UNIVERSIDADE DE SÃO PAULO

PUBLICAÇÕES

INSTITUTO DE FÍSICA
CAIXA POSTAL 66318
05389-970 SÃO PAULO - SP
BRASIL

IFUSP/P-1173

PSEUDOCLASSICAL THEORY OF RELATIVISTIC SPINNING PARTICLE

Dmitri M. Gitman
Instituto de Física, Universidade de São Paulo

Setembro/1995

Pseudoclassical Theory of Relativistic Spinning Particle

Dmitri M. Gitman

Instituto de Física, Universidade de São Paulo

Caixa Postal 66318-CEP o5389-970-São Paulo, S.P., Brasil

(September 19, 1995)

Abstract

A short survey of the pseudoclassical theory of spin one half relativistic particle in $3+1$ dimensions is given. In particular, the canonical and path integral quantizations of the theory are presented in details. Different representations of the Dirac propagator in terms of path integral are derived. Introduction of an anomalous magnetic moment in the model is considered. Massless case, theory of the Weyl particles, and reduction of the model to the $2+1$-dimensional case are discussed.

Typeset using REVTEX

I. INTRODUCTION

The systematic utilization in theoretical physics of the algebra and analysis of anticommuting variables, initiated to a great extend by Felix Berezin [1], has undoubtedly exerted considerable influence on its development. Among numerous examples of the application of these methods a visible place takes the so called pseudoclassical theory of relativistic spin one half particle. One of the originator of this theory was also F.Berezin.

Classical and pseudoclassical models of relativistic particles and their quantization where discussed lately in different contexts. One of the reason is on these simple examples to learn how to solve some typical problems which arise also in string theory, gravity and so on. On the other hand, it is a important question itself whether there exist classical models for any relativistic particles (with any spin), whose quantization reproduces, in a sense, the corresponding field theory or one particle sector of the corresponding quantum field theory.

In the frame of this paper I would like to give a short survey of the pseudoclassical theory of spin one half relativistic particle and recent development in this direction, since that can serve as convincing demonstration of fruitfulness of the methods and ideas introduced by F.Berezin.

II. ACTION OF RELATIVISTIC SPIN ONE HALF PARTICLE IN EXTERNAL ELECTROMAGNETIC FIELD

First an action of spin one half relativistic particle in $3+1$ dimensions, with spinning degrees of freedom, describing by grassmannian (odd) variables, was proposed by Berezin and Marinov [2] and just after that was discussed and investigated in papers [3–7]. In the most symmetric form the action of spinning particle in an external electromagnetic field can be written as [4,6,7]

$$S=\int_0^1\left[-\frac{\dot{x}^2}{2e}-e\frac{m^2}{2}-g\dot{x}^\alpha A_\alpha+igeF_{\alpha\beta}\psi^\alpha\psi^\beta+i\left(\frac{\dot{x}_\alpha\psi^\alpha}{e}-m\psi^5\right)\chi-i\psi_n\dot{\psi}^n\right]d\tau\,, \quad (2.1)$$

where x^α, e are ordinary (bosonic or even) variables and ψ^n, χ are odd variables dependent on a parameter τ, $\tau \in [0,1]$, the latter plays the role of the time in this theory, $A_\alpha(x)$ is an external electromagnetic field potential, $F_{\alpha\beta}(x)$ is the Maxwell strength tensor, and g the electrical charge. Greek indices run over $\overline{0,3}$ and Latin indices n,m run over $\overline{0,3},5$. The metric tensors: $\eta_{\alpha\beta} = \mathrm{diag}(1 -1 -1 -1)$ and $\eta_{mn} = \mathrm{diag}(1 -1 -1 -1 -1)$. The spinning degrees of freedom in such a model are described by odd variables ψ^n, that's why the model is called pseudoclassical. There are two type of gauge transformations in the theory with the action (2.1): reparametrizations,

$$\delta x = \dot{x}\xi\,, \quad \delta e = \frac{d}{d\tau}(e\xi)\,, \quad \delta\psi^n = \dot{\psi}^n\xi\,, \quad \delta\chi = \frac{d}{d\tau}(\chi\xi)\,, \tag{2.2}$$

and supertransformations,

$$\delta x^\alpha = i\psi^\alpha\epsilon\,, \quad \delta e = i\chi\epsilon\,, \quad \delta\chi = \dot{\epsilon}\,, \quad \delta\psi^\alpha = \frac{1}{2e}(\dot{x}^\alpha - i\chi\psi^\alpha)\epsilon\,, \; \delta\psi^5 = \frac{m}{2}\epsilon\,, \tag{2.3}$$

