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**CONTINUOUS PARTICLE EMISSION: A PROBE  
OF THERMALIZED MATTER EVOLUTION?**

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# CONTINUOUS PARTICLE EMISSION: A PROBE OF THERMALIZED MATTER EVOLUTION?

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## ABSTRACT

Continuous particle emission during the whole expansion of thermalized matter is studied and a new formula for the observed transverse mass spectrum is derived. In some limit, the usual emission at freeze out scenario (Cooper-Frye formula) may be recovered. In a simplified description of expansion, we show that continuous particle emission can lead to a sizable curvature in the pion transverse mass spectrum and parallel slopes for the various particles. These results are compared to experimental data.

## 1. INTRODUCTION

At the present moment, the theoretical description of relativistic heavy ion collisions is still quite controversial. On one extreme, one may try to describe heavy ion collisions as a superposition of nucleon-nucleon collisions. On the other extreme, one may apply a statistical description, assuming that complete thermalization has been attained. Technically, the second description has an advantage over the first one: such poorly understood details as nucleon structure or mechanisms for nucleon-nucleon or nucleus-nucleus collisions required in the first approach can be forgotten thanks to the thermalization assumption. If this assumption is true, then one can rely on more sound methods of statistical physics.

The basic origin for this ambiguity is that we do not know the thermalization time, or time needed for the particles created early in a collision to evolve from the initial state to a thermal equilibrium state. A reliable estimate of the thermalization time can only be done by a microscopic dynamical description of the reaction, which requires knowledge of interaction cross sections, initial density reached, etc, quantities that are not well established yet.

It is however thought that [1], due to the higher multiplicities and longer dense matter lifetimes available, states of thermal equilibrium should be reached (if they have not been reached yet) at the Relativistic Heavy Ion Collider (in Brookhaven) and Large Hadron Collider (at CERN) that will be in use in the future. So it is important to develop a complete hydrodynamical description of relativistic heavy ion collisions.

There is indeed a lot of activity in this direction. Full three-dimensional hydrodynamical codes (i.e. codes where the relativistic laws of conservation of momentum-energy and baryon number are numerically solved point by point) are becoming available [2-5] and transverse momentum and rapidity distributions are predicted. These codes took over more simplified solutions [6-9]. Finer details are now being studied. The effect of the freeze out criteria and initial conditions are tested using such codes or easier to handle semi-numerical approaches [10-13]. The impact of resonance decays (in particular in connection with the observed low- $p_t$  pion enhancement) is being evaluated both in static thermal models and hydrodynamical models [14-17]. In this work [18,19], we concentrate on the description of the particle emission process.

## 2. MOMENTUM DISTRIBUTION FOR CONTINUOUS PARTICLE EMISSION

In recent microscopic Monte-Carlo simulations of heavy ion collisions, the evaluation of the observables is similar to what happens experimentally. Namely, there exists a last-interaction location and time for each individual particle. This defines a scatter-plot in four dimensional space. In hydrodynamical models however, one usually introduces the notion of a sharp three-dimensional freeze out surface. Before crossing it, particles have a hydrodynamical behavior, and after, they free-stream toward the detectors, keeping memory of the conditions (flow, temperature) of where and when they crossed the three dimensional surface. Let us outline the steps leading to the commonly used formula of Cooper and Frye [20] for the produced hadron spectra in this case.

The model assumes that all hadrons are kept in thermal equilibrium until some decoupling criterium has become satisfied (e.g. a certain freeze out temperature has been reached). The space-time points of the fluid where the criterium becomes satisfied, define a three-dimensional surface called decoupling surface. Let  $d\sigma_\mu$  be the surface element of such a surface  $\sigma$  and  $f$  be the distribution function of the corresponding type of particles. The particle current density at some point  $x$  is

$$\int d^3p p^\mu / Ef(x, p), \quad (1)$$

so the particle current crossing the surface is

$$\int_\sigma d\sigma_\mu \int d^3p p^\mu / Ef(x, p), \quad (2)$$

and the invariant distribution in momentum space is

$$Ed^3N/dp^3 = \int_\sigma d\sigma_\mu p^\mu f(x, p). \quad (3)$$

We call attention to the fact that, when  $\sigma$  is time-like, equation (3) may in principle have negative contributions ( $p^\mu d\sigma_\mu < 0$ ), corresponding to particles going from the freeze-out surface into the fluid. One must not count these particles as being emitted. However this should not be a common case, as free particles crossing the freeze out surface outward have no way of changing their momentum to travel inward and re-enter into the fluid.

