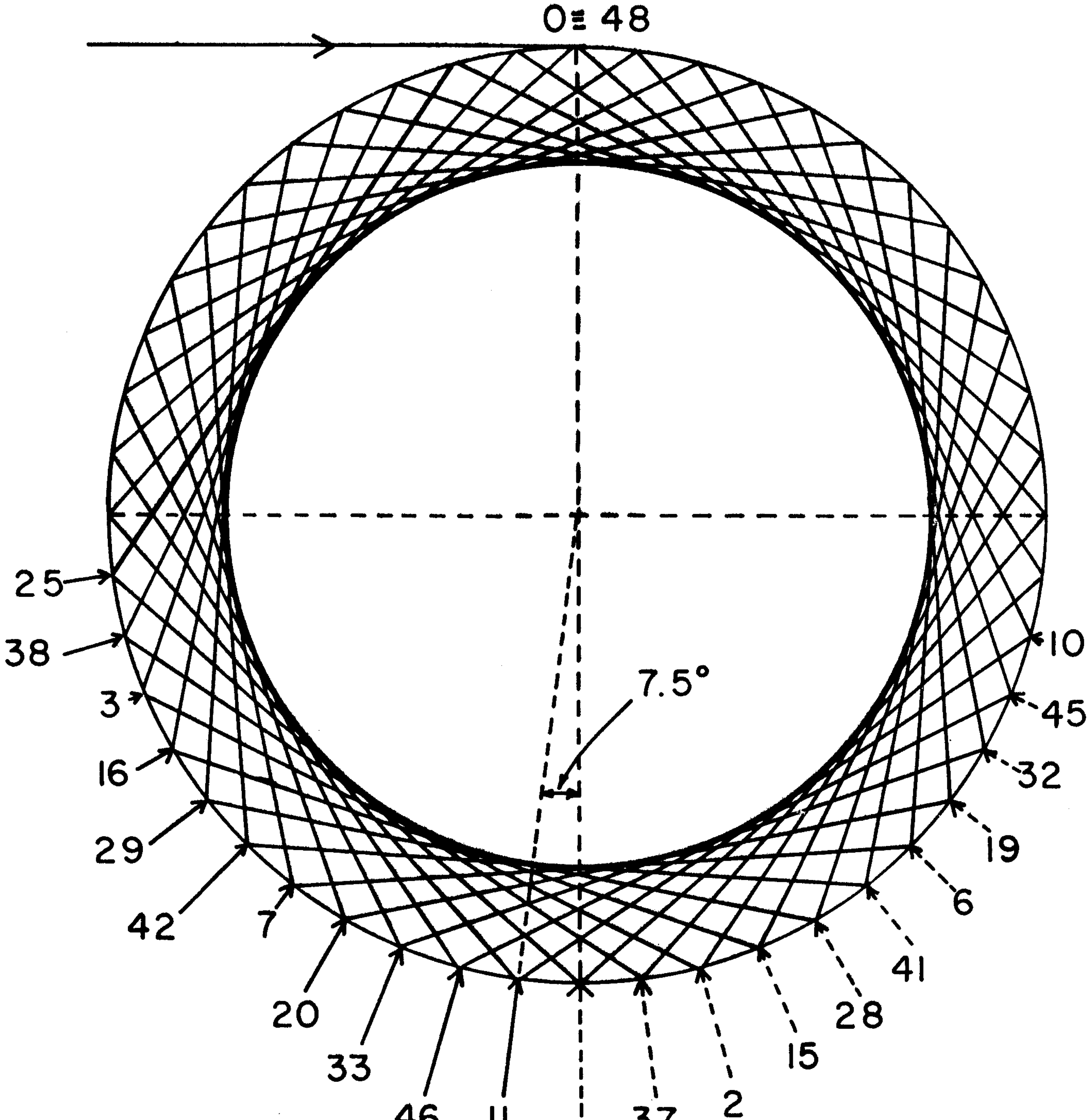
