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NEUTRINO HELICITY FLIP IN A CURVED SPACE-TIME

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In Minkowskian spaces, the helicity of a massless fermion is a conserved quantity. In principle, this property may not hold when gravitational effects are not neglected. In this context, this work proves that the helicity of a massless neutrino is not conserved in curved spaces. In order to show this fact, the time variation of the helicity in the Heisenberg picture is calculated. Also, we verify that the differential cross-section due to helicity flip of a massless neutrino in a curved space does not vanish as a result of the coupling between the spin and the curvature of space-time.

1 Introduction

Many physicists have studied the inertial effects of a fermion in a gravitational field. These works have shown the existence of a coupling[1] between the total angular momentum of a fermion and the rotation of accelerated frames. J.Anandan[2] showed that this result is more general by coupling of the spin to the curvature through the quantum interference of the de Broglie waves. As a consequence, particles with spin are sensitive to gravitation, independently of the frame, which means that, in free fall, particles with or without spin do not behave in the same way due to the absolute coupling between spin and curvature. According to this, Cai and Papini[3] have shortly shown that massless neutrinos in curved spaces can flip their helicity contrary to the expected in Minkowskian space-time. Objections of Anandan[4] have begun the discussion about this question which has not been solved yet.

Through this work, we calculate the probability of the existence of a helicity flip of a massless neutrino which interacts with a gravitational field represented by a Minkowskian asymptoticly metric. The asymptotic regions of metric are necessary in order to define, without ambiguity, the mass and the helicity of the neutrino at issue. Initially, we prove that the helicity operator is not conserved in general in the quantum formalism calculating the commutator of the helicity operator with the hamiltonian of the system in a static metric (section 2). In this paper, we calculate the differential cross-section of the neutrino helicity flip due to an external gravitational field in the semiclassical approximation (sections 3 and 4), solving the discussion mentioned previously showing that there is helicity flip of massless neutrinos in curved spaces.

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2 The time derivative of helicity

To calculate the commutator $[\hat{h}, \mathcal{H}]$ we take the helicity operator as

$$\hat{h} = \frac{1}{4k^0}\epsilon_{ijk}\sigma^{ij}p^k = \frac{1}{k^0}\hat{S}_k p^k \tag{1}$$

where the $\sigma^{ij} = \frac{i}{2}[\gamma^i, \gamma^j]$ are the commutators of Dirac matrices and the $\hat{S}_k$ are the spin operator of the neutrino (see the appendix B). In this work, the Latin indices denote the Lorentz indices of the four-vectors (tetrad $e^{(a)}_\mu$) which i, j, k are equal to $1, 2, 3$. The Greek indices are related to the space-time. The units where $c = 1$ and $\hbar = 1$ are used.

In the Heisenberg picture the dinamical evolution of the helicity operator is given by

$$i\dot{\hat{h}} = [\hat{h}, \mathcal{H}] \tag{2}$$

where $\mathcal{H}$ is the hamiltonian of a lepton in a curved space. Its calculation is given in appendix B.

Using the metric written like

$$ds^2 = d\eta^2 - f(x)dx^i dx_i \tag{3}$$

and the calculation of the spinorial connection $\Gamma^{(a)}$ (see appendix A) we see that the helicity operator is not conserved in curved spaces owing to the non-vanishing of the time derivative of the helicity operator. This derivative (see its calculus in appendix B) is

$$\dot{\hat{h}} = \frac{\Upsilon}{E}\begin{pmatrix} \sigma^1(p^3+p^2) - \sigma^3 p^1 - \sigma^2 p^1 & 0 \\ 0 & \sigma^1(p^3+p^2) - \sigma^3 p^1 - \sigma^2 p^1 \end{pmatrix} + \\ -i\frac{\partial_x \Upsilon}{2E}\begin{pmatrix} \sigma^2+\sigma^3 & 0 \\ 0 & \sigma^2+\sigma^3 \end{pmatrix} \tag{4}$$

where $\Upsilon \equiv \frac{1}{4f^{3/2}}\frac{df}{dx}$ and E is the energy of the particle.