Now let us try to make a description of particle emission that is closer to what happens experimentally. At each space-time point  $x$ , a given particle has some chance to escape the dense matter region without collision. This is due to the finite dimensions and lifetime of the thermalized matter. So we consider that the fluid has two components, a free part plus an interacting part and write

$$f(x, p) = f_{free}(x, p) + f_{int}(x, p). \quad (4)$$

$f_{free}$  counts all the particles that last scattered earlier at some point and are at time  $x^0$  in  $\vec{x}$ .  $f_{int}$  describes all the particles that are still interacting (i.e. that will suffer collisions at time  $> x^0$  and change momentum). The free particle current across some three dimensional surface at fixed  $\chi$  (for example temperature or proper time) is

$$\int_\sigma d\sigma_\mu \int d^3p p^\mu / Ef_{free}(x, p)|_\chi, \quad (5)$$

so the variation in the total number between two infinitesimally close surfaces is

$$\begin{aligned} \delta \int_\sigma d\sigma_\mu \int d^3p p^\mu / Ef_{free}(x, p) &= \int_{\sigma(x)}^{\sigma(x+\delta x)} dS_\mu \int d^3p \partial_\chi [p^\mu / Ef_{free}(x, p) \sqrt{-g}] d\chi \\ &= \int_\sigma^{\sigma+\delta\sigma} d^4x \int d^3p D_\mu [p^\mu / Ef_{free}(x, p)], \end{aligned}$$

and the new invariant momentum distribution is

$$Ed^3N/dp^3 = \int d^4x D_\mu [p^\mu f_{free}(x, p)]. \quad (6)$$

Here,  $d^4x$  is the invariant volume element i.e. it contains the relevant Jacobian. Similarly  $d\sigma_\mu = \sqrt{-g} dS_\mu$ . (Note that in the last equation, the integral is over the four dimensional space-time). The physical meaning of this expression is simple: the number of detected particles with momentum in some range is given by summing all changes in space-time, of the current of free particles with momentum in that range. This is our basic formula.

If we take  $f_{free}$  equal to zero inside some freeze out surface, and equal to a distribution function on the surface, we see that equation (6) reduces to equation (3). So the Cooper-Frye formula is a particular case of our formula.

### 3. ENERGY CONSERVATION

One may compute the total energy crossing the decoupling surface  $\sigma$  from the Cooper-Frye formula. It is

$$E_{tot} = \int dNE = \int d^3p \int_\sigma d\sigma_\mu p^\mu f(x, p) = \int_\sigma d\sigma_\mu T^{0\mu}. \quad (7)$$

The total energy emitted by the gas therefore coincides with what is expected from hydrodynamics.

Similarly using equation (6), we may compute the total energy emitted as free particles between the initial value  $\chi_i$  and some final value  $\chi_f$

$$\Delta E_{tot} = \int d^3p \int d^4x D_\mu p^\mu f_{free}(x, p) = \int d^4x D_\mu T_{free}^{0\mu} = \int_{\chi_f} d\sigma_\mu T_{free}^{0\mu} - \int_{\chi_i} d\sigma_\mu T_{free}^{0\mu}. \quad (8)$$

Again, the total energy emitted as free particles is in agreement with what is expected from hydrodynamics. Note that one can also write momentum conservation in a similar way [19].