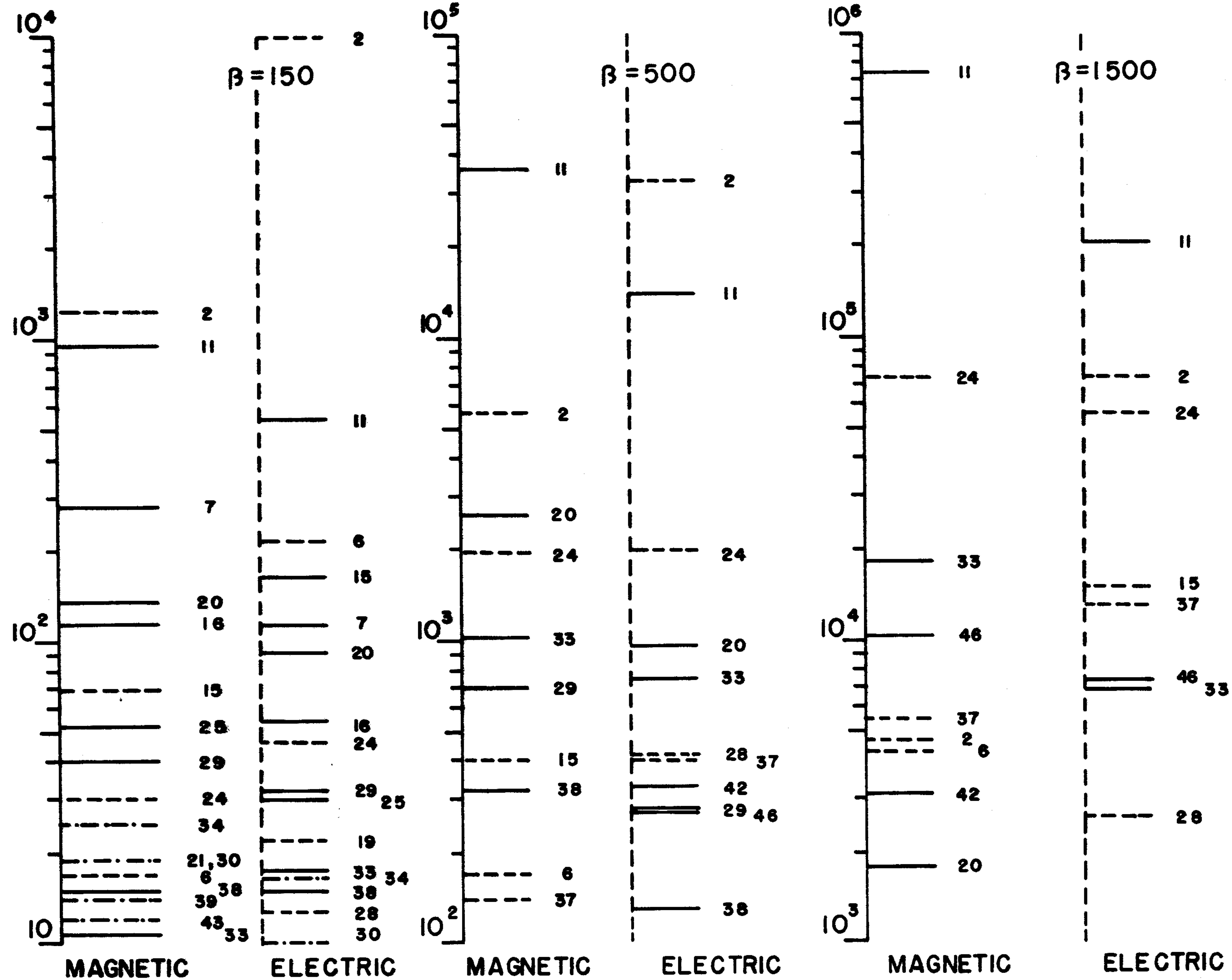


