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**Comment on "RELATIVISTICALLY
COVARIANT SYMMETRY IN QED"
by Z. TANG and D. FINKELSTEIN,
Phys. Rev. Lett. 73, 3055 (1994)**

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Abstract

We show that recently found symmetries in QED are just non-local versions of the usual BRST symmetry.

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Recently it was found a non-local and non-covariant symmetry of QED in the Feynman gauge by Lavelle and McMullan [1] which was cast in a covariant form by Tang and Finkelstein [2]. Even being nilpotent it was claimed that this symmetry generalizes the BRST symmetry. Also a non-nilpotent and non-local symmetry was found which was also claimed to generalize the BRST symmetry [2]. We would like to point out that these symmetries are essentially BRST symmetries.

The ghost Lagrangian of QED which implements the Lorentz condition, $L_{gh} = i\bar{c}\square c$, has a huge freedom when we perform field redefinitions in the ghost fields c and \bar{c} . If we consider the path integral formulation then a further requirement is that these field redefinitions leave the path integral measure invariant. If we consider, e.g., the following non-local redefinitions $c = \frac{1}{\xi}\partial_0 d$, $\bar{c} = \frac{1}{\xi_0}\nabla\bar{d}$, the Lagrangian and the path integral measure remain invariant and the usual BRST transformations become $\delta A_\mu = \partial_\mu \frac{1}{\xi}\partial_0 d$, $\delta d = 0$, $\delta\bar{d} = -\frac{i}{\xi}\frac{1}{\xi_0}\partial_0\partial_\mu A^\mu$. (we leave out the fermion fields since they are not essential for our purposes as they can be easily included). These are the covariant non-local transformations presented in Ref. [2] here written in an arbitrary gauge (arbitrary ξ). They are nilpotent because they are just the BRST transformations of the redefined fields.

Of course this procedure can be generalized to any (local or non-local) redefinition of the ghost fields. Assume that F, G, \dots are operators which possess an adjoint F', G', \dots in the sense that $\int dx \phi F\psi = \int dx (F'\phi)\psi$ (since the action $\int dx \phi\psi$ defines a bilinear metric). Then the following field redefinitions leave the action invariant $A_\mu^F = A_\mu$, $\lambda^F = \lambda$, $c^F = F^{-1}c$, $\bar{c}^F = F'\bar{c}$. Here λ is the extra bosonic ghost which makes the BRST transformations nilpotent off-shell (the ghost Lagrangian then acquires one more term $\frac{1}{2}\xi\lambda^2$). The BRST transformations for the redefined fields are $\delta A_\mu^F = \partial_\mu F d^F$, $\delta\lambda^F = \frac{1}{\xi}\square F d^F$, $\delta d^F = 0$, $\delta\bar{d}^F = iF'(\lambda^F - \frac{1}{\xi}\partial_\mu A^\mu)$. Since the field redefinitions leave the action and the path integral measure invariant we could drop the suffix F of all fields after making the redefinitions. Of course, it does not mean that the BRST symmetry of the redefined fields is a new symmetry. The BRST transformations of the redefined fields are precisely the transformations Eqs.(5) and (9) of Ref. [2] for $g = 0$.

The QED action is also invariant under anti-BRST transformations which anticommute with the BRST transformations. If we perform the following field redefinitions $A_\mu^G = A_\mu$, $\lambda^G = \lambda$, $c^G = G^t c$, $\bar{c}^G = G^{-1} \bar{c}$ which also leave the action and the path integral measure invariant, then the anti-BRST transformations are the transformations Eqs.(5) and (9) of Ref. [2] for $f = 0$.

Since the F and G field redefinitions leave the action invariant we could apply a F field redefinition and get the F transformed BRST transformations while keeping the anti-BRST transformations of the original fields. Then perform a G transformation to get the G transformed anti-BRST transformations but leaving unchanged the BRST transformations. Summarizing, we end up with a set of BRST transformations for the F field redefinitions and a set on anti-BRST transformations for the G field redefinitions. If we sum the two transformations we get precisely the Tang and Finkelstein transformations Eqs.(5) and (9) with nonvanishing f and g . Originally the BRST and anti-BRST transformations are anticommutating, but now, since they are acting after field redefinitions they no longer need to anticommute. This explains why the transformations Eqs.(5) and (9) of Ref. [2] are no longer nilpotent. In fact the anticommutator should give a new field redefinition which should also be a symmetry of the action. If we denote the anticommutator by Δ then we easily get $\Delta A_\mu = i\partial_\mu(GF^t - FG^t)(\lambda - \frac{1}{\xi}\partial_\nu A^\nu)$, $\Delta c = \Delta \bar{c} = 0$, $\Delta \lambda = i\frac{1}{\xi}\square(GF^t - FG^t)(\lambda - \frac{1}{\xi}\partial_\nu A^\nu)$. From it we can read off the new field redefinitions, which now act only in A_μ and λ leaving the ghosts c and \bar{c} unchanged $A_\mu^H = A_\mu + i\partial_\mu(H - H^t)(\lambda - \frac{1}{\xi}\partial_\nu A^\nu)$, $c^H = c$, $\bar{c}^H = \bar{c}$, $\lambda^H = \lambda + \frac{1}{\xi}\square(H - H^t)(\lambda - \frac{1}{\xi}\partial_\nu A^\nu)$. It is easily checked that the H transformations leave the action and the path integral measure invariant. Therefore the non-nilpotent transformations of Ref. [2] are just a combination of BRST and anti-BRST transformations with general field redefinitions.

The Lavelle and McMullan transformations Ref. [1] can be obtained in the Hamiltonian path integral formalism by integrating over the ghost fields instead of the ghost momenta as it is usually done. After an obvious change of variables in the ghost momenta the resulting action is the usual ghost action while the BRST transformations are the transformations

Eqs.(5) of Ref. [1]. Therefore the Lavelle and McMullan transformations are plain BRST transformations.

A non-abelian version of these non-local field redefinitions can also be found. However the action turns out to be local only in the abelian case.

REFERENCES

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[1] M. Lavelle and D. McMullan, Phys. Rev. Lett. **71**, 3758 (1993).

[2] Z. Tang and D. Finkelstein, Phys. Rev. Lett. **73**, 3055 (1994).