#### 4. CALCULATION OF $T(t, \rho)$

In our case, part of the energy is in the free particles and the rest in the interacting component of the gas. Energy conservation must therefore be written as

$$D_\mu T_{free}^{0\mu} + D_\mu T_{int}^{0\mu} = 0. \quad (9)$$

Let us write in addition  $f_{free} = \mathcal{P}f$  and  $f_{int} = (1 - \mathcal{P})f$  or simply

$$f_{free} = \mathcal{P}/(1 - \mathcal{P})f_{int}, \quad (10)$$

where  $\mathcal{P} = f_{free}/f$  is the proportion of free particles with a given four-momentum  $p$  at a given space-time point  $x$ .  $\mathcal{P}$  may also be identified (and this turns out to be more convenient later) with the probability that any particle with momentum  $p$  escapes from  $x$  without collision. (For example, if this probability equals 0.3, we expect a corresponding free particle proportion of 30 %). We now assume that approximately  $f_{int}$  is a thermalized matter distribution

$$f_{int}(x, p) = f_{th}(x, p) = g/(2\pi)^3 \times 1/\{\exp[p \cdot u(x, p)/T(x, p)] \pm 1\}, \quad (11)$$

where  $u^\mu$  is the fluid velocity and  $T$  its temperature.

In the usual freeze out scenario, there is no free particles in the fluid so one needs to solve only  $D_\mu T_{int}^{0\mu} = 0$  with  $f_{int}$  given by equation (11). Solving equation (9) is a complicated task by itself. In order to see if the continuous free particle emission process that we discuss has interesting new effects, we will therefore adopt a simplified description of the fluid evolution. Namely we are going to consider a fluid with boost invariant longitudinal expansion and compare our continuous particle emission picture with the freeze out one. In this case, the fluid velocity has the simple form [21]  $u^\mu = (t/\tau, 0, 0, z/\tau)$ . For simplicity, we suppose that the gas consists of massless pions; the calculation can easily be generalized to include all types of massive particles. In the freeze out case, the temperature is given [21] by  $T(z, t) = T(z_0, t_0) \times (\tau_0/\tau)^{1/3}$ .

We now proceed to extract the behavior of  $T$  from equation (9). At  $z = 0$ , we have for the interacting component

$$D_\mu T_{int}^{0\mu} = \partial_t \epsilon + 4/3 \times \epsilon/t, \quad (12)$$

where  $\epsilon = (\pi^2/10)T^4$ . This is the standard result for a fluid of massless particles, with boost invariant longitudinal expansion [21]. We can also compute for the free

component, either by expressing  $T^{0\mu}$  as function of  $\bar{T}^{\mu\mu}$  calculated in the rest frame, or by direct partial differentiation of  $\mathcal{P}/(1 - \mathcal{P})$  and  $f_{int}$  (for details see [19])

$$D_\mu T_{free}^{0\mu} = \partial_t(\alpha\epsilon) + (\alpha + \beta)\epsilon/t + \partial_\rho(\rho\gamma\epsilon)/\rho, \quad (13)$$

with

$$\begin{aligned} \alpha &= \int d\phi d\theta \sin \theta / (4\pi) \mathcal{P} / (1 - \mathcal{P})|_{z=0}, \\ \beta &= \int d\phi d\theta \sin \theta \cos^2 \theta / (4\pi) \mathcal{P} / (1 - \mathcal{P})|_{z=0}, \\ \gamma &= \int d\phi d\theta \cos \phi \sin^2 \theta / (4\pi) \mathcal{P} / (1 - \mathcal{P})|_{z=0}. \end{aligned}$$

The first term on the right hand side in (13) corresponds to  $\mu = 0$ , the second to  $\mu = z$  and the last to  $\mu = \rho$ . (We suppose cylindrical symmetry, so there is no angular term.) Because particles escape from the dense matter more easily if they are already close to the surface (going outward), a radial dependence will appear in  $T_{free}^{0\mu}$ , even if we start with a fluid whose initial energy density is  $\rho$  independent. Expressions (9), (12) and (13) lead to a partial differential equation (in  $t$  and  $\rho$ ) for  $\epsilon$  or  $T$  that can be solved numerically, given some initial conditions. For illustration, we take a flat initial energy density  $\epsilon(\rho, t_0) = (\pi^2/10)T_0^4$  for  $\rho < R$  and 0 outside. This allows analytical calculations (for example  $\mathcal{P}$ ) and better understanding of the physics involved.