Fig. 1



10^4
10^3
10^2
10
2
11
7
20
16
15
25
29
24
34
21,30
6
38
39
43
33
MAGNETIC
β=150
2
11
6
15
7
20
16
24
29
25
19
33
34
38
28
30
ELECTRIC
10^5
10^4
10^3
10^2
11
2
20
24
33
29
15
38
6
37
MAGNETIC
β=500
2
11
24
20
33
28
37
42
29
46
38
ELECTRIC
10^6
10^5
10^4
10^3
11
24
33
46
37
2
6
42
20
MAGNETIC
β=1500
11
2
24
15
37
46
33
28
ELECTRIC

Fig. 2

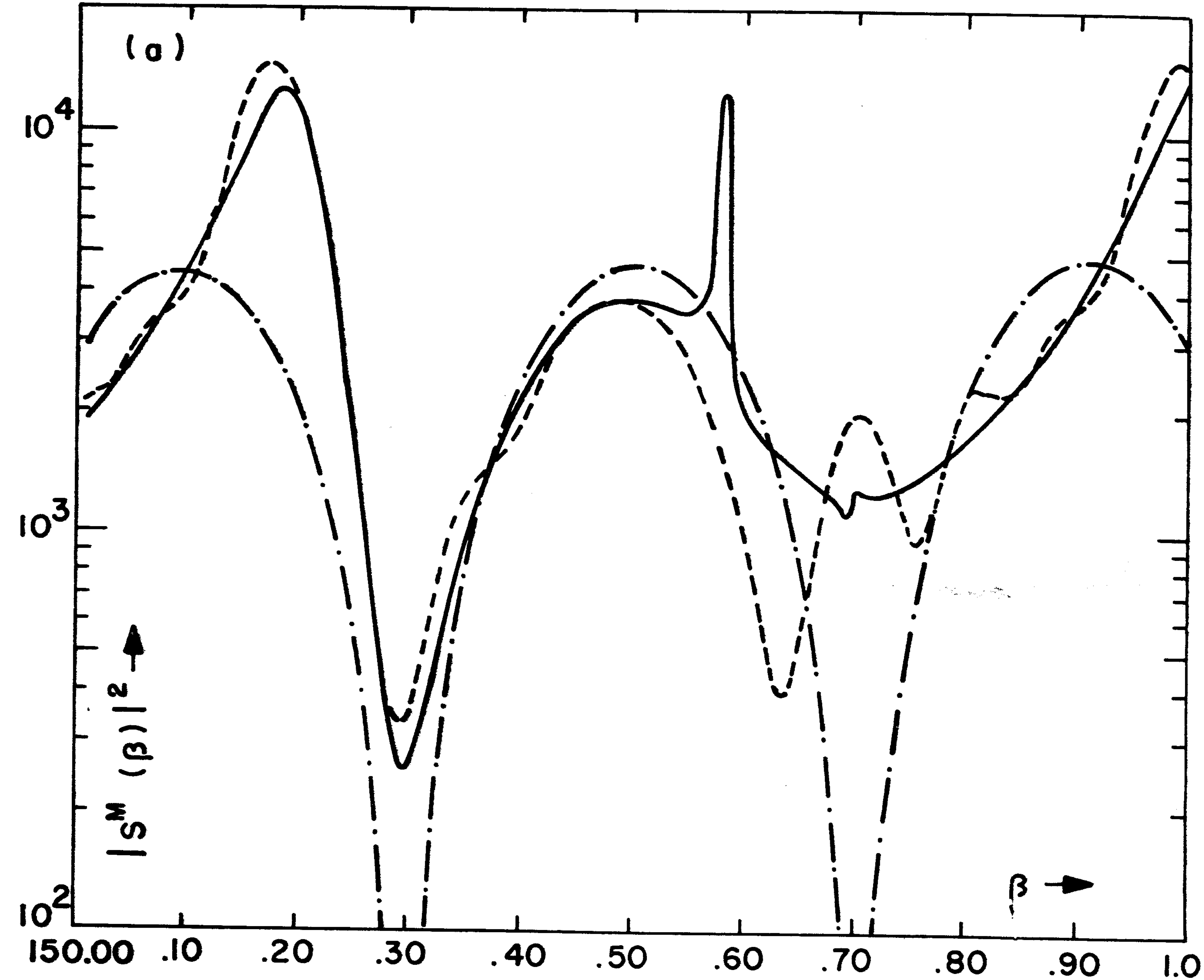


Fig. 3a

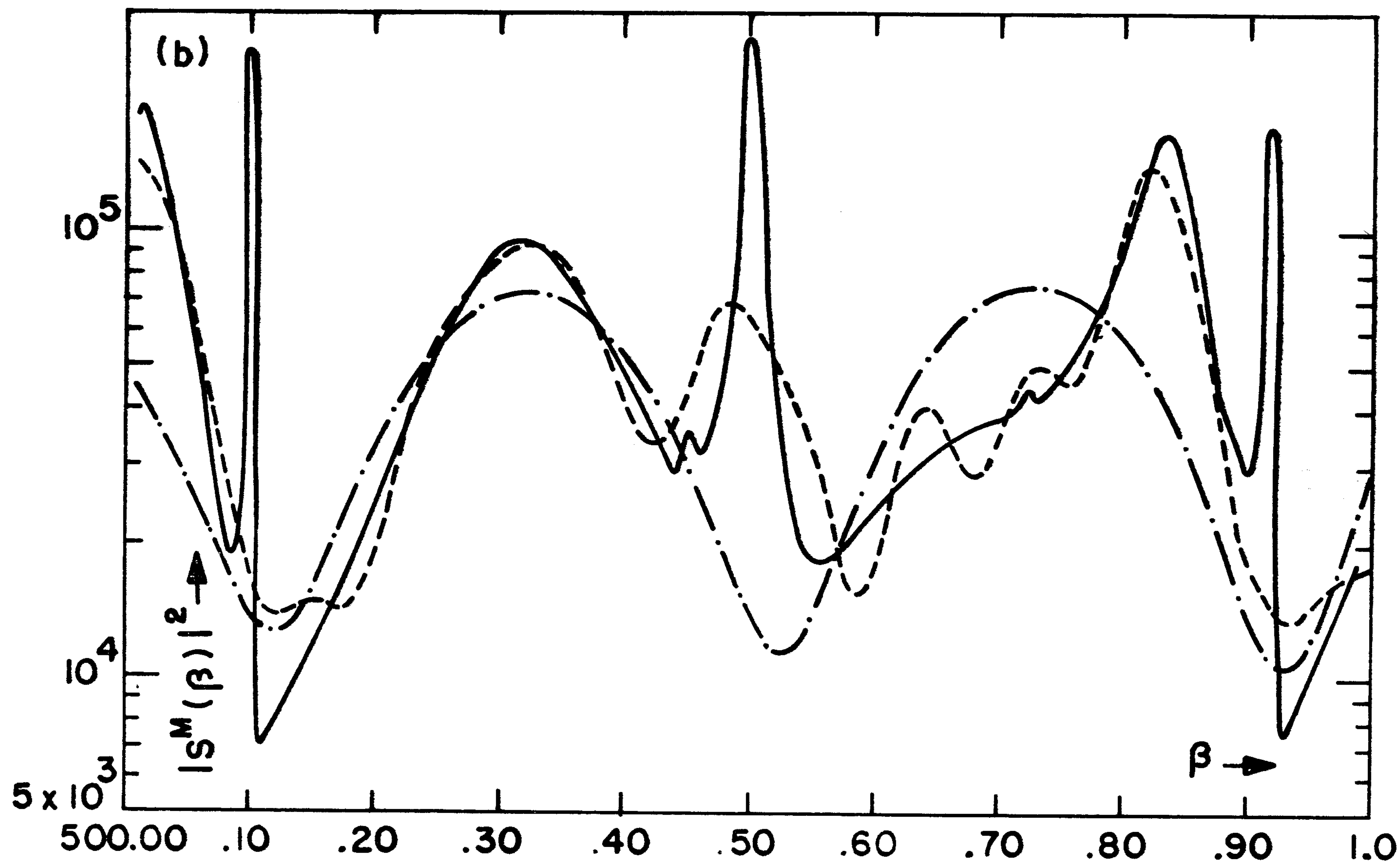

(b)
10^5
10^4
5×10^3
$|S^M(\beta)|^2$
500.00
.10
.20
.30
.40
.50
.60
.70
.80
.90
1.0
β

