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Abstract

An effective hamiltonian for heavy quarkonia is derived from QCD by separating gluonic fields in background and quantum fields and neglecting anharmonic contributions. Mesonic states with nonperturbative gluonic components are constructed. These states are invariant under gauge changes of the background fields and form an orthogonal basis. The effective hamiltonian is diagonalized in this basis in a systematic $1/m$- and short distance expansion. For very heavy quarkonia, we obtain an effective potential similar to the phenomenological funnel potential. We compare our method to 2^{nd} order perturbation theory in the background fields and demonstrate its applicability even for the relatively light charmonium system. The results to order $1/m$ for pseudoscalar meson masses and wave functions are shown and compared

with those of the Cornell model.

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I. INTRODUCTION

In phenomenological hadron models nonperturbative gluonic effects are accounted for in a variety of ways. In non-relativistic potential quark models [1], an effective interquark potential is assumed to result from them and the effective hamiltonian is diagonalized in the Hilbert space of quarks only. In flux tube models [2] one does not completely eliminate the gluons. Their net effect is to generate a color flux tube between quark and antiquark that binds them. In bag like models [3] nonperturbative gluonic effects appear in the guise of the bag constant. They are taken to generate a vacuum pressure which counterbalances that of the (perturbative) quarks in the interior of the bag. Gluons in the bag are assumed to be perturbative and responsible for a hyperfine interaction. All these models are surprisingly successful in describing the hadronic spectrum. Except for a few exotic states there seems to be no need for hybrid states, bound states of quarks and gluons. Nevertheless, even one gluon exchange, leads to intermediate states, where the (constituent) quarks are not in a color singlet representation and one may wonder, whether the nonperturbative part of their interaction (for our purposes everything that is not one gluon exchange) is really as color blind as is generally assumed. A better understanding of the relation among the various model parameters and the structure of the nonperturbative ground state would also clearly be of interest, and there have been attempts to find a relation between the bag pressure and gluon condensation and the confining potential in nonrelativistic models and the string tension obtained in lattice calculations [4].

In this paper we study the influence of a nontrivial gluonic ground state on the *structure* of heavy quarkonia. The nearly nonrelativistic nature of these mesons makes them ideal probes of ground state properties. Their large mass allows for a systematic expansion of the interaction at small distances and we gamble that the lowest dimensional (gluonic) condensate suffices for a rudimentary description of the nonperturbative vacuum structure in this case. We estimate the matrix elements of the hamiltonian in an approach very similar to the one used in QCD-sum rules [5] using background fields to describe nonperturbative gluons

and parametrizing their vacuum matrix elements. In contrast to the approach taken by Voloshin [6] and Leutwyler [7], we do not treat the nonperturbative part of the hamiltonian so obtained as small compared to the (perturbative) coulomb interaction. This is probably only the case for quarkonia heavier than botonium [7]. We will also not try to save 2^{nd} order perturbation theory by the inclusion of ad hoc correlation effects in the condensates [8]. Such correlations can be thought of as effectively including higher dimensional condensates and cannot be treated in a systematic way. Instead we calculate the short distance expansion of the hamiltonian matrix elements in a gauge invariant basis of color singlet states [9], including only terms up to order r^2 and $1/m$. We then obtain the eigenstates by (numerical) diagonalization of this hamiltonian matrix and see how far we can go in this approach. For this systematic short distance expansion we had to extended the (background) gauge invariant basis of ref. [9] to include states in which the heavy quark-antiquark pair are in a color octet representation but coupled to a nonperturbative gluonic background field [6,10] to form an overall color singlet. Diagonalization of the hamiltonian matrix leads to a "color-octet" component in the wavefunction of heavy quarkonia that takes account of the possibility of color exchange between the valence quarks and the background field. This component becomes rather large for higher excited states and its coupling to the "singlet" is the main reason for the distortion of the coulombic spectrum in this model. In the pseudoscalar meson channel only the few octet states constructed in section II couple and a numerical diagonalization of the resulting system of differential equations is still quite feasible.

In section III we derive the effective hamiltonian appropriate for the description of heavy quarkonia. We first obtain the effective Lagrangian to order $1/m$ by a Foldy-Wouthuysen [11] transformation. Using the background field formalism [12] and neglecting anharmonic quantum fluctuations one finally arrives at an effective Lagrangian [13,14] that includes background fields up to 2^{nd} order and which, in the instantaneous approximation, gives rise to an effective hamiltonian that is accurate to order $1/m$ and r^2. Since retardation effects are of order $1/m^2$, their consistent inclusion would require a much more elaborate treatment, which would only obscure the basic nonperturbative gluonic effects we want to elucidate here.

Numerical diagonalization of this effective hamiltonian in the extended basis then yields (pseudoscalar) meson masses and wave functions. Such an approach can in principle not be described by an effective potential in the singlet channel [6,10] because the elimination of the relative octet states would make it energy dependent. One can however obtain an energy independent effective potential for infinitely heavy quarks, where all terms of order $1/m$ (also the kinetic energy) can be neglected. This potential should be closely related to the static quark potential one extracts from the Wilson loop [15]. This potential, which we derive and discuss in section IV, also gives an idea how far the short distance expansion can be trusted. We show that although it is apparently linearily rising at intermediate distances $.4fm < r < .7fm$, the potential is also compatible with an effective exchange of the Gribov type [16].

In section V we compare our method to methods relying on 2^{nd}-order perturbation theory in the background field and to the phenomenological Cornell [26] potential. We first present results where all potential matrix elements of order $1/m$ are neglected. This greatly simplifies the calculations because only a few basis states couple, but pseudoscalar and vector mesons are degenerate at this level. We also give the results of a more complete calculation of the pseudoscalar quarkonia which includes the $1/m$ corrections.

Section VI is a summary and discussion of our results.

II. GAUGE INVARIANCE AND BASIS STATES

It is usually assumed that physical states are color singlets. For heavy mesons, where the non-relativistic approximation is adequate, one can represent quark and antiquark fields by 2-component spinors. Gauge invariance of the state requires that color is parallel transported from the the quark to the anti-quark along some path. With a straight path the quark and anti-quark anti-commutation relations ensure that the basis states are orthogonal [9]. For

simplicity we will restrict ourselves in the following to pseudoscalar mesons. The simplest gauge invariant pseudoscalar state is of the form

$$|\,21\rangle_S = \frac{1}{\sqrt{6}}\sum_{ab;\alpha} u^{\dagger a}_\alpha(\vec{x}_2)T_{ab}(\vec{x}_2,\vec{x}_1)v^b_\alpha(\vec{x}_1)\,|\,\Omega\rangle\,, \tag{1}$$

where $u^{\dagger a}(\vec{x}_2)$ creates a quark with color a and spin α at $\vec{x}_2$. v does the same for an antiquark. We will refer to such gauge invariant states as singlet states since the quark anti-quark pair at vanishing separation is in a color singlet representation.

We choose for the color transport operator

$$T_{ab}(\vec{x}_2,\vec{x}_1) = Pexp(-ig\int_{\vec{x}_1,t}^{\vec{x}_2,t} dx^\mu A_\mu(x))_{ab}\,, \tag{2}$$

the path ordered exponential (denoted by $Pexp$) of gluon operators $A_\mu(x)$ along a straight line from $\vec{x}_1$ to $\vec{x}_2$. It is then relatively straightforward to show that canonical anti-commutation relations for the quark and anti-quark operators imply that the singlet meson states (7) are orthogonal [9].

