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COLLISIONS**

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# PARTICLE EMISSION IN THE HYDRODYNAMICAL DESCRIPTION OF RELATIVISTIC NUCLEAR COLLISIONS

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Continuous particle emission during the whole expansion of thermalized matter is studied and a new formula for the observed transverse mass spectrum is derived. In some limit, the usual emission at freeze out scenario (Cooper-Frye formula) may be recovered. In a simplified description of expansion, we show that continuous particle emission can lead to a sizable curvature in the pion transverse mass spectrum and parallel slopes for the various particles. These results are compared to experimental data.

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At the present moment, the theoretical description of relativistic heavy ion collisions is still quite controversial. On one extreme, one may try to describe heavy ion collisions as a superposition of nucleon-nucleon collisions. On the other extreme, one may apply a statistical description, assuming that complete thermalization has been attained. The reason for this ambiguity is that we do not know the thermalization time. A reliable estimate of this time requires knowledge of quantities (e.g. density reached) that are not well established yet. It is however thought that [1] due to the higher multiplicities and longer dense matter lifetimes available, states of thermal equilibrium should be reached (if not yet reached) at the accelerators that will be in use in the future. So it is important to develop a complete hydrodynamical description of relativistic heavy ion collisions. There is indeed a lot of activity in this direction. Full three-dimensional hydrodynamical codes are becoming available [2-5] and transverse momentum and rapidity distributions are predicted. These codes took over more simplified solutions [6-9]. Finer details are now being studied. The effect of the freeze out criteria and initial conditions are tested using such codes or easier to handle semi-numerical approaches [10-13]. The impact of resonance decays (in particular in connection with the observed low- $p_t$  pion enhancement) is being evaluated both in static thermal models and hydrodynamical models [14-17]. In this paper, we concentrate on the description of the particle emission process.

In hydrodynamical models, one usually introduces the notion of a sharp three-dimensional freeze out surface  $\sigma$ . Before crossing it, particles have a hydrodynamical behavior and, after, they free-stream toward the detectors, keeping memory of the conditions (flow, temperature) of where and when they crossed the three dimensional surface. To compute the momentum distribution of freeze out hadrons, one often uses the Cooper-Frye formula [18]

$$E d^3 N / d p^3 = \int_{\sigma} d\sigma_{\mu} p^{\mu} f(x, p), \quad (1)$$

where  $d\sigma_{\mu}$  is a surface element and  $f$  a distribution function.

Let us try to make a description of particle emission that is closer to what happens experimentally. At each space-time point  $x$ , a given particle has some chance to escape the dense matter region without collision. This is due to the finite dimensions and

lifetime of the thermalized matter. So we consider that the fluid has two components, a free part plus an interacting part and write

$$f(x, p) = f_{free}(x, p) + f_{int}(x, p), \quad (2)$$

where  $f_{free}$  counts the particles at time  $x^0$  in  $\vec{x}$  which last scattered earlier and  $f_{int}$  describes all the particles that still will undergo collisions later. The variation in the total number of free particles between two infinitesimally close surfaces is

$$\delta \int_{\sigma} d\sigma_{\mu} \int d^3p p^{\mu} / E f_{free}(x, p) = \int_{\sigma}^{\sigma+\delta\sigma} d^4x \int d^3p D_{\mu} [p^{\mu} / E f_{free}(x, p)]. \quad (3)$$

So the momentum distribution of all free particles emitted at  $\tau > \tau_0$  is

$$E d^3N/dp^3 = \int d^4x D_{\mu} [p^{\mu} f_{free}(x, p)]. \quad (4)$$

This is our basic formula. The physical meaning of this expression is simple: the number of detected particles with momentum in some range is given by summing all changes in space-time of the current of free particles with momentum in that range. If we take  $f_{free}$  equal to zero inside some freeze out surface and equal to a distribution function on it, we see that (4) reduces to (1). So the Cooper-Frye formula is a particular case of our formula. For both formulas, the total energy emitted is in agreement with what is expected from hydrodynamics [19].