$\mathcal{P}$  needed in  $\alpha$ ,  $\beta$  and  $\gamma$ , can be computed with the Glauber formula

$$\mathcal{P}(t, z, \rho; E = p, p_z = p \cos \theta, p_\perp = p \sin \theta; \phi) = \exp[-\int_t^{t_{out}} \sigma v_{rel} n(x') dt'], \quad (14)$$

where  $t_{out} = t + (-\rho \cos \phi + \sqrt{R^2 - \rho^2 \sin^2 \phi})/\sin \theta$  is the time when the particle reaches the surface of the dense matter region at  $\rho = R$ ,  $x' = (t', z' = z + \cos \theta(t' - t), \rho' = \sqrt{\rho^2 + [\sin \theta(t' - t)]^2 + 2\rho \sin \theta(t' - t) \cos \phi})$  is the location of the particle at time  $t'$ , and  $n(x')$  the density of all particles (free or interacting) at  $x'$ . In principle we have a self-consistent problem in  $\mathcal{P}$  and  $T$  via equations (13) and (14). To simplify we approximate  $n$  in  $\mathcal{P}$  by the solution for a fluid without particle emission. We get

$$\mathcal{P}(t, z, \rho; \theta, \phi) = \begin{cases} \left[ \frac{|\sin \theta| \sqrt{t^2 - z^2 + \sin^2 \theta t + \cos \theta (\cos \theta t - z)}}{|\sin \theta| \sqrt{t_{out}^2 - [z + \cos \theta (t_{out} - t)]^2 + \sin^2 \theta t_{out} + \cos \theta (\cos \theta t - z)}} \right]^{\frac{a}{|\sin \theta|}} & \text{if } \theta \neq 0 \text{ or } \pi \\ \exp\{a[\sqrt{(t \pm z)/(t \mp z)} - \sqrt{(2t_{out})/(t \mp z)} - 1]\} & \text{otherwise,} \end{cases} \quad (15)$$

with  $a \sim 3 \times (1.202/\pi^2) T_0^3 \tau_0 \sigma_{v_{rel}}$ .  $\mathcal{P}$  depends on expansion through  $\tau^{-1/3}$  in  $T$ , geometry (i.e. where the particle is at  $t$ ), particle type (via scattering) and direction of motion.

In figure 1, we show the behavior of the temperature as function of the radius, for various times for a fluid with boost invariant longitudinal expansion without and with continuous particle emission.

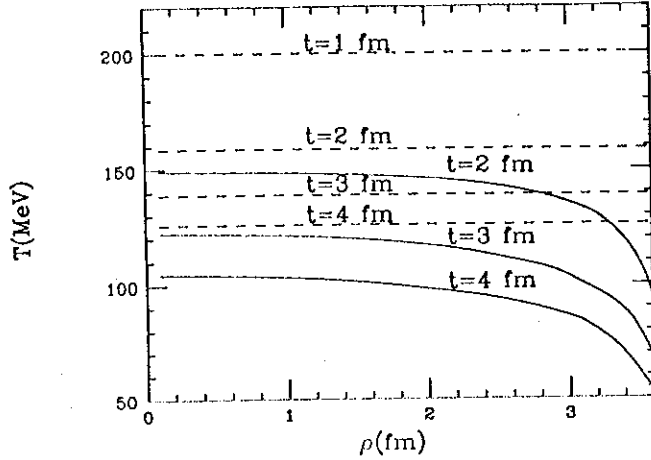


Figure 1: Temperature as a function of radius for various times. Solid (resp. dashed) line is the model with (resp. without) continuous particle emission.  $T_0 = 200$  MeV,  $\tau_0 = 1$  fm,  $R = 3.7$  fm and  $\langle \sigma_{v_{rel}} \rangle \sim 2$  fm<sup>2</sup>.

As expected, the cooling is faster in the last case since free particle emission removes energy. For times close to  $\tau_0$ , free particle emission does not affect very much the cooling - except at the edge - so our simplification for  $\mathcal{P}$  is reasonable. Later, cooling may occur much faster, so a self-consistent solution for  $\mathcal{P}$  and  $T$  should be done.