We can also construct gauge invariant basis states where the valence quark anti-quark pair is in a color octet representation at vanishing separation by coupling them to chromo-electric or -magnetic fields. We shall call these states octet states for obvious reasons. Since the chromomagnetic field $\vec{B} = \vec{B}^a\lambda^a/2$ transforms as a pseudovector and the chromoelectric field $\vec{E} = \vec{E}^a\lambda^a/2$ as a vector under rotations and according to the adjoint representation of the gauge group, we extend the basis for pseudoscalar mesons by the states

$$|\,21\rangle_B = \sum_{\alpha\beta}\frac{g}{\pi\phi}u^\dagger_\alpha(\vec{x}_2)\vec{\sigma}_{\alpha\beta}\cdot\vec{B}(\vec{x}_2,t)T(\vec{x}_2,\vec{x}_1)v_\beta(\vec{x}_1)\,|\,\Omega\rangle\,, \tag{3}$$

$$|\,21\rangle_{E1} = \sum_{\alpha}\frac{\sqrt{3}g}{\pi\phi}u^\dagger_\alpha(\vec{x}_2)\vec{E}(\vec{x}_2,t)\cdot(\vec{x}_2-\vec{x}_1)T(\vec{x}_2,\vec{x}_1)v_\alpha(\vec{x}_1)\,|\,\Omega\rangle\,, \tag{4}$$

and

$$|\,21\rangle_{E2} = \sum_{\alpha\beta}\frac{i\sqrt{3}g}{\sqrt{2}\pi\phi}u^\dagger_\alpha(\vec{x}_2)\vec{E}(\vec{x}_2,t)\cdot(\vec{\sigma}_{\alpha\beta}\times(\vec{x}_2-\vec{x}_1))T(\vec{x}_2,\vec{x}_1)v_\beta(\vec{x}_1)\,|\,\Omega\rangle\,, \tag{5}$$

the summation over color indices being implied.

The above states are seen to be mutually orthogonal and normalized by the canonical anti-commutation relations of the quark and anti-quark operators if we assume expectation values

$$\langle\Omega \mid \frac{g^2}{4\pi^2} B^{ia} B^{jb} \mid \Omega\rangle = -\langle\Omega \mid \frac{g^2}{4\pi^2} E^{ia} E^{jb} \mid \Omega\rangle$$
$$= \frac{1}{96}\delta^{ij}\delta^{ab}\langle\Omega \mid \frac{\alpha}{\pi} F^{\mu\nu c} F^{c}_{\mu\nu} \mid \Omega\rangle = \frac{1}{96}\delta^{ij}\delta^{ab}\phi^2 , \tag{6}$$

and $< E >=< B >=< EB >= 0$, which are a consequence of the Lorentz- and parity-invariance of the vacuum state $\mid \Omega\rangle$. Its nontrivial nature is reflected in a non-vanishing value for ϕ^2, which from QCD sumrule estimates should be close to $(330MeV)^4$ [5]. If we neglect expectation values of all higher dimensional gluonic operators the states (1),(3),(4) and (5) form a complete orthogonal basis for the valence quarks of heavy pseudoscalar mesons, while the singlet states (1) are only complete in this sense if one neglects the non-trivial vacuum structure altogether. Note also that within this simplified approximation to the true vacuum it is irrelevant where the chromo- electric or magnetic operators are inserted along the string-the difference being proportional to higher dimensional operators.

Note also that Lorentz invariance of the ground state $|\Omega >$ forces one to assume that the chromoelectric background field is either antihermitian or that the "E-states" have negative norm (see (6)). Both possibilities lead to a nonhermitian hamiltonian matrix, whose eigenvalues in general are not real. Already the construction of basis states for the mesons on a non-trivial ground state indicates that one can at best hope to find a few stable mesons in this approach. We assume that such a pseudoscalar meson η can be well represented by a linear combination of the above basis states

$$\mid \eta\rangle = \sum_{M=S,E1,E2,B} \int_{1,2} \psi_M(2,1) \mid 2,1\rangle_M . \tag{7}$$

III. EFFECTIVE HAMILTONIAN

We would like to use the basis constructed in section 2 to approximately diagonalize the QCD hamiltonian for heavy quarkonia. Since low lying heavy quarkonia are generally

believed to be of relatively small size, the interaction should be dominated by the perturbative gluon exchange, which leads to a coulomb-like effective potential. The experimental spectrum deviates however noticeably from a purely coulombic one and can be reasonably well reproduced by the combination of a coulomb- and a linear confining- force. We wish to emphasize here that the truly long range part of the confining force (r > 1fm) is not really tested by the observed quarkonium states [17]. Our conjecture is, that what has to be included in a systematic approach are the first short distance corrections to the (perturbative) coulomb force due to the non-trivial structure of the gluonic ground state. This was proposed previously but without any tangible results, because the corrections were found to be exceedingly large within the framework of 2^{nd}-order perturbation theory except for quarkonia beyond botonium [6,7]. This apparent failure of an idea which we believe is quite well founded, has led us to reexamine the basic procedure used in the evaluation of these effects.

In this section we outline the derivation of the effective hamiltonian for heavy quarkonia, whose matrix elements are correct up to order $\alpha, 1/m$ and r^2. To this order in the short distance and heavy quark expansion the nonperturbative aspects of the ground state can be described by the gluon condensate ϕ^2.

The nonrelativistic approximation for heavy quarks is conveniently obtained from the QCD lagrangian

$$\mathcal{L}(x) = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \bar{\psi}(i\partial\!\!\!/ + gT^a A\!\!\!/_a)\psi - m\bar{\psi}\psi , \tag{8}$$

by a Foldy-Wouthuysen transformation [11]. In terms of transformed quark fields

$$\psi \to \exp(i\vec{\gamma}\cdot\vec{D}/2m)\psi ,$$
$$\bar{\psi} \to \bar{\psi}\exp(-i\vec{\gamma}\cdot\overleftarrow{D}/2m) , \tag{9}$$

the lagrangian is

$$\mathcal{L}_{NRQCD}(x) = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \bar{\psi}(i\gamma^0 D_0 - m)\psi$$

$$
\begin{aligned}
&+\bar{\psi}(\frac{\gamma^0}{2m}[\vec{\gamma}\cdot\vec{D},D_0]+\frac{(i\vec{\gamma}\cdot\vec{D})^2}{2m}))\psi+O(1/m^2) \\
&=-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a+\bar{\psi}(i\gamma^0D_0-m)\psi+\bar{\psi}\frac{\vec{D}^2}{2m}\psi \\
&-\bar{\psi}\frac{ig\vec{\alpha}\cdot\vec{E}}{2m}\psi+\bar{\psi}\frac{g\vec{\Sigma}\cdot\vec{B}}{2m}\psi+O(1/m^2)\,,
\end{aligned} \tag{10}
$$

where $\vec{D}=\vec{\partial}-ig\vec{V}$ and $\vec{\Sigma}=\begin{pmatrix}\vec{\sigma}&0\\0&\vec{\sigma}\end{pmatrix}$ does not couple upper and lower spinor components. They are only coupled in order $1/m$ by the $\vec{\alpha}$ matrices. It can be reduced to order $1/m^2$ by another transformation

$$
\psi\rightarrow\exp(-ig\vec{\alpha}\cdot\vec{E}/4m^2)\psi\,,\text{etc..} \tag{11}
$$

The non-relativistic lagrangian to order $1/m$ finally is

$$
\mathcal{L}_{NRQCD}(x)=-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a+\bar{\psi}(i\gamma^0D_0-m)\psi+\bar{\psi}\frac{\vec{D}^2}{2m}\psi+\bar{\psi}\frac{g\vec{\Sigma}\cdot\vec{B}}{2m}\psi\,, \tag{12}
$$

which can be written in terms of uncoupled 2-component spinors by decomposing $\psi=(u,v^\dagger)$ $\bar{\psi}=(u^\dagger,-v)$ and using the Dirac representation of the γ-matrices.

