UNIVERSIDADE DE SÃO PAULO

PUBLICAÇÕES

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01452-990 SÃO PAULO - SP
BRASIL

IFUSP/P-1144

PSEUDOCLASSICAL SUPERSYMMETRICAL MODEL FOR 2+1 DIRAC PARTICLE

D.M. Gitman and **I.V. Tyutin**
Lebedev Physical Institute
117924 Moscow, Russia

A.E. Gonçalves
Instituto de Física, Universidade de São Paulo

Março/1995

Pseudoclassical supersymmetrical model for 2+1 Dirac particle

D.M.Gitman* and I.V.Tyutin

Lebedev Physical Institute, 117924 Moscow, Russia

A.E. Gonçalves

Instituto de Física, Universidade de São Paulo

P.O. Box 20516, 01452-990 São Paulo, SP, Brazil

(March 11, 1995)

Abstract

A new pseudoclassical supersymmetrical model to describe spin 1/2 particle in 2+1 dimension is proposed. Different ways of its quantization are discussed. They all lead to the minimal quantum theory of the Dirac particle (spin projection 1/2 or $-1/2$). It turns out that the model can be derived in course of a dimensional reduction from one of the Weyl particle in $3+1$ dimensions, proposed recently by the authors.

11.10.Ef, 03.65.Pm

I. INTRODUCTION

In this paper we present a new pseudoclassical model for a massive Dirac particle in 2+1 dimensions, interacting with an external Abelian gauge field. Such a model has an important meaning not only for the deeper understanding of the quantum theory of relativistic particles, but also because of a close connection with the theory of interacting anyons, which attracts in recent years great attention (see for example [1]). As it is known, the pseudoclassical supersymmetrical model for Dirac (spinning) particle in 3+1 dimensions was first proposed by Berezin and Marinov [2] and after that was discussed and investigated in the numerous papers [3–9]. Generalizations of the model for particles with arbitrary spin, for Weyl particles and so on, one can find, for example, in [10–12]. Attempts to extend the pseudoclassical description to the arbitrary odd-dimensions case had met some problems, which are connected with the absence of an analog of γ^5-matrix in odd-dimensions. For instance, in 2+1 dimensions the direct generalization of the Berezin-Marinov action (standart action) does not reproduce a minimal quantum theory of spinning particle, which has to provide only one value of the spin projection (1/2 or $-1/2$). In papers [13,14] they have proposed two modifications of the standard action to get such a minimal theory, but these models can not be considered as satisfactory solutions of the problem. The action [13], in fact, is classically equivalent to the standard action and does not provide required quantum properties in course of canonical and path-integral quantization. Moreover, it is P- and T-invariant, so that an anomaly is present. Another one [14] does not obey gauge supersymmetries and therefore loses the main attractive features in such kind of models, which allows one to treat them as prototypes of superstrings or some modes in superstring theory.

The action, we are proposing, obeys three gauge symmetries-one reparametrization symmetry and two supergauge symmetry. It is P- and T-noninvariant in full accordance with the expected properties of the minimal theory in 2+1 dimensions, which has to describe only one value of the spin projection. Dirac quantization (without explicit gauge fixing on the classical level) and quasicanonical quantization with fixation of the gauge freedom, which corresponds to two types of gauge transformations of the three existing, leads to the quantum theory of spin 1/2 Dirac particle in 2+1 dimensions. Technically, the Dirac equation in 2+1 dimensions arises in both schemes of quantization in different ways, but both quantum theories appear to be equivalent and describe a particle with spin 1/2. In conclusion we discuss a relation between the theory of the massive spinning particle in 2 + 1 dimensions and the theory of massless (Weyl) particle in 3 + 1 dimensions.