As $(\dot{\hat{h}} \neq 0)$ hence the helicity operator is not conserved in the presence of the gravitational fields contrary to the classical argument given by Anandan[4].

3 The Interaction Lagrangian

A massless fermion in a gravitational field obeys the Dirac equation coupled minimally to the gravitation (covariant form of the Pauli-Lee-Yang equation)[10] given by equation:

$$\sigma^{(a)}(\partial_{(a)} + \Gamma_{(a)})\Psi(x) = 0 \tag{5}$$

where[5] $\Gamma_{(a)} = \frac{1}{4}e^\nu_{(a)}e^\mu_{(i)}e_{\mu(j);\nu}\sigma^{(i)}\sigma^{(j)}$ are the spinorial connections, $\sigma^{(i)}$ are the Pauli matrices, $e^{(a)}_\nu$ are the field of tetrad[6] and Ψ is the usual spinor of 2 components. Hereafter, we use Pauli matrices instead of Dirac matrices.

Actually, the equations (5) are the Euler-Lagrange equations obtained from the Lagrangian density[7] which, integrated by parts and written by means of Pauli matrices ($r \equiv det(g_{\mu\nu})$ and $\sigma^\mu \equiv e^\mu_{(a)}\sigma^{(a)}$), is given by

$$\mathcal{L} = -\frac{1}{\sqrt{-r}}\{\overline{\Psi}\sigma^\mu(\partial_\mu + \Gamma_\mu)\Psi\} \tag{6}$$

from which we get the interaction Lagrangian density:

$$\mathcal{L}_I = -\frac{1}{\sqrt{-r}}\overline{\Psi}\sigma^{(a)}\Gamma_{(a)}\Psi \tag{7}$$

In this case, we use $\sigma^\mu\Gamma_\mu = \sigma^{(a)}\Gamma_{(a)}$. We get the equation (5) applying the variational principle to (6), proving that this Lagrangian density is correct.

This Lagrangian density represents the coupling between the curvature of space-time and the spin of neutrino. This fact can be seen of a better way through of square of massless Dirac equation (5) in formalism of Dirac matrices. Doing it, we get the following result:

$$\{\nabla^\mu\nabla_\mu - \frac{1}{8}R_{\mu\nu\rho\lambda}\sigma^{\mu\nu}\sigma^{\rho\lambda}\}\Psi = 0 \tag{8}$$

where $\nabla^\mu \equiv \partial^\mu + \Gamma^\mu$, $R_{\mu\nu\rho\lambda}$ is the curvature of space-time and $\sigma^{\mu\nu}$ is given by $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{(a)},\gamma^{(b)}]e^\mu_{(a)}e^\nu_{(b)}$ so that the spin angular momentum $S^{(ab)}$ is related to the spin $S^{(a)}$ of neutrino by $S_{(i)} = \frac{1}{2}\epsilon_{(ijk)}S^{(jk)}$.

In order to calculate this Lagrangian density in the context of the semi-classical gravitation we consider a static physical case where the adiabatic hypothesis is true in which the gravitational interaction can be switched off so that we can define asymptotically the usual states which define the neutrino. For this we demand that the space-time is asymptotically flat with the metric:

$$ds^2 = d\eta^2 - f(x)(dx^2 + dy^2 + dz^2) \tag{9}$$

so that $\lim_{x\to L} f(x) = 1$ and x is a specific spatial coordinate.

If we want to calculate the interaction Lagrangian, we have to get the spinorial connections calculated in the appendix A. Using these results we get finally

the following interaction Lagrangian density:

$$\begin{aligned}
\mathcal{L}_I &= \frac{-1}{\sqrt{-r}}\overline{\Psi}\sigma^{(a)}\Gamma_{(a)}\Psi \\
\mathcal{L}_I &= \frac{i}{4\sqrt{-r}f^{3/2}}\frac{df}{dx}\overline{\Psi}(\sigma^{(2)}\sigma^{(3)} - \sigma^{(3)}\sigma^{(2)})\Psi \\
\mathcal{L}_I &= \frac{-1}{2f^3}\frac{df}{dx}\overline{\Psi}\sigma^{(1)}\Psi
\end{aligned} \tag{10}$$