To effect a short distance expansion we separate the gluonic fields in slowly varying background- [12](A) and quantum- (Q) fields having high fourier components:

$$
V^a_\nu=A^a_\nu+Q^a_\nu\,. \tag{13}
$$

The division (13) can however only be defined if the gauge is fixed. In deriving the spin-dependence of nonperturbative interactions for heavy quarkonia Curci et al. [13] found a particular gauge very convenient. We essentially follow their procedure here and impose the Coulomb background gauge condition

$$
D_iQ^i=0 \tag{14}
$$

for the quantum fields, where

$$
D_\mu Q_\nu=\partial_\mu Q_\nu+gA_\mu\times Q_\nu=\partial_\mu Q_\nu+gf^{abc}A_{\mu b}Q_{\nu c}\,. \tag{15}
$$

The background fields are defined in a modified Schwinger gauge [13]

$$
A^b_j=-\frac{1}{2}F^b_{ji}x^i\qquad;\qquad A^b_0=-F^b_{0i}x^i\,, \tag{16}
$$

valid to order x^2, where we assumed that the field-strengths corresponding to the background fields are constant (or have sufficiently low momenta, such that they can be regarded as essentially constant over the extent of the meson). This definition of the background fields in terms of (practically) constant field strengths also gives a definite meaning to the separation in equation (13). It also implies that the background fields in this gauge are (practically) time independent – a property which will become useful when a Hamiltonian is required.

We next expand the nonrelativistic Lagrangian only to second order in the quantum fields and subsequently integrate them out in favor of an effective (coulombic) interaction. Although ghost terms are necessary in the gauge defined by (14), they do not contribute to quadratic order in the quantum fields. Our truncation of the interaction terms for the quantum fields eliminates all radiative corrections. To obtain them one would have to go beyond this approximation and calculate perturbative corrections before eliminating hard gluons. Fortunately, the asymptotic freedom of QCD guarantees that they are only logarithmic at short distances and could be accounted for by a running coupling constant. These logarithmic corrections to the Coulomb potential do not seem to be dramatically important for describing heavy quarkonia spectra [18] and we will not include them in this study.

The matrix elements of the slowly varying background fields (A) will however be parameterized. Their amplitude is large and an expansion in the coupling in this case not applicable.

After elimination of the quantum gluonic fields by their equations of motion the effective lagrangian in terms of the background and heavy quark fields becomes [14]:

$$
\begin{aligned}
L_{eff}=&\int d^4x\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\bar{\psi}(i\gamma^0\partial_0+g\gamma^0A_0-m)\psi \\
&+\frac{1}{2m}\bar{\psi}(\vec{\nabla}^2-ig\vec{\nabla}\cdot\vec{A}-ig\vec{A}\cdot\vec{\nabla}-g^2\vec{A}^2)\psi+\frac{g}{2m}\bar{\psi}\vec{\Sigma}\cdot\vec{B}\psi \\
&+g^2\int d^4y\psi^\dagger(x)T^a\psi(x)\mathcal{D}^{ab}(x,y)\psi^\dagger(y)T^b\psi(y) \\
&-\frac{1}{2}\int d^4yJ^a_i(x)(\bar{\mathcal{D}}^{ab}_{ij}(x,y)-\mathcal{K}^{ab}_{ij}(x,y))J^b_j(y)\,,
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
J_k^a(x) = & -\frac{ig}{2m}[(\partial_k\bar{\psi})T^a\psi - \bar{\psi}T^a\partial_k\psi - i\epsilon_{ijk}\partial_i(\bar{\psi}T^a\Sigma_j\psi)] \\
& +\frac{g^2}{2m}\bar{\psi}(T^bT^a + T^aT^b)\psi A_k^a - \frac{g^2}{2m}f^{abc}\epsilon_{ijk}A_i^b\bar{\psi}T^c\Sigma_j\psi \\
& -2g^2f^{adc}F_{k0}^d\int d^4z\mathcal{D}^{cb}(x,z)\psi^\dagger(z)T^b\psi(z)\,.
\end{aligned} \tag{18}
$$

The propagator $\mathcal{D}$ relates the Q_0 field to its source

$$
Q_0^a(x) = \int d^4y\mathcal{D}^{ab}(x,y)j_b^0(y)\,, \tag{19}
$$

with

$$
j_0^a = g\psi^\dagger T^a\psi + 2gf^{abc}F_{bi0}Q_{ci}\,, \tag{20}
$$

and satisfies

$$
(D_jD_j)^{ab}\mathcal{D}^{bd}(x,y) = \delta^{ad}\delta^4(x-y)\,. \tag{21}
$$

Similarly the propagator $\tilde{\mathcal{D}}$ appears in the elimination of the spatial components Q_i and satisfies

$$
\int d^4zM_{ij}^{ab}(x,z)\tilde{\mathcal{D}}_{jk}^{bc}(z,y) = \delta^{ac}\delta^4(x-y)\delta_{ik}\,, \tag{22}
$$

with the differential operator M_{ij}^{ab} given by

$$
\begin{aligned}
M_{ij}^{ab}(x,z) = & \ \delta^4(x-z)[-(D_\mu D_\mu)^{ab}\delta_{ij} - 2gf^{abc}F_{ij}^c \\
& -\frac{g^2}{2m}\bar{\psi}(T^aT^b + T^bT^a)\psi\delta_{ij} + \frac{g^2}{2m}f^{abc}\epsilon_{ijk}\bar{\psi}T^c\Sigma^k\psi]\,, \qquad (23) \\
& +4g^2f^{dfa}f^{ceb}F_{i0}^f(x)\mathcal{D}^{dc}(x,z)F_{j0}^e(z)\,. \qquad (24)
\end{aligned}
$$

Finally, the propagator $\mathcal{K}$ enters when the lagrange multiplier of the gauge fixing condition for the quantum fields (14) is eliminated in turn. Its equation of motion is

$$
\int d^4zD_i^{ba}(x)\tilde{\mathcal{D}}_{ij}^{ad}(x,z)D_j^{dc}(z)\mathcal{K}^{cd}(z,y) = \delta^{bd}\delta(x-y)\,. \tag{25}
$$

These rather formidable integro-differential equations for the Green functions can formally be solved order by order in the background field A. Since we will only retain terms of the hamiltonian matrix proportional to the lowest dimensional condensate $< g^2FF >$, we only keep terms up to second order in the background fields in this gauge (16). As we will see shortly, linear terms in the background field have to be retained, although we will assume that $< F >= 0$, e.g. that global colour- and lorentz- invariance is not broken.

In order to obtain a tractable hamiltonian, further approximations are necessary. We will neglect all retardation effects in the effective interaction. This instantaneous approximation is correct to order $1/m$, because retardation effects are generally expected to be of order $1/m^2$. Since the interaction is instantaneous, the effective hamiltonian becomes time independent even in the presence of the background fields (which are (nearly) time independent in the gauge (16)). This greatly simplifies the interpretation of our results.