II. LAGRANGIAN ACTION AND HAMILTONIAN FORMULATION

The new action to describe a Dirac particle in 2 + 1 dimension has the form

$$S=\int_0^1\left[-\frac{z^2}{2e}-e\frac{m^2}{2}-g\dot{x}^\mu A_\mu+\imath geF_{\mu\nu}\psi^\mu\psi^\nu-\imath m\psi^3\chi-\frac{\imath}{2}sm\kappa-\imath\psi_a\dot{\psi}^a\right]d\tau\equiv$$
$$\equiv\int_0^1 Ld\tau\ ,\quad s=\pm\ ;\quad z^\mu=\dot{x}^\mu-\imath\psi^\mu\chi-\varepsilon^{\mu\nu\lambda}\psi_\nu\psi_\lambda\kappa\ ;\qquad (1)$$

the Latin indices a, b, c, ... run over 0, 1, 2, 3, whereas the Greek (Lorentz) ones μ, ν, ... run over 0, 1, 2; x^μ, e, κ are even and ψ^a, χ are odd variables, dependent on an invariant parameter $\tau\in[0,1]$; $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the strength tensor and g is the $U(1)$-charge of the particle, interacting with an external gauge field $A_\mu(x)$, which can have the Maxwell or (and) Chern-Simons nature; $\varepsilon^{\mu\nu\lambda}$ is the totally antisymmetric tensor density of Levi-Civita in 2+1 dimensions normalized by $\varepsilon^{012}=+1$; $\eta_{ab}=\mathrm{diag}(1,-1,-1,-1)$, $\eta_{\mu\nu}=\mathrm{diag}(1,-1,-1)$. We suppose that x^μ and ψ^μ are 2 + 1 Lorentz vectors and e, κ, ψ^3, χ are scalars so that the action (1) is invariant under the restricted Lorentz transformations (but not P- and T-invariant). There are three types of gauge transformations, under which the action (1) is invariant: reparametrizations

$$\delta x^\mu=\dot{x}^\mu\xi\ ,\quad \delta e=\frac{d}{d\tau}(e\xi)\ ,\quad \delta\psi^a=\dot{\psi}^a\xi\ ,\quad \delta\chi=\frac{d}{d\tau}(\chi\xi)\ ,\quad \delta\kappa=\frac{d}{d\tau}(\kappa\xi)\ ,\qquad (2)$$

with an even parameter $\xi(\tau)$; supertransformations

$$\delta x^\mu=\imath\psi^\mu\epsilon\ ,\quad \delta e=\imath\chi\epsilon\ ,\quad \delta\psi^\mu=\frac{z^\mu}{2e}\epsilon\ ,\quad \delta\psi^3=\frac{m}{2}\epsilon,\quad \delta\chi=\dot{\epsilon}\ ,\quad \delta\kappa=0\ ,\qquad (3)$$

with an odd parameter $\epsilon(\tau)$; and additional supertransformations

$$\delta x^\mu=-\imath\varepsilon^{\mu\nu\lambda}\psi_\nu\psi_\lambda\theta\ ,\quad \delta\psi^\mu=\frac{1}{e}\varepsilon^{\mu\nu\lambda}z_\nu\psi_\lambda\theta\ ,\quad \delta\kappa=\dot{\theta},\quad \delta e=\delta\psi^3=\delta\chi=0\ ,\qquad (4)$$

with an even parameter $\theta(\tau)$. Calculating the total angular momentum tensor $M_{\mu\nu}$, whose components are Nöther currents related to the Lorentz symmetry of the action (1), and are at the same time generators of the correspondent transformations for the variables of the theory, we get

$$M_{\mu\nu}=x_\mu\pi_\nu-x_\nu\pi_\mu+\imath[\psi_\mu,\psi_\nu]\ ,\qquad (5)$$

where $\pi_\nu=\partial L/\partial\dot{x}^\nu$. Below we are going to use it to prove properties of the Lorentz symmetry of the correspondent quantum theory.

Going over to the Hamiltonian formulation, we introduce the canonical momenta

$$\pi_\mu=\frac{\partial L}{\partial\dot{x}^\mu}=-\frac{1}{e}z_\mu-gA_\mu\ ,\quad P_e=\frac{\partial L}{\partial\dot{e}}=0\ ,\quad P_\chi=\frac{\partial_r L}{\partial\dot{\chi}}=0\ ,$$
$$P_\kappa=\frac{\partial L}{\partial\dot{\kappa}}=0\ ,\quad P_a=\frac{\partial_r L}{\partial\dot{\psi}^a}=-\imath\psi_a\ .\qquad (6)$$

It follows from (6) that there exist primary constraints

$$\Phi_1^{(1)}=P_e\ ,\quad \Phi_2^{(1)}=P_\chi\ ,\quad \Phi_3^{(1)}=P_\kappa\ ,\quad \Phi_{4a}^{(1)}=P_a+\imath\psi_a\ .\qquad (7)$$