In our case, part of the energy is in the free particles and the rest in the interacting component of the gas. Energy conservation can therefore be written as

$$D_{\mu} T_{free}^{0\mu} + D_{\mu} T_{int}^{0\mu} = 0. \quad (5)$$

Let us write in addition  $f_{free} = \mathcal{P}f$  and  $f_{int} = (1 - \mathcal{P})f$  or equivalently,  $f_{free} = \mathcal{P}/(1 - \mathcal{P})f_{int}$ , where  $\mathcal{P} = f_{free}/f$  is the proportion of free particles with a given four-momentum  $p$  at a given space-time point  $x$ .  $\mathcal{P}$  may also be identified (and this turns out to be more convenient later) with the probability that any particle with momentum  $p$  escapes from  $x$  without collision. (For example, if this probability equals 0.3, we expect a corresponding free particle proportion of 30 %). We now assume that approximately  $f_{int}$  is a thermalized matter distribution

$$f_{int}(x, p) = f_{th}(x, p) = g/(2\pi)^3 \times 1/\{\exp[p \cdot u(x, p)/T(x, p)] \pm 1\}, \quad (6)$$

where  $u^{\mu}$  is the fluid velocity and  $T$  its temperature.

In the usual freeze out scenario, there is no free particles in the fluid so one needs to solve only  $D_{\mu} T_{int}^{0\mu} = 0$  with  $f_{int}$  given by (6). Solving (5) is a complicated task by itself. In order to see whether the continuous free particle emission process that we propose has interesting new effects, let us adopt a simplified description of the fluid evolution. Namely we are going to consider a fluid with boost invariant longitudinal expansion and compare our continuous emission picture with the freeze out one. In this case, the fluid velocity has the form [20]  $u^{\mu} = (t/\tau, 0, 0, z/\tau)$ . For simplicity, we suppose that the gas consists of massless pions. In the freeze out case, the temperature is given [20] by  $T(z, t) = T(z_0, t_0) \times (\tau_0/\tau)^{1/3}$ . We now proceed to extract the behavior of  $T$  from (5). At  $z = 0$ , we have for the interacting component

$$D_{\mu} T_{int}^{0\mu} = \partial_t \epsilon + 4/3 \times \epsilon/t, \quad (7)$$

where  $\epsilon = \pi^2/10T^4$ , and for the free component (for details see [19])

$$D_{\mu} T_{free}^{0\mu} = \partial_t(\alpha\epsilon) + (\alpha + \beta)\epsilon/t + \partial_{\rho}(\rho\gamma\epsilon)/\rho, \quad (8)$$

with  $\alpha = \int d\phi d\theta \sin \theta / (4\pi) \mathcal{P} / (1 - \mathcal{P})$ ,  $\beta = \int d\phi d\theta \sin \theta \cos^2 \theta / (4\pi) \mathcal{P} / (1 - \mathcal{P})$ ,  $\gamma = \int d\phi d\theta \cos \phi \sin^2 \theta / (4\pi) \mathcal{P} / (1 - \mathcal{P})$ .

Because free particles escape from the dense matter more easily if they are already close to the surface, going outward, a radial dependence will appear in  $T_{free}^{0\mu}$  even if we start with a fluid whose initial energy density is  $\rho$  independent. Expressions (5),(7) and (8) lead to a partial differential equation (in  $t$  and  $\rho$ ) for  $\epsilon$  or  $T$  that can be solved numerically, given some initial conditions. For illustration, we take a flat initial energy density  $\epsilon(\rho, t_0) = \pi^2/10 T_0^4$  for  $\rho < R$  and 0 outside. This allows analytical calculations (for example  $\mathcal{P}$ ) and better understanding of the physics involved.  $\mathcal{P}$  needed in  $\alpha$ ,  $\beta$  and  $\gamma$  can be computed with the Glauber formula,  $\mathcal{P} = \exp[-\int_{t_0}^{\infty} \sigma v_{rel} n(x') dt']$ . To simplify we approximate  $n$  in  $\mathcal{P}$  by the solution for a fluid without particle emission.  $\mathcal{P}$  depends on expansion through density, geometry (i.e. where the particle is at  $t$ ), particle type (via scattering) and direction of motion.