Fig. 3b

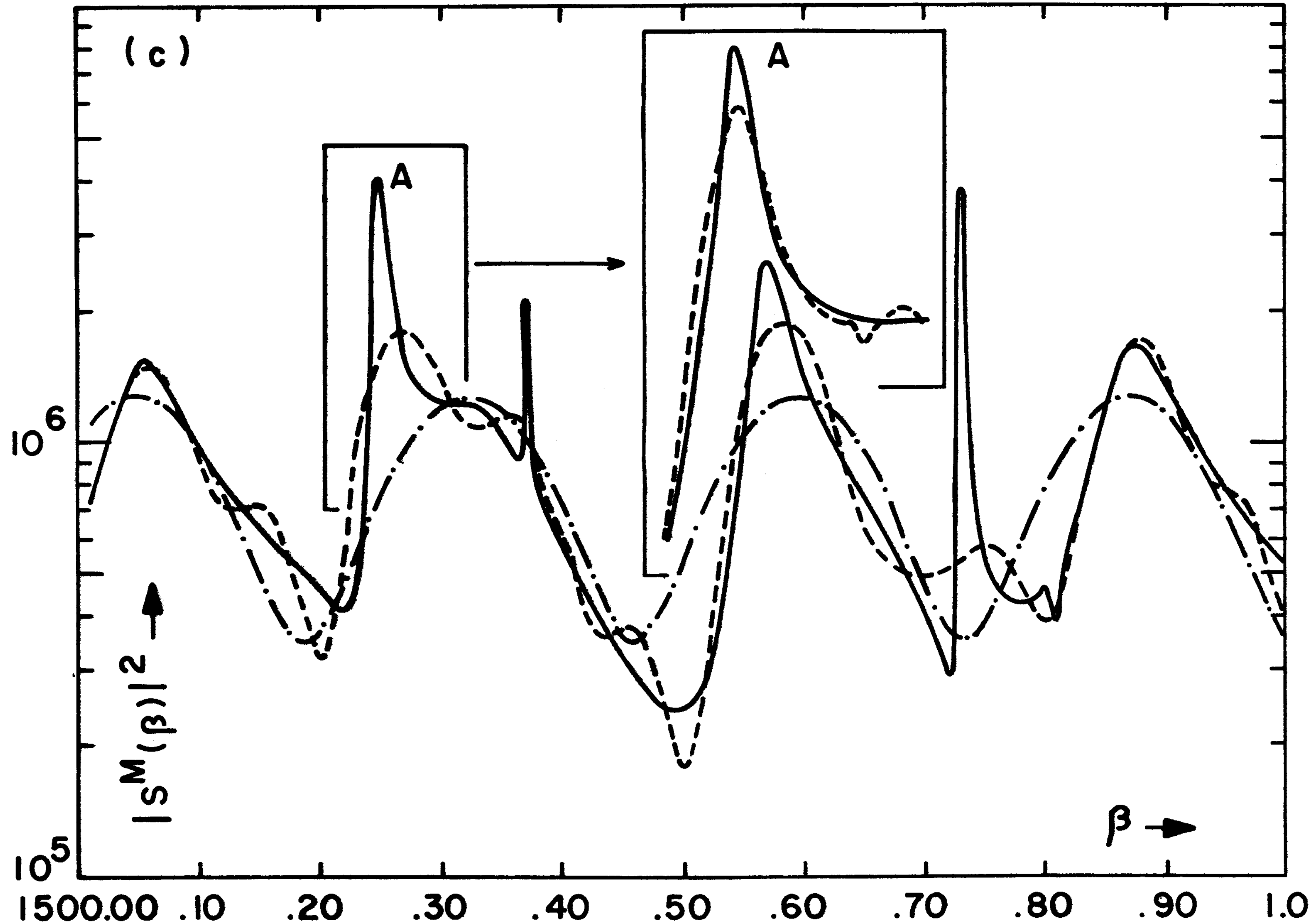


Fig. 3c