Constructing the total Hamiltonian $H^{(1)}$, according to the standard procedure [16,17], we get $H^{(1)}=H+\lambda_A\Phi_A^{(1)}$, where

$$H=-\frac{e}{2}(\Pi^2-m^2+2\imath gF_{\mu\nu}\psi^\mu\psi^\nu)+\imath(\Pi_\mu\psi^\mu+m\psi^3)\chi+(\varepsilon^{\mu\nu\lambda}\Pi_\mu\psi_\nu\psi_\lambda+\frac{\imath}{2}sm)\kappa\ ,\qquad (8)$$

where $\Pi_\mu=\pi_\mu+gA_\mu$. From the consistency conditions $\dot{\Phi}^{(1)}=\{\Phi^{(1)},H^{(1)}\}=0$ we find secondary constraints $\Phi^{(2)}=0$,

$$\Phi_1^{(2)}=\Pi_\mu\psi^\mu+m\psi^3\ ,\quad \Phi_2^{(2)}=\Pi^2-m^2+2\imath gF_{\mu\nu}\psi^\mu\psi^\nu,\quad \Phi_3^{(2)}=\varepsilon^{\mu\nu\lambda}\Pi_\mu\psi_\nu\psi_\lambda+\frac{\imath}{2}sm\ ,\qquad (9)$$

and determine λ, which correspond to the primary constraints $\Phi_4^{(1)}$. No more secondary constraints arise from the consistency conditions and the Lagrangian multipliers, correspondent to the primary constraints $\Phi_i^{(1)}$, $i = 1,2,3$, remain undetermined. The Hamiltonian (8) is proportional to the constraints as one could expect in the case of a reparametrization invariant theory. One can go over from the initial set of constraints $\Phi^{(1)}, \Phi^{(2)}$ to the equivalent ones $\Phi^{(1)}, \tilde{\Phi}^{(2)}$, where $\tilde{\Phi}^{(2)} = \Phi^{(2)}\left(\psi \to \tilde{\psi} = \psi + \frac{i}{2}\Phi_4^{(1)}\right)$. The new set of constraints can be explicitly divided in a set of the first-class constraints, which are $(\Phi_i^{(1)},\ i = 1,2,3,\ \tilde{\Phi}^{(2)})$ and in a set of second-class constraints $\Phi_4^{(1)}$. Thus, we are dealing with a theory with first-class constraints.

III. DIRAC QUANTIZATION

Let us consider first the Dirac quantization, where the second-class constraints define the Dirac brackets and therefore the commutation relations, whereas, the first-class constraints, being applied to the state vectors, define physical states. For essential operators and nonzeroth commutation relations one can obtain in the case of consideration:

$$[\hat{x}^\mu, \hat{\pi}_\nu] = i\{x^\mu, \pi_\nu\}_{D(\Phi_4^{(1)})} = i\delta_\nu^\mu\,,\quad [\hat{\psi}^a, \hat{\psi}^b]_+ = i\{\psi^a, \psi^b\}_{D(\Phi_4^{(1)})} = -\frac{1}{2}\eta^{ab}\,. \tag{10}$$

It is possible to construct a realization of the commutation relations (10) in a Hilbert space $\mathcal{R}$ whose elements $\mathbf{f} \in \mathcal{R}$ are four-component columns dependent on x,

$$\mathbf{f}(x) = \begin{pmatrix} u_-(x) \\ u_+(x) \end{pmatrix}, \tag{11}$$

where $u_\mp(x)$ are two-component columns. Then

$$\hat{x}^\mu = x^\mu \mathbf{I}\,,\quad \hat{\pi}_\mu = -i\partial_\mu \mathbf{I}\,,\quad \hat{\psi}^a = \frac{i}{2}\gamma^a, \tag{12}$$

here $\mathbf{I}$ is 4x4 unit matrix and γ^a, $a = 0,1,2,3$ are γ-matrices in four-dimensions, which we select in the spinor representation $\gamma^0 = \mathrm{antidiag}(I,\ I)$, $\gamma^i = \mathrm{antidiag}(\sigma^i,\ -\sigma^i)$, $i = 1,2,3$, where σ^i are the Pauli matrices and I is 2x2 unit matrix.