## 5. CALCULATION OF $dN/dy_{p_{\perp}} dp_{\perp}$

Now that we have the fluid evolution (namely  $u^{\mu}(x)$  and  $T(x)$  are known), we can turn to the actual calculation of particle spectra. In the case of a fluid with freeze out at a constant temperature, the Cooper-Frye formula can be re-written [22] as

$$\frac{dN}{dy_{p_{\perp}} dp_{\perp}} = \frac{g}{(2\pi)^2} \int d\phi d\eta \frac{m_{\perp} \cosh(y - \eta) \tau_{f_0} \rho d\rho - p_{\perp} \cos \phi \rho_{f_0} \tau d\tau}{\exp[m_{\perp} \cosh \eta_t \cosh(y - \eta)/T - p_{\perp} \sinh \eta_t \cos \phi/T] \pm 1}, \quad (16)$$

where  $\eta$  (resp.  $\eta_t$ ) is the longitudinal (resp. transverse) fluid rapidity. When ignoring transverse expansion, equation (16) reduces to

$$\frac{dN}{dy_{p_{\perp}} dp_{\perp}} = \frac{gR^2}{2\pi} \left(\frac{T_0}{T_{f_0}}\right)^3 \tau_0 m_{\perp} \sum_{n=1}^{\infty} (\mp)^{n+1} K_1\left(\frac{nm_{\perp}}{T_{f_0}}\right) \quad (17)$$

For continuous emission, we can rewrite equation (6) as

$$\frac{dN}{dy_{p_{\perp}} dp_{\perp}} = 2\pi \int d\phi d\eta \{m_{\perp} \cosh(\eta - y) [(\tau_{f_{free}})_{\tau_{\infty}} - (\tau_{f_{free}})_{\tau_0}] \rho d\rho + p_{\perp} \cos \phi (\rho_{f_{free}})_{|R} d\tau\}. \quad (18)$$

We see that both momentum distributions (16) and (18) depend only on the combination  $y - \eta$  (and not just  $y$ ) as it should for a boost invariant evolution (note that  $\mathcal{P}(t = \tau \cosh \eta, z = \tau \sinh \eta, \rho; p_{\perp}, p_z; \phi) = \mathcal{P}(\tau, \rho; p_{\perp}, p_z = m_{\perp} \sinh(y - \eta); \phi)$ ). The apparent difference in sign for the  $p_{\perp}$  term comes from the fact that in equation (16) the integral in  $\tau$  is along the freeze out curve with a definite orientation while in (17) it is for increasing  $\tau$  (see reference [19] for more details). To account also for free particles already present at  $\tau = \tau_0$ , the term at  $\tau_0$  in (18) must be removed.

Now, we expect that the approximation  $f_{int} = f_{th}$  breaks down when there is a big proportion of free particles. We will therefore consider in the integrals only those space-time points for which  $\mathcal{P} \leq 0.5$ . The  $\tau$  integral is then cut at  $\tau_F$  and the  $\rho$  integral at  $\rho_F$ , where  $\mathcal{P} = 0.5$ .  $\tau_F(\rho, \phi, \eta; v_{\perp}) = [-\rho \cos \phi + \sqrt{R^2 - \rho^2 \sin^2 \phi}] / [v_{\perp} (A(\eta) - \cosh \eta)]$ .  $\rho_F(\tau, \phi, \eta; v_{\perp}) = -v_{\perp} \tau (A(\eta) - \cosh \eta) \cos \phi \pm \sqrt{R^2 - v_{\perp}^2 \tau^2 \sin^2 \phi} (A(\eta) - \cosh \eta)$  with positive sign for  $0 \leq \phi \leq \pi/2$  and negative sign for  $\pi/2 \leq \phi \leq \pi$ .  $A(\eta) = [(1 + \cosh \eta)^2 / 0.5^{2/a} + \sinh^2 \eta] / [2(1 + \cosh \eta) / 0.5^{1/a}]$ . The momentum distribution of all free particles is then

$$\frac{dN}{dy_{p_{\perp}} dp_{\perp} |_{tot}} \sim \frac{2g}{(2\pi)^2} \int_{\mathcal{P}=0.5} d\phi d\eta \frac{m_{\perp} \cosh(y - \eta) \tau_F \rho d\rho + p_{\perp} \cos \phi \rho_F \tau d\tau}{\exp[m_{\perp} \cosh(y - \eta)/T] \pm 1} \quad (19)$$