After that, it is straightforward to build the S-Matrix in the tree-approximation. Therefore using the fact that there are no derivative couplings we can write the action in this approximation as

$$\begin{aligned}
S_1 &= -i\int d^4\xi\sqrt{-r}\mathcal{L}_I \\
S_1 &= i\int d^4\xi\frac{1}{2f^{3/2}}\frac{df}{dx}\overline{\Psi}(\xi)\sigma^{(1)}\Psi(\xi)
\end{aligned} \tag{11}$$

where $\xi \equiv (t, x, y, z)$ is the coordenate of the space-time.

4 Helicity Flip

Consider the following physical situation: a left-handed neutrino is emitted from a source placed in a region where no gravitational field exists. After a short time T it crosses a region of dimension L which contains a gravitational field and reaches a detector situated out of this region where there is no gravitational field. Taking these considerations, we will calculate the differential cross-section due to the neutrino helicity flip. This flip occurs because in the interaction Lagrangian density (7) the helicity couples to the curvature of space-time causing a non-vanishing probability of flipping the neutrino helicity.

In order to get this flip, we consider initially the metric bellow that satisfies the adiabatic hypothesis mentioned previously

$$ds^2 = d\eta^2 - \frac{1}{[1 - \frac{\Lambda(L^2 - x^2)}{L^2}]^2}(dx^2 + dy^2 + dz^2) \tag{12}$$

so that $0 < \Lambda << 1$ at region $|x| \leq L$, $|y| \leq L$ and $|z| \leq L$ and $\Lambda = 0$ at the rest of space-time.

Using this metric, the element of the S-matrix (11) to first order in Λ is

$$S_1 = -2i\frac{\Lambda}{L^2}\int d^4\xi \overline{\Psi} x \sigma^{(1)} \Psi \tag{13}$$

This element of the matrix S_1 is the leading term of the Scattering-Matrix due to this flip because Λ is a very small parameter and hence other terms become negligible. Thus Λ simulates a coupling constant for the gravitational interaction.

The incoming state $|in\rangle$, a left-handed neutrino with an energy $E = |\vec{p}|$, is given[8] by

$$|e\rangle \equiv |in\rangle = \frac{1}{\sqrt{2E}} u_p(p) \exp(-ipx) \tag{14}$$

and the outcoming state $|out\rangle$, a right-handed neutrino with energy $E' = |\vec{p}'|$:

$$|d\rangle \equiv |out\rangle = \frac{1}{\sqrt{2E'}} v_{p'}(p') \exp(-ip'x) \tag{15}$$

where $v_{p'} = -\sigma^{(2)} u^*_{p'}$, $u^\dagger_p u_p = 2E$ and $v^\dagger_p v_p = 2E$.

The state $|out\rangle$ represents a right-handed neutrino but not an antineutrino due to the fact that the hamiltonian density of a massless neutrino, (see equation (39)) in a curved space in the bispinor formalism, commutes with the leptonic

number operator $L \equiv \int d^3x : \Psi^\dagger(x)\Psi(x) :$ which implies the conservation of the leptonic number[9]. Therefore, by consistency, if a change of helicity happens without changing the leptonic number then a flip of helicity means that a left-handed neutrino flipped to a right-handed neutrino and not to a antineutrino.

The amplitude of probability of this flip to first order in Λ is obtained combining the equations (13) to (15) giving

$$\langle d|S_1|e\rangle = 2\frac{\Lambda}{L^2}\int_{|x|<L} d^3\vec{x} \int dt \langle d|\overline{\Psi}\sigma^{(1)} x \Psi|e\rangle \tag{16}$$

so that the wave equations are normalized in a box of volume V. Integrating this amplitude in a finite range of time T we get

$$\langle d|S_1|e\rangle = \frac{\Lambda}{\sqrt{E'E}VL^2}\delta_T(E' - E)\int d^3\vec{x} x e^{-i(\vec{p}' - \vec{p})\cdot\vec{x}}(v^\dagger_{p'}\sigma^{(1)} u_p) \tag{17}$$

where $\delta(E' - E) = \frac{1}{2\pi}\lim_{T\to\infty}\delta_T(E' - E)$.