where ξ is even and ϵ is odd τ-dependent parameters. The lagrangian equations of motion have the form:

$$\begin{aligned}
&\frac{d}{d\tau}\left[\frac{1}{e}(\dot{x}_\alpha - i\psi_\alpha\chi)\right] + g\dot{x}^\beta F_{\beta\alpha} + iegF_{\beta\gamma,\alpha}\psi^\beta\psi^\gamma = 0\,,\\
&\frac{1}{2e^2}(\dot{x}^\alpha - i\psi^\alpha\chi)^2 - \frac{m^2}{2} + igF_{\alpha\beta}\psi^\alpha\psi^\beta = 0\,, \;\; \dot{x}_\alpha\psi^\alpha - me\psi^5 = 0\,,\\
&2\dot{\psi}_\alpha + 2egF_{\beta\alpha}\psi^\beta - \frac{\dot{x}_\alpha}{e}\chi = 0\,, \;\; 2\dot{\psi}^5 - m\chi = 0\,.
\end{aligned} \tag{2.4}$$

Calculating the total angular momentum, corresponding to the action (2.1), we get

$$\begin{aligned}
&M_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu}\,,\\
&L_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu\,, \;\; S_{\mu\nu} = i(\psi_\mu\psi_\nu - \psi_\nu\psi_\mu)\,.
\end{aligned}$$

The spatial part of $S_{\mu\nu}$ forms a tree-dimensional spin vector $\mathbf{s} = (s_k)$

$$s_k = \frac{1}{2}\epsilon_{kjl}S_{ij} = i\epsilon_{kjl}\psi_l\psi_j\,, \tag{2.5}$$

where ϵ_{kjl} is tree-dimensional Levi-Civita symbol. To demonstrate that this vector really behaves like a spin one can use, for example, the nonrelativistic approximation $dx^i/dx^0 \ll 1$

and consider the case of a magnetic field only. In such a case $F_{0i} = 0$, and $F_{ij} = -\epsilon_{ijk}B_k$, where B_k are components of a magnetic field $\mathbf{B}$. Selecting the gauge $\chi = 0$ and $x^0 = \tau$, the latter leads effectively to $e = 1/m$ in the approximation in question, one can find from the equations (2.4)

$$\frac{d\mathbf{s}}{dt} = \frac{g}{m}[\mathbf{s}\times\mathbf{B}]\,, \;\; m\frac{d^2\mathbf{x}}{dt^2} = g\left[\frac{d\mathbf{x}}{dt}\times\mathbf{B}\right] + \frac{g}{m}\nabla(\mathbf{s}\cdot\mathbf{B})\,. \tag{2.6}$$

The equations (2.6) describe [8] a nonrelativistic motion of a particle with total spin momentum $\mathbf{s}$ and total magnetic momentum $g\mathbf{s}/m$, what confirms the interpretation of the action (2.1).

Going over to the hamiltonian formulation, we introduce the canonical momenta

$$\begin{aligned}
&p_\alpha = \frac{\partial L}{\partial\dot{x}^\alpha} = -\frac{1}{e}(\dot{x}_\alpha - i\psi_\alpha\chi) - gA_\alpha\,,\\
&P_e = \frac{\partial L}{\partial\dot{e}} = 0, \quad P_\chi = \frac{\partial_r L}{\partial\dot{\chi}} = 0, \quad P_n = \frac{\partial_r L}{\partial\dot{\psi}^n} = -i\psi_n\,.
\end{aligned} \tag{2.7}$$

It follows from the equation (2.7) that there exist primary constraints $\Phi^{(1)} = 0$,

$$\Phi^{(1)} = \left(\Phi_1^{(1)} = P_\chi, \;\; \Phi_2^{(1)} = P_e, \;\; \Phi_{3n}^{(1)} = P_n + i\psi_n\right)\,. \tag{2.8}$$