This equation is almost the same as the Cooper-Frye formula except for an overall factor of 2. We also remind that the integration surface depends on the particle

momentum. Counting all the particles that become free inside the surface  $\mathcal{P}=0.5$  gives (19) without factor 2. However we must account for the remaining interacting matter. When  $\mathcal{P}=0.5$  is reached, little matter is interacting. We suppose that it is so rarefied that later changes in its spectrum are negligible. Consequently we may apply the Cooper-Frye formula for this component on the surface  $\mathcal{P}=0.5$ , hence the factor 2.

In figure 2a, we show a plot of the pion transverse mass distribution computed with (19), and compare with two thermal distributions (17) respectively at  $T_{fo}=150$  MeV and  $T_{fo} = T_0=200$  MeV.

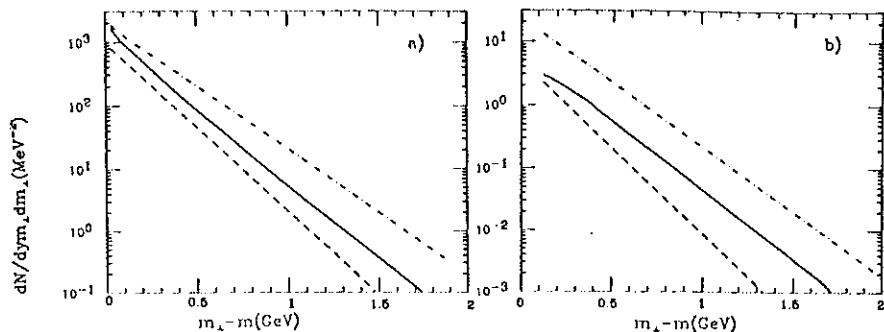


Figure 2: Transverse mass spectrum for a) the pion b) the nucleon. Solid line is our model with continuous particle emission. Dash-dotted and dashed lines are (scaled) thermal distributions respectively at  $T_0$  and  $T_{fo}=150$  MeV. (Same values of the parameters as for figure 1).

The interesting feature of the spectrum in the continuous emission scenario is its concave shape. The high  $p_{\perp}$  tail has a slope close to that of a thermal distribution at  $T_0$ , showing the existence of fast particles escaping while the temperature is high (when the density is high, the probability to escape is small but on the other side, if a small percentage of the high density matter escapes, it means a lot of particles). The low  $p_{\perp}$  part of the spectrum has a slope reflecting low temperatures, and is more similar to a thermal distribution at  $T_{fo}$ . It corresponds to the fact that low  $p_{\perp}$  particles get trapped and can be considered free when matter has become diluted. Figure 2b shows the same distributions than 2a but for more massive particles,

nucleons, with assumed null overall baryonic number. We see that their spectrum is now similar to the thermal distribution at  $T_0$ , showing a strong suppression as the temperature decreases. For that reason, their spectrum has no concave curvature and reflects only the highest temperature, when a large number of particles (but a small percentage of the density) is freed. Observe that our distribution is not a simple superposition of thermal distributions and the convex form at low  $m_{\perp}$  simply means that low  $p_{\perp}$  particles (at high  $T$ ) hardly escape. (If the baryonic number is not zero, a term  $\exp(\mu_b(\vec{x}, t)/T(\vec{x}, t))$  should be included in the integrals of equation (19). Due to its space-time dependence, in principle it does not amount to a simple multiplicative factor in the evaluation of  $dN/dym_{\perp} dm_{\perp}$ .)

## 6. CONCLUSION

Since we have worked with a simplified model, it would be unwise to use it to fit data. However it is interesting to see if its qualitative features go in the right direction and are quantitatively sizable. Data on transverse mass spectra have been obtained by most experiments. NA34, NA35 and EMU05 seem to agree that the pion spectrum has a concave curvature [23]. For heavy particles, NA35, NA36 and WA85 obtained approximately constant slopes [24]. This is qualitatively in agreement with what our simple model predicts. Even quantitatively, as shown in figure 3, the agreement is reasonably good.