Taking the limit $T \to \infty$ above we obtain the following flip probability per unit of time:

$$w = 2\pi\Lambda^2 \frac{\delta(E' - E)}{E'EV^2L^4}|M|^2 \tag{18}$$

so that $|M| \equiv \left|\int d^3\vec{x} x e^{-i\vec{q}\cdot\vec{x}}(v^\dagger_{p'}\sigma^{(1)} u_p)\right|$.

After these calculations, the differential cross-section for this process may be obtained by multiplying w by the final states density $\frac{V d^3\vec{p}'}{2\pi}$ and dividing by the incident flux $\frac{1}{V}$ we yields

$$\frac{d\sigma}{d\Omega'} = \frac{2\pi\Lambda^2}{E'EL^4}\delta(E' - E)|M|^2 E'^2 dE' \tag{19}$$

Without loss of generality, we can set $p_y = p_z = 0$ and hence $E = p_x$ as initial conditions. This choice simplifies the calculation of quantity $|M|^2$. Using the value of $|M|^2$ obtained in the appendix C and integrating over the energy E the above equation becomes

$$\frac{d\sigma}{d\Omega'} = \frac{256\pi\Lambda^2 E}{L^4}(\frac{\sin^2 q_y L}{q_y^2})(\frac{\sin^2 q_z L}{q_z^2})(\frac{L^2 q_x^2 \cos^2 q_x L - L q_x \sin 2q_x L + \sin^2 q_x L}{q_x^3 L}) \tag{20}$$

where $\vec{q}$ is the transfered momentum of particle and (Θ', φ') are the polar angles of momentum $\vec{p}'$ in the rest frame of the detector.

Introducing $q_x = E(1 - \sin\Theta' \cos\varphi')$, $q_y = E \sin\Theta' \sin\varphi'$ and $q_z = E\cos\Theta'$ in this expression we can rewrite it and therefore obtain finally the differential cross-section concerning to the flip as being ($c \neq 1$ and $\hbar \neq 1$)

$$\begin{aligned}\frac{d\sigma}{d\Omega'} &= \frac{256\pi\Lambda^2 L^2}{\Delta^6}\frac{\sin^2\Delta \sin\Theta' \sin\varphi'}{\sin^2\Theta' \sin^2\varphi'}\frac{\sin^2\Delta\cos\Theta'}{\cos^2\Theta'(1-\sin\Theta'\cos\varphi')} \\ & [\frac{\sin^2\Delta(1-\sin\Theta'\cos\varphi')}{(1-\sin\Theta'\cos\varphi')^2} + \Delta^2\cos^2\Delta(1-\sin\Theta'\cos\varphi') - \\ & \frac{\Delta(\sin 2\Delta(1-\sin\Theta'\cos\varphi')}{1-\sin\Theta'\cos\varphi'}]\end{aligned} \tag{21}$$

where $\Delta \equiv \frac{EL}{\hbar c}$.

This cross-section is a finite and non-vanishing quantity. We can prove this assertion integrating numerically the above expression using the values: $E = 10^{-4}eV$, $\Lambda = 0.1$ and $L = 0.988$cm. Doing this, we obtain the following non-null cross-section:

$$\sigma = 2.76cm^2 \tag{22}$$

This cross-section is large when compared with the dimension L^2 of region of interaction. Integrating numerically with other parameters we obtain values finite and different of zero and, in this way, this calculation exhibits that the

cross-section due to the flip of helicity does not vanish and grows with the energy. Though this probability is non-null, it vanishes when we remove ($\Lambda \to 0$) the gravitational field as expected. According to this fact, we do not observe right-handed neutrino in our experiences because as the gravitational effects are negligible then the coupling to this neutrino is very small and so the right-handed neutrino is not observed.