We construct the Hamiltonian $H^{(1)}$, according to the standard procedure [9,10] (we use the notations of the book [10]),

$$H^{(1)} = H + \lambda_a\Phi_a^{(1)}, \;\; H = -\frac{e}{2}\left(\mathcal{P}^2 + 2igF_{\alpha\beta}\psi^\alpha\psi^\beta - m^2\right) - i\left(\mathcal{P}_\alpha\psi^\alpha - m\psi^5\right)\chi\,,$$

where $\mathcal{P} = -p_\mu - gA_\mu(x)$. From the conditions of the conservation of the primary constraints $\Phi_{1,2}^{(1)}$ in time τ, $\dot{\Phi}_{1,2}^{(1)} = \left\{\Phi_{1,2}^{(1)}, H^{(1)}\right\} = 0$, we find the secondary constraints $\Phi^{(2)} = 0$,

$$\Phi^2 = \left(\Phi_1^{(2)} = \mathcal{P}_\alpha\psi^\alpha - m\psi^5 = 0, \;\; \Phi_2^{(2)} = \mathcal{P}^2 + 2igF_{\alpha\beta}\psi^\alpha\psi^\beta - m^2 = 0\right)\,, \tag{2.9}$$

and the same conditions for the constraints $\Phi_{3n}^{(1)}$ give equations for the determination of λ_{3n}. Thus, the Hamiltonian H appears to be proportional to constraints, as one can expect in the case of a reparametrization invariant theory, $H = i\chi\Phi_1^{(2)} - \frac{e}{2}\Phi_2^{(2)}$. No more secondary constraints arise from the Dirac procedure, and the Lagrange's multipliers λ_1 and λ_2 remain undetermined, in accordance with the fact that the number of gauge transformations parameters equals two for the theory in question [10]. One can go over from the initial set of constraints $\left(\Phi^{(1)},\Phi^{(2)}\right)$ to the equivalent one $\left(\Phi^{(1)},T\right)$, where $T=\Phi^{(2)}-(i/2)\Phi^{(1)}_{3n}\partial_r\Phi^2/\partial\psi_n$. The new set of constraints can be explicitly divided in a set of first-class constraints, which is $\left(\Phi^{(1)}_{1,2},\ T\right)$ and in a set of second-class constraints, which is $\Phi^{(1)}_{3n}$.

III. CANONICAL QUANTIZATION

Consider here the canonical operator quantization of the theory in question. Because of it contains first-class constraints, well known problems arise in course of quantization. Moreover, in this particular case, due to the reparametrization invariance, an additional problem appears, namely, Hamiltonian equals zero on the constraint surface. Usually they try to avoid this difficulty using the so called Dirac method of quantization [9], in which one considers first-class constraints in sense of restrictions on the state vectors. In the problem in question the Dirac wave equation arises namely in such a way [3,6]. Unfortunately, this scheme of quantization creates many questions, e.g. with Hilbert space construction, what is Schrödinger equation and so on. A consistent, but more complicated technically way is to work in the physical sector, namely, first, on the classical level, one has to impose gauge conditions to reduce the theory to one with second-class constraints only, and then quantize by means of Dirac brackets. We present here this way of quantization [10,11], which gives the Dirac equation as Schrödinger one. For simplicity, we restrict ourselves with the free particle case.

We fix a gauge, imposing preliminary three additional conditions

$$\Phi^G_1=x_0-\zeta\tau=0\,,\quad \Phi^G_2=\chi=0\,,\quad \Phi^G_3=\psi^5=0\,,$$

where $\zeta=-\text{sign}\,p_0$. The gauge $x_0-\zeta\tau=0$ was first proposed in [11] as a conjugate gauge condition to the constraint $p^2=m^2$ in the case of scalar and spinning particles. In contrast with the gauge $x_0=\tau$, which, together with the continuous reparametrization symmetry,

breaks the time reflection symmetry and therefore fixes the variables ζ, the former gauge breaks only the continuous symmetry, so that the variable ζ remains in the theory to describe states of particles $\zeta=+1$ and states of antiparticles $\zeta=-1$. Namely this circumstance allowed one to get Klein-Gordon and Dirac equations as Schrödinger ones in course of the canonical quantization. From the condition of the consistency $\dot\Phi^G=0$ we find an additional condition

$$\Phi^G_4=e-|p_0|^{-1}\,.$$

The total set of constraints $\Phi=(\Phi^1,\Phi^2,\Phi^G)$ is already of second-class one. However, now we are dealing with constraints, which depend on time. In this case the canonical way of quantization by means of Dirac brackets, generally speaking, has to be modified [10,12]. To avoid this new problem one can go over to a time-independent set of constraints, making the canonical transformation $x_0'=x_0-\zeta\,\tau,\quad x'^i=x^i,\quad p'_\mu=p_\mu$, with the generating function, having the form $W=x^\mu p'_\mu+\tau|p'_0|+W_0$, where W_0 is the generating function of the identity transformation with respect to all the variables except x_0,p_0. We change, in fact, only the coordinate x_0, and therefore the primes on the other variables are henceforth omitted. The transformed Hamiltonian $H'^{(1)}$ is of the form $H'^{(1)}=H^{(1)}+\partial W/\partial\tau=H+\{\Phi\}$, where H is the physical Hamiltonian,