According to the scheme of quantization selected, the operators of the first-class constraints have to be applied to the state vectors to define physical sector, namely, $\hat{\Phi}^{(2)}\mathbf{f}(x) =$

0, where $\hat{\Phi}^{(2)}$ are operators, which correspond to the constraints (9). There is no ambiguity in the construction of the operator $\hat{\Phi}_1^{(2)}$ according to the classical function $\Phi_1^{(2)}$. Taken into account the realization (11), (12) one can get a set of equations $\hat{\Phi}^{(2)}\mathbf{f}(x) = 0$ in the two component form,

$$[(i\partial_\mu - gA_\mu)\gamma^\mu - m\gamma^3]\mathbf{f}(x) = 0 \iff \begin{cases} [(i\partial_\mu - gA_\mu)\Gamma_+^\mu - m]u_+(x) = 0\,, \\ [(i\partial_\mu - gA_\mu)\Gamma_-^\mu - m]u_-(x) = 0\,, \end{cases} \tag{13}$$

where two sets of γ-matrices Γ_s^μ, $s = \pm$, in 2+1 dimensions are introduced,

$$\Gamma_s^0 = \sigma^3,\ \ \Gamma_s^1 = si\sigma^2,\ \ \Gamma_s^2 = -si\sigma^1\,,\ \ \Gamma_-^\mu = \Gamma_{+\mu}\,,\ \ [\Gamma_s^\mu, \Gamma_s^\nu]_+ = 2\eta^{\mu\nu}\,. \tag{14}$$

As to the construction of the operator $\hat{\Phi}_2^{(2)}$ according to the classical function $\Phi_2^{(2)}$ from the eq. (9), here we meet an ordering problem since the constraint $\Phi_2^{(2)}$ contains terms with the product of the momenta and a function of the coordinates, namely terms of the form $\pi_\mu A^\mu(x)$. For such terms we choose the symmetrized (Weyl) form of the correspondent operators, $\pi_\mu A^\mu(x) \ \ \to\ \ \frac{1}{2}[\hat{\pi}_\mu, A^\mu(\hat{x})]_+$, which provides, in particular, the consistency of two equations $\hat{\Phi}_1^{(2)}\mathbf{f} = 0$ and $\hat{\Phi}_2^{(2)}\mathbf{f} = 0$, because of in this case we have $\hat{\Phi}_2^{(2)} = (\hat{\Phi}_1^{(2)})^2$. The operator $\hat{\Phi}_3^{(2)}$ may be constructed without the ordering problem, and the equation $\hat{\Phi}_3^{(2)}\mathbf{f}(x) = 0$ can be presented in the following form

$$[\varepsilon^{\mu\nu\lambda}(i\partial_\mu - gA_\mu)\gamma_\nu\gamma_\lambda + ism]\mathbf{f}(x) = 0 \iff \begin{cases} [(i\partial_\mu - gA_\mu)\Gamma_+^\mu - sm]u_+(x) = 0\,, \\ [(i\partial_\mu - gA_\mu)\Gamma_-^\mu - sm]u_-(x) = 0\,. \end{cases} \tag{15}$$

Combining eq. (13) and (15), we get

$$[(i\partial_\mu - gA_\mu)\Gamma_s^\mu - sm]u_s(x) = 0\,,\ \ u_{-s}(x) \equiv 0\,,\ \ s = \pm\,. \tag{16}$$

To interpret the result obtained one has to calculate also the operators $\hat{M}_{\mu\nu}$ correspondent to the angular momentum tensor (5),

$$\hat{M}_{\mu\nu} = -i(x_\mu\partial_\nu - x_\nu\partial_\mu) - \frac{i}{4}\begin{pmatrix} [\Gamma_{-\mu}, \Gamma_{-\nu}] & 0 \\ 0 & [\Gamma_{+\mu}, \Gamma_{+\nu}] \end{pmatrix}.$$

Thus, in the quantum mechanics constructed, the states with $s = +$ are described by the two component wave function $u_+(x)$, which obeys the Dirac equation in 2+1 dimensions and is transformed under the Lorentz transformation as spin $+1/2$ (see for example [19]). For $s = -$ the quantization leads to the theory of 2+1 Dirac particle with spin $-1/2$ and wave function $u_-(x)$.