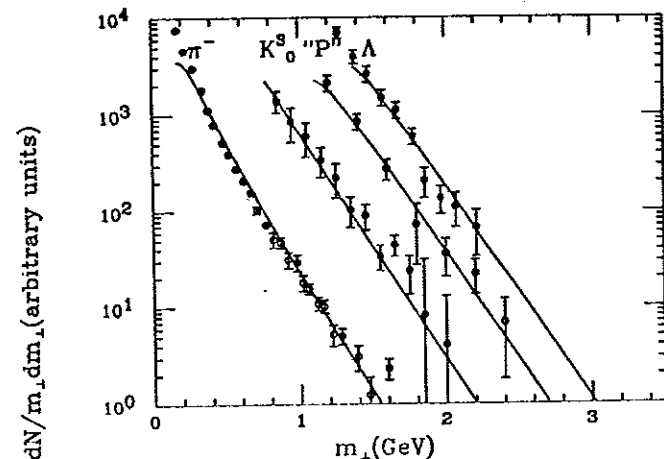


Figure 3: Transverse mass spectra computed with our model of continuous particle emission. Experimental points are NA35 S+S (all-y) data. This is not a least square fit and just show the general trend. (Same value of the parameters as in figure 1 except  $\langle \sigma_{rel} \rangle \sim 5 \text{ fm}^2$ ).

It is fair to recall that the usual freeze out scenario can also reproduce these data: see for example Lee et al. [11] for a model with spherical expansion (these authors also argue that the first points in the pion spectrum were not well predicted and suggested that this was due to the neglect of resonance decays) and Ornik & Weiner [7] for a model with longitudinal plus transverse expansion and resonance decays. In these models, the large value of the temperature seen in the high  $p_{\perp}$  tails of the various spectra comes from the fact that the low freeze out temperature is blue shifted due to transverse expansion. So this is an apparent temperature, not the real temperature of the fluid. In addition, resonance decays populate the low  $p_{\perp}$  part of the pion spectrum and provide a sizable curvature. On the basis of these data, it is not possible to see which mechanism, freeze out or continuous particle emission, provides a better description.

There exist however some data where continuous particle emission might be crucial, namely particle ratios such as those obtained [24] at Cern by NA35, NA36 and WA85, because transverse expansion affects the slope of the distributions but not the particle ratios. Various groups (see e.g. references [25,26]) have shown that to reproduce the WA85 ratios  $\bar{\Lambda}/\Lambda$ ,  $\bar{\Xi}^{-}/\Xi^{-}$ ,  $\Xi^{-}/\Lambda$  and  $\bar{\Xi}^{-}/\bar{\Lambda}$  and NA35 ratios  $\bar{\Lambda}/\Lambda$  and  $K_S^0/\Lambda$ , temperatures of order 200 MeV are needed. Such temperatures are hard to reconcile with the conventional freeze out scenario. In our description, since these experimental ratios concern heavy particles, their spectra should exhibit naturally a high temperature. This high temperature problem with the standard freeze out scenario, among other reasons, lead some authors [27] to conclude that the only hydrodynamical scenario consistent with data is one where a quark-gluon phase has been reached. We think that another explanation might be that the process for particle emission is continuous as in our model, which cure the high temperature problem. However, these are just qualitative arguments, we cannot be more quantitative yet about these ratios because this requires the knowledge of the chemical potentials.

Our hydrodynamical description has been very simplified (transverse expansion was not considered, longitudinal boost invariance was assumed, resonance decays were not included). Our aim was to see if new and interesting features emerge in our scenario for particle emission. We saw it could lead to a sizable curvature of the pion spectrum and affect the heavy particle spectrum as well (high non-“apparent” temperatures). Also, if hydrodynamical flow has indeed been established in current relativistic nuclear collisions, our scenario may lead to a more consistent description

of experimental data: it may reproduce not just the shape of the spectra, but the ratios of particle abundances. On the basis of this work, we think that it is necessary to develop a hydrodynamical numerical code incorporating this continuous particle emission process.