The effective hamiltonian, correct to order $1/m$, r^2 and α_s therefore is

$$
\begin{aligned}
H = \int d^3x\Big\{ & u^\dagger(\vec{x})mu(\vec{x}) + v(\vec{x})mv^\dagger(\vec{x}) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& -u^\dagger(\vec{x})T^AgE_i^Ax_iu(\vec{x}) - v(\vec{x})\bar{T}^AgE_i^Ax_iv^\dagger(\vec{x}) \\
& -u^\dagger(\vec{x})\frac{\vec{\nabla}_x^2}{2m}u(\vec{x}) - v(\vec{x})\frac{\vec{\nabla}_x^2}{2m}v^\dagger(\vec{x}) \\
& +\alpha\int d^3yu^\dagger(\vec{x})T^Au(\vec{x})\frac{1}{r}v(\vec{y})\bar{T}^Av^\dagger(\vec{y}) \\
& +\frac{\alpha}{2}\epsilon_{ijk}gB_k^Cf^{ACB}\int d^3yu^\dagger(\vec{x})T^Au(\vec{x})\frac{y_jx_i}{r}v(\vec{y})\bar{T}^Av^\dagger(\vec{y}) \\
& -\frac{\pi^2\phi^2\alpha}{64}\int d^3yu^\dagger(\vec{x})T^Au(\vec{x})\frac{(\vec{x}\times\vec{y})^2}{r}v(\vec{y})\bar{T}^Av^\dagger(\vec{y}) \\
& +\frac{1}{m}\Big[\frac{\pi^2\phi^2}{128}u^\dagger(\vec{x})u(\vec{x})\vec{x}^2 + \frac{\pi^2\phi^2}{128}v(\vec{x})v^\dagger(\vec{x})\vec{x}^2 \\
& -\frac{i}{2}u^\dagger(\vec{x})T^A\epsilon_{ijk}gB_k^Ax_j\partial_{x_i}u(\vec{x}) - \frac{i}{2}v(\vec{x})\bar{T}^A\epsilon_{ijk}gB_k^Ax_j\partial_{x_i}v^\dagger(\vec{x}) \\
& -\frac{1}{2}u^\dagger(\vec{x})\sigma_iT^AgB_i^Au(\vec{x}) - \frac{1}{2}v(\vec{x})\sigma_i\bar{T}^AgB_i^Av^\dagger(\vec{x}) \\
& +\frac{i\alpha}{8}f^{ADC}gE_j^D\Big[(\partial_{x_i}u^\dagger(\vec{x}))T^Au(\vec{x}) - u^\dagger(\vec{x})T^A(\partial_{x_i}u(\vec{x})) \\
& \quad -i\epsilon_{ilk}\partial_{x_l}(u^\dagger(\vec{x})T^A\sigma^ku(\vec{x})\Big]\cdot\int d^3y(-3\delta_{ij}r + \frac{r_ir_j}{r})v(\vec{y})\bar{T}^Cv^\dagger(\vec{y}) \\
& -\frac{i\alpha}{8}f^{ADC}gE_j^D\Big[-(\partial_{x_i}v(\vec{x}))\bar{T}^Av^\dagger(\vec{x}) + v(\vec{x})\bar{T}^A(\partial_{x_i}v^\dagger(\vec{x}))
\end{aligned} \tag{26}
$$

$$+i\epsilon_{ilk}\partial_{x_l}(v(\vec{x})\bar{T}^A\sigma^k v^\dagger(\vec{x})\Big] \cdot \int d^3y(-3\delta_{ij}r + \frac{r_ir_j}{r})u^\dagger(\vec{y})T^C u(\vec{y})\Big]\Big\}.$$

Here $u(\vec{x})$ and $v(\vec{x})$ denote the anihilation operators for a quark and antiquark of mass m respectively whose spin and color indices have been suppressed, $\vec{r} = \vec{x} - \vec{y}$ and $T^A, \bar{T}^A$ are the hermitian generators of the SU(3) color Lie-algebra in the 3 and $\bar{3}$ representation.

This is essentially the Hamiltonian to order $1/m$ derived previously by Curci et al. [13] in position space, except for a term which can be regarded as a long range correction of the Coulomb potential

$$\frac{3\pi^2\phi^2\alpha}{64}\int d^3x d^3y u^\dagger(\vec{x})T^A u(\vec{x})\, r^3\, v(\vec{y})\bar{T}^A v^\dagger(\vec{y})\,. \tag{27}$$

We have disregarded this term because it is of order r^3 and our gauge fixing condition (16) is not valid at this order.

The terms of the Hamiltonian (26) linear in the chromo -electric and -magnetic fields as well as those proportional to $\vec{x}^2$ are obviously also not translationally invariant and therefore apparently depend on the chosen gauge fixing point in (16). This gauge dependence of (26) is however absent [14] in its matrix elements in the basis (1),(3),(4) and (5). It cancels against terms which appear when derivatives act on the color transport matrix of the states. This cancelation of course only works to a certain order in the short distance expansion and only works if the Hamiltonian and the basis in which it is diagonalized are defined consistently. This is not a big surprise in any gauge theory, but a gauge invariant scheme to include nonperturbative aspects of the ground state in a hamiltonian formulation without losing gauge invariance was only first proposed in [9], but not used to its full extent there. From the above we see that a short distance expansion in the construction of the basis as well as the hamiltonian to order r^2 can be performed and the effects from a nontrivial expectation value ϕ^2 consistently included.

The straightforward but lengthy computation of the matrix elements of the Hamiltonian (26) in the basis for pseudoscalar mesons (7) to obtain the coupled differential equations (30) for the wavefunctions [14] will not be exhibited here. We do however also have to account for matrix elements of the purely gluonic part of the Hamiltonian

$$H_G = \int d^3x(E^2 + B^2)/2 \tag{28}$$

in the basis states (1),(3),(4) and (5).

From rotational symmetry we conclude that only diagonal matrix elements can be non-vanishing. Since we neglect in our approximation higher order expectation values, matrix elements in "octet"-states (3),(4),(5) are naturally assumed to vanish.

From (6) we might infer that the matrix elements of H_G between "singlet" states vanish as well.

Phenomenologically we do however need an energy splitting between the "singlet"- and "octet"- states. Leutwyler [19] attributes it to an effective mass of the low frequency gluonic modes. The trace anomaly [20]

$$\epsilon = -\frac{11}{32}\phi^2, \tag{29}$$

relates the energy density of the nonperturbative gluonic ground state, ϵ, to ϕ^2 and allows for a more quantitative estimate of the splitting between "singlet"- and "octet"- states: since we neglect higher dimensional operators, the energy of the singlet states is reduced by the vacuum energy density relative to the octet states over the volume of the meson.

From a typical radius of 0.62fm for heavy quarkonia and using the value $\phi^2 = (360\text{MeV})^4$ in (29), this argument would give an estimated contribution of about -780MeV in the "singlet"-channel, which is quite close to the -756MeV we find best fits the phenomenology. This is only a rough estimate and might depend somewhat on the actual size of the meson. The complementary point of view taken by Leutwyler [19] would increase the energy of "octet"-matrix elements by an effective mass m_G for the low frequency modes. Since the overall normalization of the energy is ambiguous anyhow, our best fits would imply $m_G \sim$ 756MeV. This value is not in conflict with the fact that the lightest glueball has at least two such excitations and a mass of $\sim$ 1700MeV [21], although a somewhat smaller glueball mass would seem more natural.

Including this energy shift between "singlet" and "octet" components by a constant C, the coupled set of differential equations for flavor singlet pseudoscalar mesons becomes

$$\Big[2m - E + C - \frac{1}{m}\frac{\partial^2}{\partial r^2} - \frac{4}{3}\frac{\alpha_s}{r} + \frac{\pi^2\phi^2 r^2}{36m}\Big]S(r) =$$
$$+\Big(-\frac{\pi\phi r}{3\sqrt{2}} + \frac{g^2\phi}{4\sqrt{2}m} + \frac{g^2\phi r}{8\sqrt{2}m}\frac{\partial}{\partial r}\Big)\ E_1(r) - \frac{5g^2\phi}{64m}E_2(r)$$
$$\Big[2m - E - \frac{1}{m}\Big(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\Big) + \frac{1}{6}\frac{\alpha_s}{r}\Big]E_1(r) = \Big(\frac{\pi\phi r}{3\sqrt{2}} + \frac{g^2\phi}{8\sqrt{2}m} - \frac{g^2\phi r}{8\sqrt{2}m}\frac{\partial}{\partial r}\Big)S(r)$$
$$\Big[2m - E - \frac{1}{m}\Big(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\Big) + \frac{1}{6}\frac{\alpha_s}{r}\Big]E_2(r) = \frac{5g^2\phi}{64m}S(r), \tag{30}$$

where E is the mass eigenvalue of the quarkonium and m is the mass of the constituent quarks.The functions $S(r)$, $E_1(r)$ and $E_2(r)$ are related to the wave function components in the expansion (7) via

$\psi_S(r) = \frac{1}{r}S(r)$; $\psi_{E_1}(r) = \frac{1}{r^2}E_1(r)$; $\psi_{E_2}(r) = \frac{1}{r^2}E_2(r)$.