IV. CANONICAL QUANTIZATION

To quantize the theory canonically we have to impose as much as possible supplementary gauge conditions to the first-class constraints. In the case under consideration, it turns out to be possible to impose gauge conditions to all the first-class constraints, excluding the constraint $\tilde{\Phi}_3^{(2)}$. Thus, we are fixing the gauge freedom, which corresponds to two types of gauge transformations (3) and (4). As a result we remain only with one first-class constraint, which is a reduction of $\Phi_3^{(2)}$ to the rest of constraints and gauge conditions. It can be used to specify the physical states. All the second-class constraints form the Dirac brackets. We consider below, for simplicity, the case without an external field. The following gauge conditions $\Phi^G = 0$ are imposed: $\Phi_1^G = e+\zeta\pi_0^{-1}$, $\Phi_2^G = \chi$, $\Phi_3^G = \kappa$, $\Phi_4^G = x_0-\zeta\tau$, $\Phi_5^G = \psi^0$, where $\zeta = -\text{sign}\,\pi^0$. (The gauge $x_0 - \zeta\tau = 0$ was first proposed in [8,17] as a conjugated gauge condition to the constraint $\pi^2 - m^2 = 0$). Using the consistency condition $\dot{\Phi}^G = 0$, one can determine the Lagrangian multipliers, which correspond to the primary constraints $\Phi_i^{(1)}$, $i = 1,2,3$. To go over to a time-independent set of constraints (to use standart scheme of quantization without any modifications [17,18]) we introduce the varialble x_0', $x_0' = x_0 - \zeta\tau$, instead of x_0, without changing the rest of the variables. That is a canonical transformation in the space of all variables with the generating function $W = x_0\pi_0' + \tau|\pi_0'| + W_0$, where W_0 is the generating function of the identity transformation with respect to all variables except x^0 and π_0. The transformed Hamiltonian $H^{(1)\prime}$ is of the form

$$H^{(1)\prime} = H^{(1)} + \frac{\partial W}{\partial \tau} = \omega + \{\Phi\} \;, \quad \omega = \sqrt{\pi_d^2 + m^2} \;, \quad d = 1,2 \;, \tag{17}$$

where $\{\Phi\}$ are terms proportional to the constraints and ω is the physical Hamiltonian. All the constraints of the theory, can be presented after this canonical transformation in the following equivalent form: $K = 0$, $\phi = 0$, $T = 0$, where

$$K = (e - \omega^{-1}\;,\; P_e\;;\; \chi\;,\; P_\chi\;;\; \kappa\;,\; P_\kappa\;;\; x_0'\;,\; |\pi_0| - \omega\;;\; \psi^0\;,\; P_0)\;;$$
$$\phi = (\pi_d\psi^d + m\psi^3, P_k + i\psi_k)\;,\quad d = 1,2\;,\quad k = 1,2,3\;;\quad T = \zeta\omega[\psi_2,\psi_1] + \frac{i}{2}sm\;. \tag{18}$$

The constraints K and ϕ are of the second-class, whereas T is the first-class constraint. Besides, the set K has the so called special form [17]. In this case, if we eliminate the variables e, P_e, χ, P_χ, κ, P_κ, x_0', $|\pi_0|$, ψ^0, and P_0, using the constraints $K = 0$, the Dirac brackets with respect to all the second-class constraints (K,ϕ) reduce to ones with respect to the constraints ϕ only. Thus, on this stage, we will only consider the variables x^d, π_d, ζ, ψ^k, P_k and two sets of constraints - the second-class ones ϕ and the first-class one T. Nonzeroth Dirac brackets for the independent variables are

$$\{x^d,\pi_r\}_{D(\phi)} = \delta_r^d\;,\quad \{x^d,x^r\}_{D(\phi)} = \frac{i}{\omega^2}[\psi^d,\psi^r]\;,\quad \{x^d,\psi^r\}_{D(\phi)} = -\frac{1}{\omega^2}\psi^d\pi_r\;,$$
$$\{\psi^d,\psi^r\}_{D(\phi)} = -\frac{i}{2}(\delta_r^d - \omega^{-2}\pi_d\pi_r)\;,\quad d,r = 1,2\;. \tag{19}$$