$$H=\omega=(\mathbf{p}^2+m^2)^{1/2}\,,\quad \mathbf{p}=(p_k)\,, \tag{3.1}$$

and $\{\Phi\}$ are terms proportional to constraints Φ. We present the constraints Φ in an equivalent form, dividing them into two groups K and ϕ, each of which is a set of second-class constraints

$$K=\left(e-\omega^{-1},\ P_e,\ \chi,\ P_\chi,\ \psi^5,\ P_5,\ x_0',\ |p_0|-\omega\right),\quad \phi=(P_\mu+i\psi_\mu,\ p\psi)\ . \tag{3.2}$$

In (3.2) and below $p_0=-\zeta\omega$. Next we eliminate the variables $e,P_e,\chi,P_\chi,\psi^5,P_5,x_0'$ and $|p_0|$ from the consideration, using the constraints K. These constraints have a special form [10], that means that for the rest of variables $\eta=(x^k,p_k,\zeta,\psi^\mu,P_\mu)$ the Dirac brackets with respect to all the constraints Φ reduce to ones with respect to the constraints ϕ only. Calculating the Dirac brackets between the variables η, we obtain[1]

$$\{x^j, x^k\}_{D(\phi)} = -(i/m^2)[R^j, R^k]_- , \quad \{x^j, p_k\}_{D(\phi)} = \delta^j_k,$$
$$\{\psi^\mu, \psi^\nu\}_{D(\phi)} = (i/2)(\eta^{\mu\nu} - p^\mu p^\nu m^{-2}), \ \{x^j, \psi^\mu\}_{D(\phi)} = -R^j p^\mu m^{-2},$$
$$\{p_j, p_k\}_{D(\phi)} = \{p_j, \psi^\mu\}_{D(\phi)} = \{\zeta, \eta\}_{D(\phi)} = 0 \ , \quad R^j = \psi^j - \psi^0 p^j p_0^{-1} \ . \tag{3.3}$$

According to the recipes of quantization of theories with second-class constraints [9,10], the operators $\hat{\eta}$ corresponding to the variables η must satisfy the relations[2]

$$[\hat{\eta}, \hat{\eta}'\} = i\{\eta, \eta'\}_{D(\phi)}\Big|_{\eta\to\hat{\eta}} \ , \quad \hat{P}_\mu + i\hat{\psi}_\mu = 0, \quad \hat{p}\hat{\psi} = 0 \ . \tag{3.4}$$

Besides, we assume the operator $\hat{\zeta}$ to have the eigenvalues $\zeta = \pm 1$ by analogy with the classical theory, so that $\hat{\zeta}^2 = 1$. We can construct the realization of the algebra (3.4) of the operators $\hat{\eta}$ in the Hilbert state space $\mathcal{R}$ whose elements $\mathbf{f} \in \mathcal{R}$ are four-component columns

$$\mathbf{f} = \begin{pmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{pmatrix} ,$$

where $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are two component columns. We seek all the operators in the block-diagonal form, in particular, the operators $\hat{\zeta}$ and $\hat{p}_k$ we immediately choose in the form[3]

$$\hat{\zeta} = \gamma^0, \quad \hat{p}_k = -i\partial_k \mathbf{I}, \tag{3.5}$$

where I and $\mathbf{I}$ are 2×2 and 4×4 unit matrices. By the assumption concerning block diagonality, the general form of the operators $\hat{\psi}^\mu$ should be $\hat{\psi}^\mu = A^\mu \mathbf{I} + B^\mu_k \Sigma^k$, where $\mathbf{\Sigma} = \mathrm{diag}(\sigma, \sigma)$ and σ^k, $k = 1,2,3$ are Pauli matrices. The commutation relations involving

[1]We mean a generalized Dirac bracket whose form depends on parities of variables involved [10]

[2]In (3.4) $[..., ...\}$ stands for the generalized commutator which is either a commutator or an depending on parities of the operators involved.

[3]We use the standard representation for the γ matrices, $[\gamma^\mu, \gamma^\nu]_+ = 2\eta^{\mu\nu}$.