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## DISCUSSION

D.Srivastava

1) It may be better to call your "evaporation" by some other name, as evaporation by definition is only from the surface. 2) I will show results with transverse expansion. They are not large at least at SPS. 3) While talking of chemical potential and applicability of hydrodynamics, we should check chemical potential divided by the total number of particles produced. If the number of particles produced is large, we feel that we can use hydrodynamics with  $\mu = 0$  with confidence.

F.Grassi

Originally, I called our emission mechanism "evaporation". Since it generated a lot of confusion, I now use the expression "continuous particle emission".

H.Ströbele

1) What is the duration of particle emission in your model ? 2) What does it mean that p+A light particle spectra have similar slopes than those of A+A ?

F.Grassi

1) At most 2 fm, in perfect agreement with NA35 findings ! Seriously, I am aware that one gets bounds on the duration of particle emission by looking at interferometry data and we are planning to study this. 2) This is true, the shapes are similar and the amount of curvature changes. It may point toward a similar mechanism producing curvature. It may also mean that thermalization of matter does not modify the primary spectrum very much, in particular there is not enough time to build up very much transverse flow.

G.Odyniec

Your model takes into account only pions (no baryons included) and nevertheless it describes fairly well NA35 S+S baryon data ???

F.Grassi

NA35 has shown that the rapidity distribution of positive minus negative particles (identified as protons) is non-zero even at mid-rapidity, hence I believe this question. The real question is, as underlined by D.Srivastava, does the presence of baryons affect the hydrodynamical behavior of the fluid substantially? The answer is not very clear for me, particularly in our description. However I hope it is clear that we did not try to make a fit of NA35 data. We study a new mechanism for particle emission in a simplified description of the fluid evolution and compare it with the standard emission at freeze out.



K.Werner 1) You do not consider resonances ? 2) Comment: you show calculations for  $\mu = 0$ , but the S+S data you refer to indicate  $\mu \neq 0$  !

F.Grassi 1) No, I made no "embroidery" but compared continuous particle emission and emission at freeze out in a simplified description of hydrodynamics. Since the results are promising, it is worth trying to include chemical potentials, transverse expansion, resonances, etc. 2) See the comment by D.Srivastava and my answer to G.Odyniec.

M.Gorenstein The Cooper-Frye formula becomes unapplicable for time-like parts of the freeze out hypersurface: it gives negative contributions to particle number. I am afraid that your formula written in covariant form has the same difficulty.

F.Grassi Our formula for the emission spectrum, in cartesian coordinates to simplify, is:  $E d^3 N / dp^3 = \int d^4 x p^\mu \partial_\mu f_{free}$ . Once we have chosen a direction  $p^\mu$ , we expect the scalar  $p^\mu \partial_\mu f_{free}$  to be positive. This is because for us "free" means that a particle free at  $t, \vec{x}$  will remain so at all later times and locations.

S.Gavin 1) Cascade people calculate distributions of last times that particles interacted. This gives a distribution of freeze-out times that is as broad as its mean value. How does your formulation compare to this ? 2) Does the evaporation effect become smaller for big systems ?

F.Grassi 1) This is a calculation I may be able to do but I have not done yet. In terms of last scattering radius, taking a more physical initial profile for matter (e.g. Woods-Saxon), I find that most of the emission comes from  $\rho = 2-3$  fm - as one would guess, in agreement I believe with cascade results. 2) Continuous particle emission still is important with increasing  $A$  but several factors come into play: larger surface, larger inner volume, eventually larger initial temperature, etc. We discuss this in detail in reference [19].

R.Opher

You said that you made the calculations near the  $z$ -axis and that the gradient of density was zero. But evaporation occurs from the surface, and the surface is defined as the region where a steep gradient exists.

F.Grassi

There is a misunderstanding: we do not do the calculations near the  $z = 0$  axis but in the  $z = 0$  plane (for energy conservation). Now for the density gradient: in the oral contribution, I presented partial results (e.g. I neglected terms such as  $\partial_\rho \mathcal{P}$  inside the interaction region) but in the written version, all terms are included. Of course the fact that there is a surface, i.e. the system has finite dimensions is crucial, but the profile of matter inside is not.