Going over to the quantum theory, we have to calculate the commutation relations between the operators $\hat{x}^d$, $\hat{\pi}_d$, $\hat{\psi}^d$ by means of the Dirac brackets (19),

$$[\hat{x}^d,\hat{\pi}_r] = i\delta_r^d\;,\quad [\hat{x}^d,\hat{x}^r] = -\frac{1}{\hat{\omega}^2}[\hat{\psi}^d,\hat{\psi}^r]\;,$$
$$[\hat{x}^d,\hat{\psi}^r] = -\frac{i}{\hat{\omega}^2}\hat{\psi}^d\hat{\pi}_r\;,\quad [\hat{\psi}^d,\hat{\psi}^r]_+ = \frac{1}{2}(\delta_r^d - \hat{\omega}^{-2}\hat{\pi}_d\hat{\pi}_r)\;. \tag{20}$$

We assume as usual [8,17] the operator $\hat{\zeta}$ to have the eigenvalues $\zeta = \pm 1$ by analogy with the classical theory, so that $\hat{\zeta}^2 = 1$, and also we assume the equations of the second-class constraints $\hat{\phi} = 0$. Then one can realize the algebra (20) in a Hilbert space $\mathcal{R}$, whose elements $\mathbf{f} \in \mathcal{R}$ are four-component columns dependent on $\mathbf{x} = (x^d)$, $d = 1,2$,

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} f_+(\mathbf{x}) \\ f_-(\mathbf{x}) \end{pmatrix}\;, \tag{21}$$

so that $f_+(\mathbf{x})$ and $f_-(\mathbf{x})$ are two-component columns. A realization has the form

$$\hat{x}^d = x^d\mathbf{I} + \frac{1}{2\hat{\omega}^2}\epsilon^{dr}[\hat{\pi}_r\Sigma^3 - m\Sigma^r]\,,\ \ \hat{\pi}_d = -\imath\partial_d\mathbf{I}\,,$$

$$\hat{\psi}^d = \frac{1}{2}(\delta_r^d - \hat{\omega}^{-2}\hat{\pi}_d\hat{\pi}_r)\Sigma^r - \frac{1}{2\hat{\omega}^2}m\hat{\pi}_d\Sigma^3,\ \ \hat{\zeta} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{22}$$

where $\mathbf{I}$ and I are 4x4 and 2x2 unit matrices, $\Sigma^k = \text{diag}(\sigma^k, \sigma^k)$, and σ^k are Pauli matrices. The commutator $[\hat{\psi}_2, \hat{\psi}_1]$, can be calculated

$$[\hat{\psi}_2, \hat{\psi}_1] = -\frac{\imath m}{2\hat{\omega}^2}(m\Sigma^3 + \hat{\pi}_d\Sigma^d)\,,$$

so that the operator $\hat{T}$ correspondent to the first-class constraint T (see (18)) appears to be

$$\hat{T} = \frac{\imath s m}{2\hat{\omega}}\hat{\zeta}\Sigma^3\left[\hat{\zeta}\hat{\omega}\Sigma^3 + \imath\partial_1(\imath s\Sigma^2) + \imath\partial_2(-\imath s\Sigma^1) - sm\right]. \tag{23}$$

The latter operator specifies the physical states according to scheme of quantization accepted, $\hat{T}\mathbf{f} = 0$. On the other hand, the state vectors $\mathbf{f}$ have to obey the Schrödinger equation, which defines their "time" dependence, $(\imath\partial/\partial\tau - \hat{\omega})\mathbf{f} = 0$, $\hat{\omega} = \sqrt{\hat{\pi}_d^2 + m^2}$, where the quantum Hamiltonian $\hat{\omega}$ corresponds the classical one ω (17). Introducing the physical time $x^0 = \zeta\tau$ instead of the parameter τ [8,17], we can rewrite the Schrödinger equation in the following form (we can now write $\mathbf{f} = \mathbf{f}(x)$, $(x = x^0,\ \mathbf{x})$),

$$(\imath\frac{\partial}{\partial x^0} - \hat{\zeta}\hat{\omega})\mathbf{f}(x) = 0\,. \tag{24}$$