Note that there is no coupling to the pseudoscalar B-states, because it is proportional to the total spin in (26), which vanishes for pseudoscalar mesons. As indicated above, these equations only depend on the relative coordinate r and all reference to a gauge fixing point-present in (26)-has disappeared in the evaluation of the effective Hamiltonian in the gauge invariant basis.

Apart from the constituent quark mass, the only parameters that enter equations (30) are the (reasonably well known) condensate ϕ^2 and the constant C, whose value we have tried to estimate above, as well as the strong coupling constant α_s.

IV. "EFFECTIVE POTENTIAL"

Before solving the coupled set of equations (30) by numerical methods, it is instructive to consider the limit of infinitely heavy quarkonia. In this case, all terms proportional to $1/m$ (including the kinetic energy) in (30) can be dropped, and the resulting equations give the binding energy $V = E - 2m$ for states where the quark and anti-quark are localized a distance r apart (i.e. for wave-functions $S(r), E1(r)$ and $E2(r)$ all proportional to $\delta(r - r_0)$. It is natural to compare this mass-independent binding energy to phenomenological potentials and those extracted from the expectation values of Wilson loops in numerical lattice simulations.

In this limit the $E2$-component decouples and one has to solve the algebraic equations

$$\Big(-V + C - \frac{4}{3}\frac{\alpha_s}{r}\Big)S(r) = -\frac{\pi\phi r}{3\sqrt{2}}E_1(r)$$
$$\Big(-V + \frac{1}{6}\frac{\alpha_s}{r}\Big)E_1(r) = \frac{\pi\phi r}{3\sqrt{2}}S(r)\,. \tag{31}$$

The eigenfunctions are obviously localized and the eigenvalue or effective potential, $V(r)$, given by

$$V(r) = -\frac{1}{2}\Big(\frac{7}{6}\frac{\alpha_s}{r} + \sqrt{\frac{9}{4}\frac{\alpha_s^2}{r^2} - \frac{9}{3}\frac{\alpha_s C}{r} - \frac{2\pi^2\phi^2 r^2}{9} + C^2}\Big) \tag{32}$$

Figure 1(a) shows this effective potential and its Coulomb part for parameters $\phi^2 = (360MeV)^4$, $C = -756$ and $\alpha_s = 0.39$, which we found appropriate for charmonium.

Although this potential is certainly no longer valid for $r > 0.9$fm, where the root in (32) becomes purely imaginary it does show a nearly linear behaviour for intermediate distances $0.4\text{fm} < r < 0.8\text{fm}$, with a correspondingly constant force of ~ 840MeV/fm, which compares favorably with a string tension of about $800 - 1000$MeV/fm extracted from recent lattice calculations of the potential [22]. In Fig. 1(b) we compare our effective potential (32) to that extracted from lattice data [22] and to the phenomenological potential used by ref [26]. It is perhaps also of some theoretical interest, that this potential closely resembles that derived from an "instantaneous" Gribov type gluon exchange [16]

$$V_{Gribov}(r) := -\frac{4}{3}g^2\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{k^4+\kappa^4}e^{i\vec{k}\vec{r}} = -\frac{4}{3}\frac{\alpha_s}{r}e^{\kappa r/\sqrt{2}}cos(\kappa r/\sqrt{2})\,, \tag{33}$$

in the limited range of interest $r < .9$fm with $\kappa = 500$MeV. For comparison we also show the potential (33) in Fig 1(b).

It is encouraging that our rather crude approximations to the vacuum structure seem to qualitatively reproduce the potential for very heavy quarks at small distances. The analytical results of this section justify the numerical calculation of "octet"-components in heavy quarkonia which we now present.

V. NUMERICAL RESULTS

Going beyond the static approximation by including the kinetic energy of the quarks but still neglecting coupling terms of order $1/m$ in (30), the quarkonium spectrum becomes discrete and the following coupled set of differential equations must be solved numerically

$$\begin{aligned}\left[2m - E + C - \frac{1}{m}\frac{\partial^2}{\partial r^2} - \frac{4}{3}\frac{\alpha_s}{r}\right]S(r) &= -\frac{\pi\phi r}{3\sqrt{2}}E_1(r)\\ \left[2m - E - \frac{1}{m}\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right) + \frac{1}{6}\frac{\alpha_s}{r}\right]E_1(r) &= \frac{\pi\phi r}{3\sqrt{2}}S(r)\,.\end{aligned} \tag{34}$$

This is essentially Leutwyler's [7] approximation to the problem, who obtained the perturbation of the Coulomb spectrum in 2^{nd}-order of ϕ and found that it is exceedingly large for canonical values of the condensate. Note that the Coulomb force is repulsive in the "octet" channel and 1/8-th in strength compared to the "singlet" channel (this is just the ratio of $T_1^a\bar{T}_2^a$ in the two representations) and the E_2-state still decouples in this approximation. In this approximation, vector- and pseudoscalar- quarkonia are furthermore still degenerate. This is expected, since the spin splitting is of order $1/m$. It however is another nontrivial consistency check of our method, because the basis states for vector mesons are of course quite different. Nevertheless equivalent equations to (34) result, if $1/m$ potential terms are neglected.

The relative minus sign of the two off-diagonal coupling terms in (34) shows that this Hamiltonian is not hermitian and that its eigenvalues will generally not be real. This effect is completely missed if the coupling is treated as a perturbation. To any finite order in perturbation theory the correction to the Coulomb spectrum is real. Perturbation theory does however show that a few (low lying) eigenvalues of (34) are real for a sufficiently small ϕ. For the canonical value $\phi^2 = (300 - 380\mathrm{MeV})^4$ we find numerically $2-4$ stable bound quarkonia in the pseudoscalar channel, depending on the heavy quark mass m.

But even for these low states, the deviation of the exact correction to the coulomb eigenvalue from the 2^{nd}-order estimate is large for the charmonium and bottonium systems as shown in table I. We conclude with Leutwyler [7] that perturbation theory is not applicable

for canonical values of ϕ, but that the effective non-hermitian Hamiltonian equations (34) do yield reasonable corrections in an exact solution.

The value $C = -756\mathrm{MeV}$ used by us was ajusted to reproduce the correct splitting between η_c and η_c' [23] when $1/m$ terms are included (see below). The corrections to the coulomb spectrum are however not extremely sensitive to the value of C once it is large enough. Using $C = -1400\mathrm{MeV}$ instead would only reduce the nonperturbative contributions for the ground and first excited bottonium-states in table I to 6 and 75MeV respectively.

The fact that a perturbative evaluation in ϕ is not appropriate in (34) can also not be circumvented by including loop corrections (higher orders in α_s than we have treated so far) to the perturbative coulomb potential. Titard and Ynduráin [25] recently proposed to modify the perturbative part of the interaction in the following manner

$$\frac{\alpha_s}{r} \to \frac{\alpha_s(\mu^2)\left[1 + (a_1 + \gamma_E\beta_0/2)\alpha_s(\mu^2)/\pi\right]}{r} + \frac{\beta_0\alpha_s^2(\mu^2)}{2\pi}\frac{\log r\mu}{r}\,, \tag{35}$$

where the appropriate constants for the SU(3) color group with 4 light quark flavors are $\beta_0 = 8.33$ and $a_1 = 1.47$. The first term of (35) which contains a piece of one-loop radiative corrections was taken by Titard and Ynduráin as an effective Coulomb potential and solved exactly. The effective coupling constant is defined as

$$\tilde{\alpha}_s(\mu^2) = \left[1 + (a_1 + \gamma_E\beta_0/2)\frac{\alpha_s(\mu^2)}{\pi}\right]\alpha_s(\mu^2)\,. \tag{36}$$

The second term in (35) was treated to first order in perturbation theory. A new scale parameter μ was introduced which depends on the quarkonium system under consideration. Taking the effective Coulomb potential we obtain the deviation of the eigenvalues from 2^{nd}-order perturbation theory in ϕ shown in table II for two sets of parameters in the bottonium system (still neglecting $1/m$ corrections).