$\hat{\psi}^\mu$ imply that for non-zero B^μ_k the equation $A^\mu = 0$ must hold and that B^μ_k may depend on the operators $\hat{p}_k$ and $\hat{\zeta}$ only. Assuming that under spatial rotations the $\hat{\psi}^0$ behaves as a scalar and the $\hat{\psi}^k$ as a 3-vector, one can write $\hat{\psi}^0 = a\mathbf{\Sigma}\hat{\mathbf{p}}$, $\hat{\psi}^k = b\Sigma^k + c\hat{p}^k\mathbf{\Sigma}\hat{\mathbf{p}} + d\varepsilon^{kjl}\hat{p}^j\Sigma^l$, where a, b, c, d depend on $\hat{\mathbf{p}}^2$ and $\hat{\zeta}$ only. From (3.4) and (3.3) it follows that $(\hat{\psi}^0)^2 = \hat{\mathbf{p}}^2/4m^2$. Hence, we can choose $a = 1/2m$. Besides, we immediately set $d = 0$ since this coefficient can always be made equal to zero using the similarity transformation $exp(r\hat{\mathbf{p}}\mathbf{\Sigma})\hat{\psi}^\mu exp(-r\hat{\mathbf{p}}\mathbf{\Sigma})$ for a certain r. The coefficients b and c are uniquely determines (up to the similarity transformation mentioned above) from the relations (3.4) and (3.3). We are finally led to

$$\hat{\psi}^0 = \frac{1}{2m}\mathbf{\Sigma}\hat{\mathbf{p}}, \quad \hat{\psi}^k = \frac{\gamma^0}{2}\left(\Sigma^k + \frac{\hat{p}^k\mathbf{\Sigma}\hat{\mathbf{p}}}{m(\hat{\omega}+m)}\right) \ , \quad \hat{\omega} = (-\partial_k^2 + m^2)^{1/2} \ . \tag{3.6}$$

One can similarly find the operator $\hat{x}^k$ to be

$$\hat{x}^k = x^k\mathbf{I} + \frac{\epsilon^{kij}\Sigma^i\hat{p}^j}{2m(\hat{\omega}+m)} \ .$$

The evolution within the time τ of the state vectors from $\mathcal{R}$ is described by the Schrödinger equation $(i\partial/\partial\tau - \hat{H})\mathbf{f} = 0$, where, according to (3.1) and (3.5), $\hat{H} = \hat{\omega}\mathbf{I}$. Going over in the equation to the physical time $x_0 = \zeta\tau$, we obtain [11],

$$i\frac{\partial}{\partial x_0}f = \gamma^0\hat{\omega}f \ . \tag{3.7}$$

We interpret $f_+(x) = f_1(x)$ as the wavefunction of a particle and $f_-(x) = \sigma^2 f_2^*(x)$ as that of an antiparticle and define accordingly the scalar product in $\mathcal{R}$,

$$(f, g) = \int [f_1^+ g_1 + g_2^+ f_2] d\mathbf{x} = \int f_\zeta^+ g_\zeta d\mathbf{x}, \quad \zeta = \pm \ .$$

The operators $\hat{H}$, $\hat{\psi}^\mu$ and $\hat{x}^k$ are self-conjugate with respect to this scalar product. The equation for $f_\zeta(x)$ follows from (3.7), $(i\partial/\partial x_0 - \hat{\omega})f_\zeta = 0$. In this case, the equations for the wavefunctions of a particle and antiparticle have an equivalent form due to the absence of an external electromagnetic field. In the rest frame the spin operator $\hat{s}_k$ acts upon the wavefunctions as $\frac{1}{2}\sigma^k$ and coincide with $\hat{\psi}^k$.

The quantum mechanics constructed is completely equivalent to the Dirac theory. Indeed, (3.7) is simply the Dirac equation in the Foldy-Wouthuysen (FW) representation [13]. Making unitary FW transformation, we come to the usual Dirac equation,

$$f = U\varphi,\quad U = \frac{\hat{\omega} + m + \gamma\hat{\mathbf{p}}}{[2\hat{\omega}(\hat{\omega}+m)]^{1/2}}\,,\quad (i\gamma^\mu\partial_\mu - m)\varphi = 0\,.$$

Applying the FW transformation to the operators $\hat{\psi}^\mu$ and $\hat{x}^k$, we obtain

$$U^+\hat{\psi}^0 U = \hat{\Psi}^0 = \frac{1}{2m}\gamma^5\hat{p}_k\sigma^{k0},\quad U^+\hat{\psi}^k U = \hat{\Psi}^k = \frac{1}{2m}\gamma^5\left[\hat{p}_j\sigma^{jk} + H_D\sigma^{0k}\right]\,,$$
$$U^+\hat{x}^k U = \hat{X}^k = x^k\mathbf{I} + \frac{i}{2m}(\gamma^k - \hat{p}^k\gamma\hat{\mathbf{p}}\hat{\omega}^{-2})\,.$$

The operators $\hat{x}^k$ and $\hat{X}^k$ are the operators of middle position in the FW and Dirac pictures, respectively. The expression for these operators were obtained from covariance considerations by Pryce [14].