Using (24) in the eq. $\hat{T}\mathbf{f} = 0$, namely replacing there the combination $\hat{\zeta}\hat{\omega}\mathbf{f}$ by $\imath\partial_0\mathbf{f}$, one can verify that both components $f_\pm(x)$, of the state vector (21) obey one and the same equation

$$(\imath\partial_\mu\Gamma_s^\mu - sm)f_\zeta(x) = 0\,,\ \ \zeta = \pm1\,, \tag{25}$$

which is the 2+1 Dirac equation for a particle of spin $s/2$ whereas $f_\pm(x)$ can be interpreted (taken into account (24)) as positive and negative frequency solutions to the equation respectively. Substituting the realization (22) into the expression (5), we get the generators of the Lorentz transformations

$$\hat{M}_{\mu\nu} = -\imath(x_\mu\partial_\nu - x_\nu\partial_\mu) - \frac{\imath}{4}\begin{pmatrix} [\Gamma_{s\mu}, \Gamma_{s\nu}] & 0 \\ 0 & [\Gamma_{s\mu}, \Gamma_{s\nu}] \end{pmatrix}, \tag{26}$$

which have the standard form for both components $f_\zeta(x)$. Thus, a natural interpretation of the components $f_\zeta(x)$ is the following: $f_+(x)$ is the wave function of a particle with spin $s/2$ and $f_-^*(x)$ is the wave function of an antiparticle with spin $s/2$. Such an interpretation can be confirmed if we switch on an external electromagnetic field. In this case the coupling constants with the external field in the equations for $f_\zeta(x)$ are ζg, i.e. have different signs for particle and antiparticle.

V. CONCLUSION

As is known, the method of dimensional reduction [21] appears to be often useful to construct models (actions) in low dimensions using some appropriate models in higher dimensions. In fact, such kind of ideas began from the works [22]. One can also mention that the method of dimensional reduction was used to interpret masses in supersymmetric theories as components of momenta is space of higher dimensions, which are frozen in course of the reduction. It is interesting that the model of 2+1 Dirac particle proposed in the present paper can also be derived in course of a dimensional reduction from a model [12] of the Weyl particle in 3+1 dimensions. The action, Hamiltonian, and constraints of the latter model have the form

$$S = \int_0^1 \left[-\frac{1}{2e}(\dot{x}^\mu - \imath\psi^\mu\chi - \varepsilon^{\mu\nu\lambda\sigma}\kappa_\nu\psi_\lambda\psi_\sigma + \frac{\imath s}{2}\kappa^\mu)^2 - \imath\psi_\mu\dot{\psi}^\mu\right]d\tau\,,$$
$$H = -\frac{e}{2}\pi^2 + \imath\pi_\mu\psi^\mu\chi - (\varepsilon_{\mu\nu\lambda\sigma}\pi^\nu\psi^\lambda\psi^\sigma + \frac{\imath s}{2}\pi_\mu)\kappa^\mu\,,$$
$$P_e = P_\chi = P_\mu + \imath\psi_\mu = P_{\kappa_\mu} = \pi^2 = \pi_\mu\psi^\mu = 0\,,$$
$$T_\mu = \varepsilon_{\mu\nu\lambda\sigma}\pi^\nu\psi^\lambda\psi^\sigma + \frac{\imath s}{2}\pi_\mu = 0\,,\ \ \mu,\nu,\lambda,\sigma = 0,1,2,3\,,\ \ s = \pm\,. \tag{27}$$

In the gauge $\psi^0 = 0$ (or in any gauge linear in ψ^μ) one can see that, in fact, among the four constraints T_μ only one is independent. That means, that, in principle, one can use only one component of κ^μ and all others put to zero. In 3+1 dimensions this violates the explicit Lorentz invariance on the classical level [20]. However, it is possible to do this in 2+1 dimensions without any violation of the Lorentz invariance. Let us fulfil a dimensional reduction $3+1 \to 2+1$ in the Hamiltonian and constraints (27), putting $\pi_3 = m$. Besides, let us consider only one component of κ^μ, namely $\kappa^3 \equiv \kappa$, whereas $\kappa^0 = \kappa^1 = \kappa^2 = 0$; how was said, this does not reduce the number of the independent constraints and does not violate the Lorentz invariance in 2+1 dimensions. As a result of such a procedure we just obtain the expression (8) (at $A = 0$) for the Hamiltonian of the massive Dirac particle in 2+1 dimensions and all the constraints of the latter model. In the presence of an electromagnetic field one has also to put $A_3 = 0$, $\partial_3 A_\mu = 0$ to get the same result. An equivalent model one can get, puting, for example, after the dimensional reduction only $\kappa^3 = 0$.