In figure 1, we show the behavior of the temperature as function of the radius, for various times for a fluid with boost invariant longitudinal expansion without and with

continuous particle emission. As expected, the cooling is faster in the last case since free particle emission removes energy. For times close to  $\tau_0$ , free particle emission does not affect very much the cooling - except at the edge - so our simplification for  $\mathcal{P}$  is reasonable. Later, cooling may occur faster, so a self-consistent solution for  $\mathcal{P}$  and  $T$  should be done.

Now that we have the fluid evolution (namely  $u^\mu(x)$  and  $T(x)$  are known), we can turn to the actual calculation of particle spectra. In the case of freeze out at a constant temperature, the Cooper-Frye formula can be rewritten as [21]

$$\frac{dN}{dy p_\perp dp_\perp} = \frac{g}{(2\pi)^2} \int d\phi d\eta \frac{m_\perp \cosh(y-\eta) \tau_{f_0} \rho d\rho - p_\perp \cos \phi \rho f_0 \tau d\tau}{\exp[m_\perp \cosh \eta_\perp \cosh(y-\eta)/T - p_\perp \sin \eta_\perp \cos \phi/T] \pm 1}, \quad (9)$$

where  $\eta$  (resp.  $\eta_\perp$ ) is the longitudinal (resp. transverse) fluid rapidity. When ignoring transverse expansion, (9) reduces to

$$\frac{dN}{dy p_\perp dp_\perp} = \frac{gR^2}{2\pi} \left(\frac{T_0}{T_{f_0}}\right)^3 \tau_0 m_\perp \sum_{n=1}^{\infty} (\mp)^{n+1} K_1\left(\frac{nm_\perp}{T_{f_0}}\right). \quad (10)$$

For continuous emission, we can rewrite (4) as

$$\frac{dN}{dy p_\perp dp_\perp} = 2\pi \int d\phi d\eta \{m_\perp \cosh(\eta-y) [(\tau f_{f_{rec}})_{\tau_\infty} - (\tau f_{f_{rec}})_{\tau_0}] \rho d\rho + p_\perp \cos \phi (\rho f_{f_{rec}})_{R\tau} d\tau\}. \quad (11)$$

The apparent difference in sign for the  $p_\perp$  term comes from the fact that in (9) the integral in  $\tau$  is along the freeze out curve with a definite orientation while in (11) it is for increasing  $\tau$  (see [19] for more details). To account also for free particles already present at  $\tau = \tau_0$ , the term at  $\tau_0$  in (11) must be removed.

Now, we expect that the approximation  $f_{int} = f_{th}$  breaks down when there is a big proportion of free particles. We will therefore consider in the integrals only those space-time points for which  $\mathcal{P} \leq 0.5$ . The  $\tau$  integral is then cut at  $\tau_F(\rho, \phi, \eta; v_\perp)$  and the  $\rho$  integral at  $\rho_F(\tau, \phi, \eta; v_\perp)$ , where  $\mathcal{P} = 0.5$ . The momentum distribution of all free particles is then

$$\frac{dN}{dy p_\perp dp_\perp}_{tot} \sim \frac{2g}{(2\pi)^2} \int_{\mathcal{P}=0.5} d\phi d\eta \frac{m_\perp \cosh(y-\eta) \tau_F \rho d\rho + p_\perp \cos \phi \rho_F \tau d\tau}{\exp(m_\perp \cosh(y-\eta)/T) \pm 1}. \quad (12)$$

This equation is almost the same as the Cooper-Frye formula except for an overall factor of 2. We also remind that the integration surface depends on the particle momentum.

Counting all the particles that become free inside the surface  $\mathcal{P}=0.5$  gives (12) without factor 2. However we must account for the remaining interacting matter. When  $\mathcal{P}=0.5$  is reached, little matter is interacting. We suppose that it is so rarefied that later changes in its spectrum are negligible. Consequently we may apply the Cooper-Frye formula for this component on the surface  $\mathcal{P}=0.5$ , hence the factor 2.

