Dynamics of the Kicked Logistic Map

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Abstract - The modulation of the logistic map by a sequence of periodic kicks brings up a three-parameter kicked logistic map (klm) with new distinct dynamic features. Thus, its parameter space structure exhibits highly interleaved sets with different attractors and complex basins of attraction are created. Additional roots to chaos, and abrupt attractor changes are identified in the parameter space. The observed intermittency route to chaos is distinct to those typical of spatially discontinuous unidimemsional maps, with a characteristic power-law dependence of the average laminar length on the control parameters. This behaviour is verified for both the internal and the transfer crisis induced intermittency.

1. INTRODUCTION

Periodic and chaotic non-linear dissipative systems can show peculiar response behaviour to applied forces of various types [1,2,3,4]. Thus, for relevant control parameter sets of these systems, striking changes in their dynamics are induced even for weak perturbations [5,6]. Some of these alterations identified in the modified bifurcation diagrams are: control of chaos, new roads to chaos, abrupt attractor changes, and new types of crisis. These features have been found in numerical experiments performed with several dynamical systems modelled either with differential or difference equations modulated by random as well as periodic forcing. Furthermore, these properties have also been observed experimentally in several dissipative dynamical systems in many scientific disciplines [2,3,4].

In dynamics, problems described by nonlinear differential equations are often reduced to discrete maps by considering their Poincaré sections. In particular, unidimensional maps have been the object of increasing interest, both due to their intrinsic mathematical richness, and to the large number of dynamical systems that experimentally display transitions into chaos through the universal bifurcation scenario [2,3,7]. The most known, and extensively studied of these maps is the logistic map.

In the last years, studies of the effect of forcing on systems simulated by the logistic map have been reported [8-11]. Thus, a new type of crisis, associated to the hysteresis followed from the coexistence of two attractors, was found in this map with a periodic modulation [11]. The effect of additive and multiplicative noise on the first bifurcations of the logistic model was analytically and numerically studied [10]. Periodic entrainment of chaotic trajectories was discussed in [9]. For additive periodic forcing two noncomplememtary attractors were found [8]. However, since most of these interesting results have been obtained by changing single parameters, this subject is still far from being fully explored. In particular, it seems worthwhile to look for new dynamic properties regarding relevant parameter sets for which this system behaves similarly concerning one or several properties of interest such as, for example, periodicity of attractors and basins of attraction. Thus, new fundamental dynamic properties may be recognised and interpreted performing the analysis in the parameter space, as it has been recently done for other systems [12,13].

The aim of the present paper is to numerically study the main topological changes of the logistic map attractors, caused by a sequence of constant kicks. Examples of structures formed by the domain of the chaotic, and periodic attractors in the space of the control parameters (the kick amplitude and the control parameter of the logistic map) are presented for given kick periods. These pictures reveal highly interleaved regions corresponding to parameter sets for which finite attractors exist. Another remarkable alteration induced by the kicks appears in the intermittency route to chaos (characterised by dynamical intermittent changes between laminar periods and chaotic bursts). For the kicked logistic map (klm), i e, the perturbed map considered in this paper, besides the interior crises [14], tranfer crises [11] has also been observed. For this transfer crisis, it is numerically obtained a power dependence of the average length of the laminar phase on the control parameters, in contrast to the spatially discontinuous maps [17,18,19]. Other dynamic properties of this kicked logistic map, as suppression of chaos, periodic entrainment, routes to chaos, and bifurcation diagrams were considered elsewhere [15,16].

Section 2 presents attractor regions in the parameter space of the kicked logistic map for different kick periods. Section 3 presents examples of interior and transfer crises and describes numerical experiments