The deviation of the exact correction from the perturbative estimate is reduced somewhat, especially in the second case, but still far from negligible. In assessing the quality of a perturbative evaluation in this case, one should also keep in mind that the scale parameter μ of Ref. [25] was chosen in such a way that the 2^{nd} order correction in ϕ is precisely canceled by the correction terms to the effective Coulomb potential in (35). For the first set of parameters this cancellation occurs for the ground state energy. The second set was chosen so that the splitting between the first excited state and the ground state is not affected to 2nd order. This is obviously a quite arbitrary procedure which requires an additional parameter and furthermore does not cure the problem that perturbation theory simply does not apply (as table II clearly indicates).

We therefore will not include these modifications to the Coulomb force in our discussion of the numerical solution to the full set of coupled equations (30). The inclusion of $1/m$ potential terms lifts the degeneracy of pseudoscalar- and vector- quarkonia and we restrict our discussion here to the pseudoscalar case.

The E_2-states now couple in, but generally have small (negative) norms, because the coupling is of order $1/m$. We nevertheless solved the full set of coupled equations numerically, although a perturbative treatment of the E_2-state admixture would probably have been sufficient. The eigenvalues we obtained are summarized in table III and compared to those obtained by Eichten et al. [26] with the phenomenological funnel potential. Note that we only found 3 or 4 stable solutions in the charmonium and bottonium systems respectively. We used the same values for the quark masses and the coupling constant as Eichten et al. [26].The constant C was chosen to reproduce the experimental splitting (not confirmed [23]) between η_c and η_c'. It was not adjusted in the bottonium system. The gluon condensate value $\phi^2 = (360\text{MeV})^4$ we used is within QCD-sumrule estimates [24] for this nonperturbative quantity. All eigenvalues were finally shifted by $E_0 = 98\text{MeV}$ to give the correct η_c mass. (This small shift in the overall energy normalization can be eliminated by a slight change of $\sim$ 50MeV in the quark masses used by Eichten et al. [26] and a corresponding small adjustment of the other parameters. To have a more direct comparison of the wavefunctions and spectra, we refrained from making these adjustments here.)

In Figs. 2 and 3 we show the eigenfunctions for the various components of our quarkonium states (7) as well as the corresponding eigenfunction of Eichten et al. [26]. The singlet component of our ground state wave functions are very similar to those of the funnel po-

tential. At small radii this is true also for the excited states, since the coupling to "octet" components is proportional to r in (30). The "octet"-components increase with increasing excitation energy of the quarkonium and lead to the appearance of extra nodes in the higher lying "singlet" wave functions at large radii (since this is a coupled channel problem, the extra nodes do not mean that we missed some bound states). As noted earlier, the E-components of the meson state contribute negatively to its norm. All the stable quarkonia states we found are however positive norm states. We could not obtain any stable state where the octet components are dominant.

Let us speculate at this point on the fact that only very few stable quarkonium states were found. This is of course quite in line with the experimental observation that only a few heavy quarkonia are stable against decay by strong interactions. For reasons which we had not anticipated, this basic property seems already to be incorporated in the nonhermitian coupling to E-components.

The strength of this coupling in our model is however determined by the gluon condensate ϕ^2, which is not expected to vanish even in the purely gluonic theory. From table III we see that the instability sets in at an excitation energy of between $1-1.3\text{GeV}$ in this model. Since we cannot account for the decay into light $q\bar{q}$-mesons with a parameter which is essentially independent of the number of (light) flavours, we speculate that the nonhermitian coupling proportional to ϕ effectively accounts for the possible decay channel

$$\text{Quarkonium}^* \longrightarrow \text{Quarkonium} + \text{Gluonium}\,. \tag{37}$$

in this model. From the fact that we do not seem to find any stable quarkonium states more than 1.3GeV above the ground state, we would estimate this to be the threshold for the production of the lightest gluonium. This estimate of the lightest gluonium mass $m_G \gtrsim 1.3\text{GeV}$, is in almost perfect agreement with our previous interpretation of the energy shift $-C \sim m_G/2$.

Since the production of light $q\bar{q}$-pairs has a much lower threshold, it is this process which limits the stability of physical quarkonia. We therefore expect this model, which does not

(not even effectively) incorporate this decay channel, to still predict more stable states than are experimentally observed.

VI. CONCLUSION

We developed a hamiltonian formalism, which enabled us to estimate the effect of a nonperturbative gluonic ground state on quarkonia in a systematic short distance, weak coupling and $1/m$ expansion of the effective hamiltonian. The gauge invariant basis [9] was extended to include color octet quark-antiquark pairs coupled to vacuum fluctuations. Hamiltonian matrix elements in this basis are gauge invariant to the order in the short distance expansion we considered.

After separating hard and soft gluons in the gauge (13), we obtained the effective hamiltonian neglecting radiative corrections to the coulomb interaction from hard gluons. We thus neglected the logarithmic corrections to the effective coupling strength at very short distances. The correct behaviour of the potential for $r < 0.2fm$ can in principle be included by "hand" in a modification of the coulomb part of the interaction [17]. Although the quarkonium spectrum is not very sensitive to this correction at small distances, it could become important for the evaluation of decay widths, which depend on the wave function at the origin.

In the limit of very heavy quark masses, where all terms of order $1/m$ can be neglected, an energy independent effective potential was obtained, which shows an approximately linear behaviour at intermediate distances $0.4fm < r < 0.8fm$ with an effective string tension of $\sim 840MeV$. In our approach this behaviour of the potential arises due to the nonperturbative structure of the gluonic vacuum parametrized by its gluon condensate and was not assumed from the outset as in most phenomenological quark models. This potential compares favourably with recent lattice results [22], the discrepancies at very small $r < 0.2fm$ being due to our neglect of radiative corrections. Surprisingly, our potential is almost exactly reproduced by an instantaneous interaction derived from the effective gluon exchange

proposed by Gribov [16]. Our short distance expansion for the effective potential however is only valid for $r < 0.9fm$, beyond which the potential aquires an imaginary part. An extension of the model to larger distances would require a more detailed knowledge of the vacuum structure in the form of higher dimensional condensates, or some other effective parametrization of this structure. The approach in this case would become increasingly phenomenological and also more complicated in this case. Its predictive power is therefore probably limited to heavy quarkonia, where a detailed knowledge of the potential for very large radii does not seem necessary.

We showed that the numerical solution of the coupled channel problem for vector- and pseudoscalar- quarkonia (they are degenerate to order $1/m$) is feasible and an exact diagonalization of the hamiltonian in the extended basis therefore possible. The resulting exact spectrum does not show the far too rapid increase of the eigenvalues with the principal quantum number of the perturbative approach to the vacuum effects [6] [7]. In addition to the usual "singlet" wave-functions describing the quark and anti-quark of the quarkonium when they are coupled in a colour singlet, we also obtain the "octet" components of quarkonium states describing the quarks in the octet configuration when an additional (soft) gluon is around. Since hadronic decays mainly proceed from this "octet" configuration with the creation of an additional octett $q\bar{q}$-pair from a hard gluon, this approach opens the possibility of estimating nonperturbative contributions to hadronic decays.