In figure 2a, we show a plot of the pion transverse mass distribution, computed with (12), and compare with two thermal distributions (10) respectively at  $T_{f_0}=150$  MeV and  $T_{f_0} = T_0=200$  MeV. The interesting feature of the spectrum in the continuous emission scenario is its concave shape. The high  $p_\perp$  tail has a slope close to that of a thermal distribution at  $T_0$ , showing the existence of fast particles escaping while the temperature is high. The low  $p_\perp$  part of the spectrum has a slope reflecting low temperatures, and is more similar to a thermal distribution at  $T_{f_0}$ . It corresponds to the fact that low  $p_\perp$  particles get trapped and can be considered free when matter has become diluted. Figure 2b shows the same distributions as 2a but for more massive particles, nucleons, with assumed null overall baryonic number. The spectrum is now similar to a thermal distribution at  $T_0$ , showing a strong suppression when the temperature decreases. Observe that our distribution is not a simple superposition of thermal distributions and the convex shape at low  $m_\perp$  simply means that low  $p_\perp$  particles (at high  $T$ ) hardly escape.

Since we have worked with a simplified model, it would be unwise to use it to fit data. However it is interesting to see whether its qualitative features go in the right direction and are quantitatively sizable. Data on transverse mass spectra have been obtained by most experiments. NA34, NA35 and EMU05 seem to agree that the pion spectrum has a concave curvature [22]. For heavy particles, NA35, NA36 and WA85 obtained approximately constant slopes [23]. This is qualitatively in agreement with what our simple model predicts. Even quantitatively, as shown in figure 3, the agreement is reasonably good. It is fair to recall that the usual freeze out scenario can also reproduce these data [11,17]. In these models, the large value of the temperature seen in the high  $p_\perp$  tails of the various spectra comes from the fact that the low freeze out temperature is blue shifted due to transverse expansion. So this is an apparent temperature, not the real fluid temperature. On the basis of these data, it is not possible to see which

mechanisms, freeze out or continuous particle emission provides a better description.

There exist however some data where continuous particle emission might be crucial, namely particle ratios such as those obtained [23] at Cern by NA35, NA36 and WA85, because transverse expansion affects the slope of the distributions but not the particle ratios. Various groups (see e.g. references [24,25]) have shown that to reproduce the WA85 ratios  $\bar{\Lambda}/\Lambda$ ,  $\Xi^-/\Xi^+$ ,  $\Xi^-/\Lambda$  and  $\Xi^-/\bar{\Lambda}$  and NA35 ratios  $\bar{\Lambda}/\Lambda$  and  $K_S^0/\Lambda$ , temperatures of order 200 MeV are needed. Such high temperatures are hard to reconcile with the conventional freeze out scenario. In our description, since these experimental ratios concern heavy particles, their spectra should exhibit naturally a high temperature. (This high temperature problem with the standard freeze out scenario, among other reasons, lead some authors [26] to conclude that the only hydrodynamical scenario consistent with data is one where a quark-gluon phase has been reached.) However, these are still qualitative arguments, we cannot be more quantitative yet about these ratios because this requires the knowledge of the chemical potentials.

Our hydrodynamical description has been very simplified (transverse expansion was not considered, longitudinal boost invariance was assumed, resonance decays were not included). Our aim was to see whether new and interesting features emerge in our scenario for particle emission. We saw it could lead to a sizable curvature of the pion spectrum and affect the heavy particle spectrum as well (high non-“apparent” temperatures). Also, if hydrodynamical flow has indeed been established in current relativistic nuclear collisions, our scenario may lead to a more consistent description of experimental data: it may reproduce not only the shape of the spectra but the ratios of particle abundances. On the basis of this work, we think that it is necessary to develop a hydrodynamical numerical code incorporating this continuous particle emission process.