ACKNOWLEDGMENTS

Two authors, D. M. Gitman and A. E. Gonçalves, thank Brazilian foundations FAPESP, CNPq and CAPES for support and I.V. Tyutin thank the European Community Commission which is supporting him in part under the contract INTAS-94-2317.

REFERENCES

* Permanent Address: Instituto de Física, Universidade de São Paulo, P.O. Box 20516, 01452-990 São Paulo, São Paulo, Brazil.

[1] C.Hagen, Ann. Phys. (N.Y.) **157**, 342 (1984); Phys. Rev. D **31**, 848 (1985); **31**, 2135 (1985); D. Arovas, J. Schrieffer, F. Wilczek and A. Zee, Nucl. Phys. **B251**, 117 (1985); R. Jackiw, MIT Report No. CTP 1824, 1989 (unpublished); S. Deser and A. N. Redlich, Phys. Rev. Lett. **61**, 1541 (1988); P. Gerbert, Int. J. Mod. Phys. A **6**, 173 (1991); H.O. Girrotti, M. Gomes, J.L. de Lira, R.S. Mendes, J.R.S. Nascimento, and A.J. da Silva, Phys. Rev. Lett. **69**, 2623 (1992); G. Scharf, W.F. Wreszinski, B.M. Pimentel, and J.L. Tomazelli, Ann. of Phys. **231**, 185 (1994).

[2] F. A. Berezin and M.S. Marinov, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 678 (1975) [JETP Lett. **21**, 320 (1975); Ann. Phys. (N.Y.) **104**, 336 (1975).

[3] R. Casalbuoni, Nuovo Cim. **A33**, 115 (1976).

[4] A. Barducci, R. Casalbuoni and L. Lusanna, Nuovo Cim. **A35**, 377 (1976).

[5] L. Brink, S. Deser, B. Zumino, P. Di Vechia and P. Howe, Phys. Lett. **64B**, 435 (1976).

[6] L. Brink, P. Di Vechia and P. Howe, Nucl. Phys. **B118**, 76 (1977).

[7] A.P. Balachandran, P. Salomonson, B. Skagerstam and J. Winnberg, Phys. Rev. D **15**, 2308 (1977).

[8] D.M. Gitman and I.V. Tyutin, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 188 (1990); Class. and Quantum Grav. **7**, 2131 (1990).

[9] G.V. Grigoryan and R.P. Grigoryan, Yad. Fiz. **53**, 1737 (1991) [Sov J. Nucl. Phys. **53**, 1062 (1991)].

[10] P.P. Srivastava, Nuovo Cim. Lett. **19**, 239 (1977).

[11] V.D. Gershun and V.I. Tkach, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 320 (1979).

[12] D.M. Gitman, A.E. Gonçalves and I.V. Tyutin, Pys. Rev. D**50**, 5439 (1994).

[13] M.S. Plyushchay, Mod. Phys. Lett. **A8**, 937 (1993).

[14] J.L. Cortes, M.S. Plyushchay and L. Velazquez, Phys. Lett. B **306**, 34 (1993).

[15] D.M. Gitman, A.E. Gonçalves and I.V. Tyutin, Int. J. Mod. Phhys A **10**, (1995).

[16] P.M.A. Dirac, *Lectures on Quantum Mechanics* (Yeschiva University, 1964).

[17] D.M. Gitman and I.V. Tyutin, *Quantization of Fields with Constraints* (Springer, Berlin, 1990).

[18] S.P. Gavrilov and D.M Gitman, Class. Quantum Grav. **10**, 57 (1993).

[19] R. Jackiw and V.P. Nair, Phys. Rev. D **43**, 1933 (1991).

[20] D.M. Gitman, A.E. Gonçalves and I.V. Tyutin, Preprint IFUSP/P - 1094, December (1993).

[21] M.J. Duff, B.E. Nilsson, C.N. Pope, Phys. Rep. **130**, 1 (1986); M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory* V. 1, 2. (Cambridge Univ. Press, 1988).

[22] Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin. Math. Phys. **K1**, 966 (1921); O. Klein, Z. Phys. **37**, 895 (1926).