5 Conclusion

We show that the cross-section due to helicity flip of massless neuthinos in curved spaces can be different of zero and finite. In order words, there is a non-vanishing probability in order that a left-handed neutrino flips to a right-handed one and not to a antineutrino since this transition obeys the conservation of lepton number. Looking at the structure of equation (8), we can infer that this flip exists as consequence of the coupling between the spin of the neutrino and the curvature of space-time. Therefore, in general, the helicity of massless neutrinos can be changed owing to the presence of weak gravitational fields in contrary to expected in Minkowskian space-time, thus the issue among Cai and Papini and Anandan about this flip is resolved. According to this conclusion, this flip is other evidence of that a new physics beyond of Standard Model need be found when gravitational effects are important in quantum context.

Appendix A

The spinorial connection[5] $\Gamma^{(a)}$, in the context of Pauli matrices $\sigma^{(i)}$, can be written as

$$\Gamma_{(a)} = \frac{1}{4}\Omega_{a(i)(j)}\sigma^{(i)}\sigma^{(j)} \tag{23}$$

so that $\Omega_{a(i)(j)} = e^{\nu}_{(a)}e^{\mu}_{(i)}e_{\mu(j);\nu}$

Accordingly, we have to get covariant derivative of tetrad field $(e_{\mu(j);\nu})$ corresponding to the metric $g_{\mu\nu}$ extracted from (9). Therefore, the Levi-Cevita connection must be calculated and their values obtained from this metric are

$$\begin{aligned}
\Xi^0_{\nu\mu} &= 0 \\
\Xi^1_{\nu\mu} &= -\frac{1}{2f}\frac{df}{dx}(2\delta_{\nu 1}\delta_{\mu 1} - \delta_{\nu 2}\delta_{\mu 2} - \delta_{\nu 3}\delta_{\mu 3}) \\
\Xi^2_{\nu\mu} &= -\frac{1}{2f}\frac{df}{dx}(\delta_{\nu 1}\delta_{\mu 2} + \delta_{\nu 2}\delta_{\mu 1}) \\
\Xi^3_{\nu\mu} &= -\frac{1}{2f}\frac{df}{dx}(\delta_{\nu 1}\delta_{\mu 3} + \delta_{\nu 3}\delta_{\mu 1})
\end{aligned} \tag{24}$$

Putting these results in the equation (23) we obtain the spinorial connections:

$$\Gamma_0 = 0 \tag{25}$$

$$\Gamma_1 = 0 \tag{26}$$

$$\Gamma_2 = \frac{-i}{4f^{3/2}}\frac{df}{dx}\sigma^{(3)} \tag{27}$$

$$\Gamma_3 = \frac{i}{4f^{3/2}}\frac{df}{dx}\sigma^{(2)} \tag{28}$$

These connections refer to a neutrino immersed in a curved space with a metric (9).

The spinorial connection[6] $\Gamma^{(a)}$, in the context of Dirac matrices $\sigma^{(i)}$, can be written as

$$\Gamma_{(a)} = \frac{1}{4}\Omega_{(a)(b)(c)}\gamma^{(b)}\gamma^{(c)} \tag{29}$$

so that $\Omega_{(a)(b)(c)} = e^{\nu}_{(a)}e^{\mu}_{(b)}e_{\mu(c);\nu}$

Joining the results (24) to the equation (29) we yield the spinorial connection:

$$\Gamma_0 = 0 \tag{30}$$

$$\Gamma_1 = 0 \tag{31}$$

$$\Gamma_2 = \frac{i}{4f^{3/2}}\frac{df}{dx}\begin{pmatrix} \sigma^{(3)} & 0 \\ 0 & \sigma^{(3)} \end{pmatrix} \tag{32}$$

$$\Gamma_3 = \frac{-i}{4f^{3/2}}\frac{df}{dx}\begin{pmatrix} \sigma^{(2)} & 0 \\ 0 & \sigma^{(2)} \end{pmatrix} \tag{33}$$

(34)

Appendix B

The spin vector of Pauli-Lubanski[11] is given by

$$W_\mu = \frac{1}{2}\epsilon_{\mu\rho\nu\alpha}J^{\rho\alpha}p^\nu \tag{35}$$

with $J^{\mu\nu}$ as infinitesimal generator of Lorentz transformation.