We compare our results for the spectrum and wavefunctions with those of the Cornell potential [26] for pseudoscalar quarkonia in order $1/m$. With the standard value for the gluon condensate and quark masses and coupling constant used by the Cornell group we obtain the correct splitting between η_c and η_c' and make predictions for the η_b's. Our main concern was however a better theoretical understanding and justification of the phenomenological ingredients common to most nonrelativistic models for heavy quarkonia and we refrained from adjusting the few parameters of this approach to optimally reproduce the experimental data. A better description of the potential at short distances with the inclusion of radiative corrections to the coulomb force and the consideration of hadronic decay channels is clearly desireable before a detailed comparison with phenomenology is attempted.

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FIGURES

FIG. 1. (a) Effective potential (solid curve) from (32) and Coulomb potential (dotted curve) with $\alpha_s = 0.39$, $C = -756$ MeV and $\phi^2 = (360 \text{ MeV})^4$.

(b) Effective potential (solid curve) with the same parameters as in (a). The potential extracted from lattice data [22] with $\sqrt{\sigma} = 365$ MeV (dot-dashed curve) and $\sqrt{\sigma} = 505$ MeV (dashed curve). Gribov potential [16] with $\kappa = 500$ MeV (crosses) and Cornell potential [26] (dotted curve) with $\alpha_s = 0.39$ and $a = 2.34 \text{ GeV}^{-1}$.

FIG. 2. The wavefunctions of the ground, 1^{st}- and 2^{nd}- excited pseudoscalar states of charmonium are shown in figures (a), (b) and (c) respectively. The dashed curve is the wavefunction for the funnel potential [26] for comparison. The solid curve is the singlet component $S(r)$ of the quarkonium state in our calculation. The dot-dashed and dotted curves are the "octet" components $E_1(r)$ and $E_2(r)$ of equation (30) respectively. The solution was obtained with the parameters $m_c = 1840$ MeV, $\alpha_s = 0.39$, $\phi^2 = (360 \text{ MeV})^4$ and $C = -756$ MeV.

FIG. 3. The wavefunctions of the ground, 1^{st}-, 2^{nd}- and 3^{nd} excited pseudoscalar states of bottonium are shown in figures (a), (b), (c) and (d), respectively. The dashed curve is the wavefunction for the funnel potential [26] for comparison. The solid curve is the singlet component $S(r)$ of the quarkonium state in our calculation. The dot-dashed and dotted curves are the "octet" components $E_1(r)$ and $E_2(r)$ of equation (30) respectively. The solution was obtained with the parameters $m_b = 5170$ MeV, $\alpha_s = 0.39$, $\phi^2 = (360 \text{ MeV})^4$ and $C = -756$ MeV.

TABLES

TABLE I. The Coulomb energy in MeV is presented in the first column for the ground state and first excitation of $c\bar{c}$ and $b\bar{b}$ (pseudoscalar or vector). Second and third columns contain the nonperturbative contributions in MeV calculated respectively within perturbation theory and with our method. We used $m_c = 1840$ MeV, $m_b = 5170$ MeV, $\alpha_s = 0.39$, $\phi^2 = 0.012$ GeV^4 and $C = -756$ MeV.

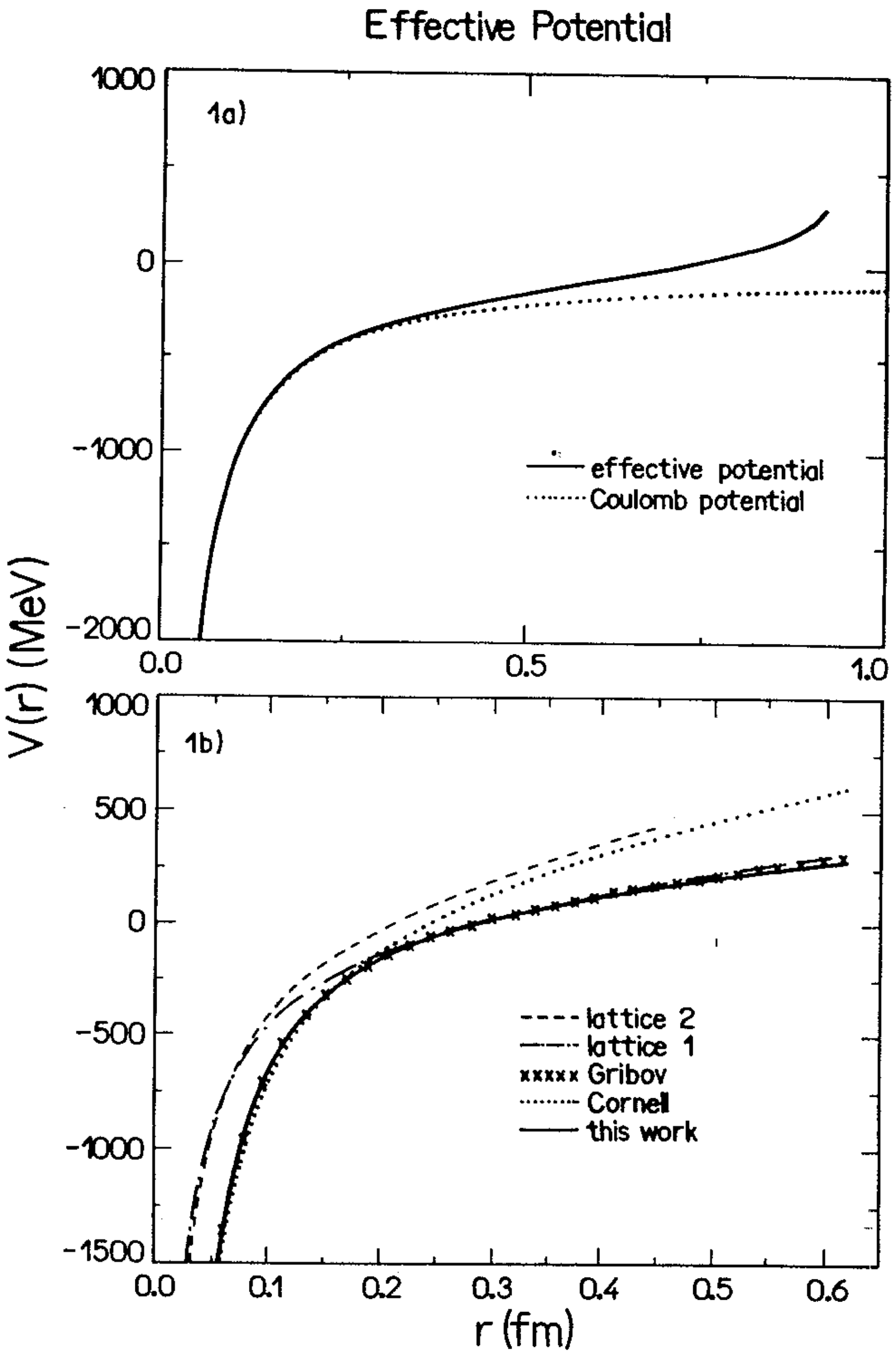
	Coulomb	Pert. theory	Non-pert.
η_c, J/ψ	-124.4	311	66
η_c', ψ'	-31.1	21525	424
η_b, Υ	-349.5	14	8
η_b', Υ'	-87.4	970	111