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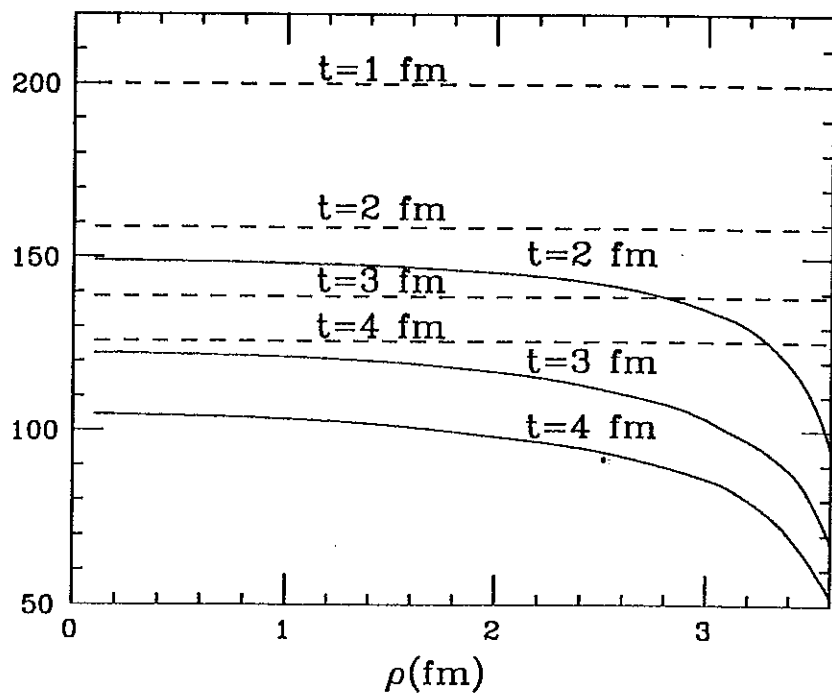
## Figure captions

Figure 1: Temperature as a function of radius for various times. Solid (resp. dashed) line is the model with (resp. without) continuous particle emission.  $T_0 = 200$  MeV,  $\tau_0 = 1$  fm,  $R = 3.7$  fm and  $\langle \sigma v_{rel} \rangle \sim 2$  fm<sup>2</sup>.

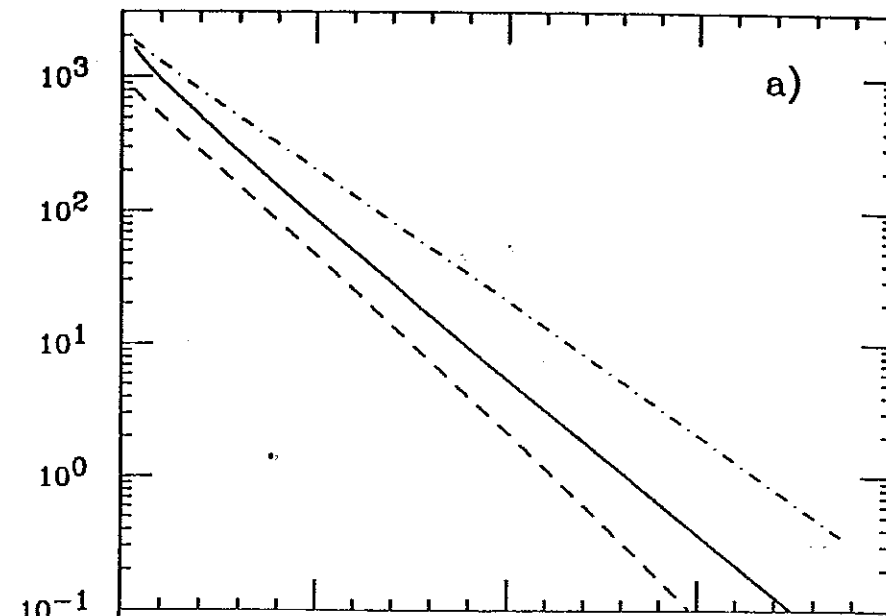
Figure 2: Transverse mass spectrum for a) the pion b) the nucleon. Solid line is our model with continuous particle emission. Dash-dotted and dashed lines are (scaled) thermal distributions respectively at  $T_0$  and  $T_{fo} = 150$  MeV. (Same values of the parameters as for figure 1)

Figure 3: Transverse mass spectra computed with our model of continuous particle emission. Experimental points are NA35 S+S (all-y) data. This is not a least square fit and just show the general trend. (Same value of the parameters as in figure 1 except  $\langle \sigma v_{rel} \rangle \sim 5$  fm<sup>2</sup>).

T (MeV)



$dN/dy dm_{\perp} (MeV^{-2})$



$dN/dy m_{\perp} dm_{\perp} (MeV^{-2})$