Defining the helicity h as $W_\mu = hp_\mu$ and taking t_μ so that $p_\mu t^\mu = 1$ we get

$$\begin{aligned}
h &= W_\mu t^\mu \\
h &= = \frac{1}{4}\epsilon_{\mu\rho\nu\alpha}\sigma^{\rho\alpha}p^\nu t^\mu
\end{aligned} \tag{36}$$

In order to simplify the previous expression we choose $t_\mu = (\frac{1}{k_0}, 0), k_0 = E$ (energy of neutrino) and, taking the spin $\hat{S}_i = \frac{1}{2}\epsilon_{ijk}\sigma^{ij}, \sigma \equiv \frac{i}{2}[\gamma^i, \gamma^j]$, we get the usual expression to the helicity operator of neutrino as

$$\hat{h} = \frac{1}{|\vec{p}|}\hat{S}_k p^k \tag{37}$$

The hamiltonian operator $\mathcal{H}$ can be obtained from massless Dirac equation written conveniently as

$$\partial_0 \Psi = -i\alpha^j \partial_j \Psi + \alpha^{(a)}\Gamma_{(a)}\Psi \tag{38}$$

and thus

$$\mathcal{H} = \alpha^j p_j + \alpha^{(a)}\Gamma_{(a)} \tag{39}$$

Putting all these results together, the time variation of helicity can be achieved as

$$\begin{aligned} i\dot{\hat{h}} &= [\hat{h}, \mathcal{H}] \\ i\dot{\hat{h}} &= \frac{1}{4E}\epsilon_{ijk}[\sigma^{ij}p^k, \alpha^{(a)}\Gamma_{(a)}] \end{aligned} \tag{40}$$

Using the results of appendix A and employing the properties of the Dirac matrices, we get the result:

$$\begin{aligned} i\dot{\hat{h}} = \frac{i\Upsilon}{E} &\begin{pmatrix} \sigma^1(p^3+p^2) - \sigma^3 p^1 - \sigma^2 p^1 & 0 \\ 0 & \sigma^1(p^3+p^2) - \sigma^3 p^1 - \sigma^2 p^1 \end{pmatrix} + \\ + \frac{\partial_x \Upsilon}{2E} &\begin{pmatrix} \sigma^2 + \sigma^3 & 0 \\ 0 & \sigma^2 + \sigma^3 \end{pmatrix} \end{aligned} \tag{41}$$

Appendix C

The spinors u_p and v'_p normalized as $u_p^\dagger u_p = 2E$ and $v_p^\dagger v_p = 2E$ are given by

$$u_p = \sqrt{E + p_z}\begin{pmatrix} \frac{p_z - E}{p_x + ip_y} \\ 1 \end{pmatrix} \tag{42}$$

$$v_{p'} = -\sigma^{(2)}u^*_{p'} = i\sqrt{E' + p'_z}\begin{pmatrix} 1 \\ \frac{E' - p'_z}{p'_x - ip'_y} \end{pmatrix} \tag{43}$$

Using these spinors in expression $|M|^2$:

$$|M| \equiv \left| \int_{|\vec{x}|<L} d^3\vec{x} x e^{-i\vec{q}\cdot\vec{x}}(v^\dagger_{p'}\sigma^{(1)}u_p) \right| \tag{44}$$

and setting $p_y = p_z = 0$ and $E = p_x \neq 0$ we get

$$|v^\dagger_{p'}\sigma^{(1)}u_p|^2 = 2E(E' - p'_z) \tag{45}$$

Integrating over the spatial coordinates we arrive at

$$|M|^2 = 2E(E' - p'_z)(\frac{4\sin^2 q_y}{q_y^2})(\frac{4\sin^2 q_z}{q_z^2})(\frac{L^2 q_x^2 \cos^2 q_x L - Lq_x \sin 2q_x L + \sin^2 q_x L}{q_x^4}) \tag{46}$$

where $q_x \equiv p_x - p'_x$, $q_y = p'_y$, $q_z = p'_z$ and (Θ', φ') are the polar angles of momentum $\vec{p}'$ in the rest frame of the detector.

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