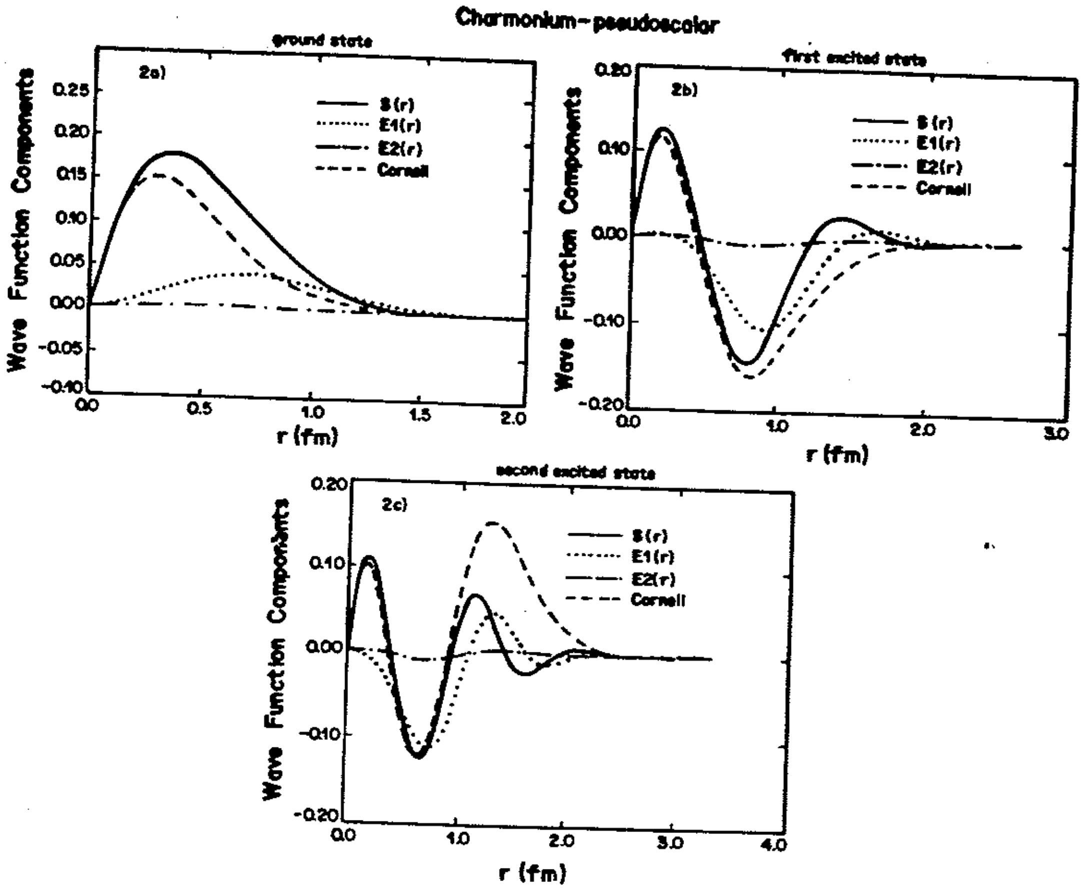
TABLE II. The Coulomb energy in MeV is presented in the first column for the ground state and first excitation of $b\bar{b}$ (pseudoscalar or vector) at two different scales μ. Second and third columns contain the nonperturbative contributions in MeV calculated respectively within perturbation theory and with our method. We used $C = -756$ MeV, $\phi^2 = 0.042$ GeV^4 and for $\mu = 1.44$ GeV: $m_b = 4866$ MeV, $\tilde{\alpha}_s = 0.38$. For $\mu = 0.99$ GeV: $m_b = 5010$ MeV, $\tilde{\alpha}_s = 0.54$.

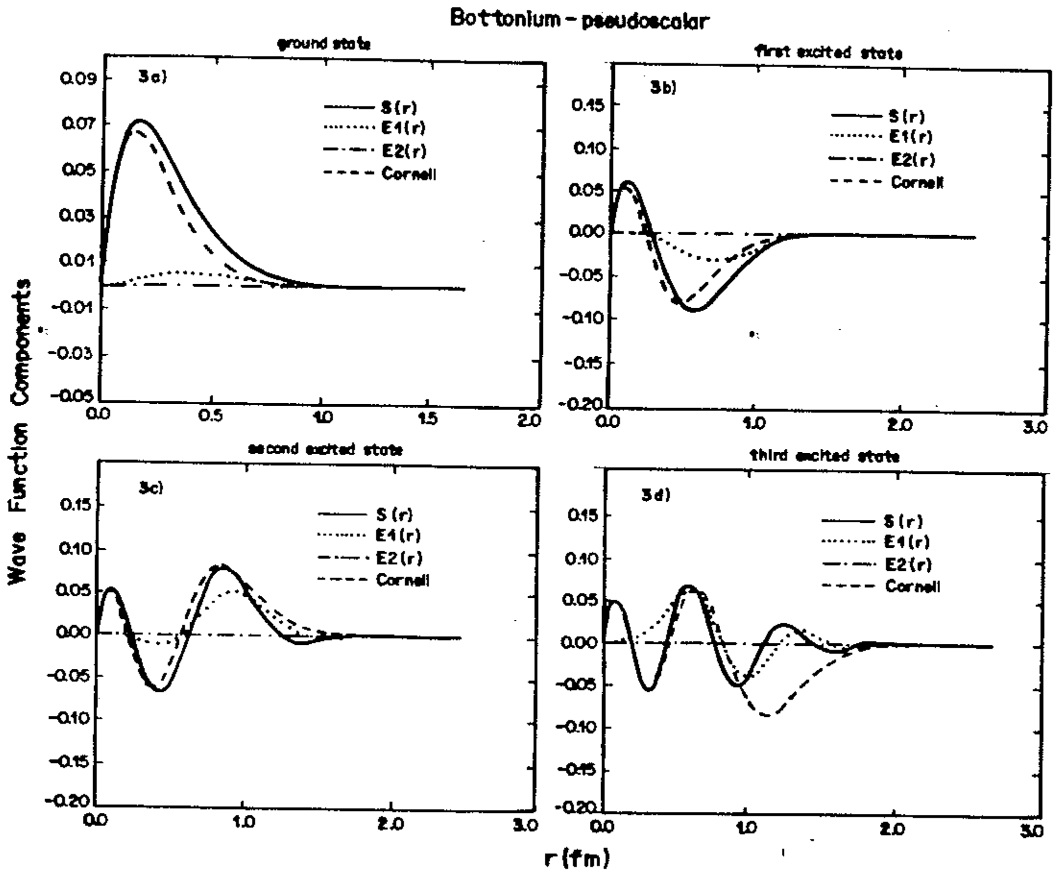
	Coulomb	Pert. theory	Non-pert.
η_b, Υ ($\mu = 1.44$ GeV)	-312	25	11
η_b', Υ' ($\mu = 1.44$ GeV)	-78	1762	129
η_b, Υ ($\mu = 0.99$ GeV)	-649	6	3
η_b', Υ' ($\mu = 0.99$ GeV)	-162	396	72

TABLE III. Masses in MeV of pseudoscalar quarkonia with the funnel potential and with our effective nonperturbative hamiltonian. Parameters used: $m_c = 1840$ MeV, $m_b = 5170$ MeV, $\alpha_s = 0.39$, $\phi^2 = (360\text{ MeV})^4$, $C = -756$ MeV.

	funnel	nonpert.
η_c	2980	2980
η_c'	3571	3594
η_c''	3994	3993
η_b	9213	9344
η_b'	9805	9739
η_b''	10150	10084
η_b'''	10427	10610


Effective Potential
1a)
effective potential
Coulomb potential
V(r) (MeV)
1000
0
-1000
-2000
0.0
0.5
1.0
1b)
500
-500
-1500
lattice 2
lattice 1
xxxxx Gribov
Cornell
this work
0.1
0.2
0.3
0.4
0.6
r (fm)


Charmonium-pseudoscalar
ground state
2a)
S(r)
E1(r)
E2(r)
Cornell
Wave Function Components
r (fm)
first excited state
2b)
second excited state
2c)



Bottonium - pseudoscalar
ground state
first excited state
second excited state
third excited state
3a)
3b)
3c)
3d)
S(r)
E1(r)
E2(r)
Cornell
Wave Function Components
r(fm)