Let us discuss the quantization of massless spinning particle [15,16]. In this connection, one can consider the limit $m = 0$ of the massive case and compare it with an independent quantization of a classical action, describing massless particle in the beginning. To consider the limit it is convenient to use a different gauge at $m \neq 0$, namely, to use instead of gauge condition $\psi^5 = 0$ another one $\psi_0 = 0$. After the gauge fixing and eliminating of a part of variables, we arrive to the theory with the variables $x^i,\ p_i,\ \zeta,\ \psi^l,\ P_l,\ l = (i,5)$, and second-class constraints $\phi = 0$,

$$\phi = (p_i\psi^i + m\psi^5,\quad P_l + i\psi_l). \tag{3.8}$$

It is useful to introduce the transversal $\psi^{i\perp} = \Pi^i_j\psi^j$, $\Pi^i_j = \delta^i_j - p^{-2}p_ip_j$, $p = |\mathbf{p}|$, and the longitudinal $\psi^\parallel = p_i\psi^i$ parts of ψ^i, because of it is convenient to treat in these variables both cases $m \neq 0$ and $m = 0$ on the same foot. The first constraint (3.8) is, in fact, a relation between $\psi^\parallel$ and ψ^5, $\psi^\parallel = -m\psi^5$, whereas $\psi^{i\perp}$ are not constrained. Nonzero Dirac brackets between all the variables have the form

$$\left\{x^k, x^j\right\}_{D(\phi)} = \frac{i}{\omega^2}\left[\psi^{k\perp},\psi^{j\perp}\right]_- + \frac{im}{\omega^2p^2}\left(p_k\left[\psi^{j\perp},\psi^5\right]_- - p_j\left[\psi^{k\perp},\psi^5\right]_-\right)\,,$$

$$\left\{x^i, \psi^{j\perp}\right\}_{D(\phi)} = -\frac{\psi^{i\perp}p_j}{p^2} + \frac{m}{p^2}\Pi^i_j\psi^5\,,\quad \left\{x^i,\psi^5\right\}_{D(\phi)} = -\frac{m}{\omega^2}\psi^{i\perp} + \frac{m^2}{\omega^2p^2}p_i\psi^5\,,$$
$$\left\{\psi^{i\perp},\psi^{j\perp}\right\}_{D(\phi)} = -\frac{i}{2}\Pi^i_j\,,\quad \left\{\psi^5,\psi^5\right\}_{D(\phi)} = -\frac{i}{2}\frac{p^2}{\omega^2}\,,\quad \left\{x^k,p_j\right\}_{D(\phi)} = \delta^k_j\,. \tag{3.9}$$

One can introduce new variables θ^i and X^k instead of x^k and ψ^μ, ψ^5,

$$X^k = x^k - \frac{i}{\omega+m}\left[\psi^{k\perp},\psi^5\right]_-\,,\quad \theta^i = \psi^{i\perp} - \frac{\omega}{p^2}p_i\psi^5\,;$$
$$x^i = X^i - \frac{i}{\omega(\omega+m)}\left[\theta^i,\theta^\parallel\right]\,,\quad \psi^{i\perp} = \theta^{i\perp}\,,\quad \psi^5 = -\frac{1}{\omega}\theta^\parallel\,. \tag{3.10}$$

which are independent with respect to the second-class constraints (3.8). Using (3.9), one gets for nonzero Dirac brackets

$$\left\{X^k,p_j\right\}_{D(\phi)} = \delta^k_j\,,\quad \left\{\theta^k,\theta^j\right\}_{D(\phi)} = -\frac{i}{2}\delta_{kj}\,. \tag{3.11}$$

Now the commutation relations between the corresponding operators $\hat{X}^i$, $\hat{p}_i$, $\hat{\zeta}$, $\hat{\theta}^k$ have to be calculated by means of Dirac brackets (3.11), so that the nonzero commutators are

$$\left[\hat{X}^k,\hat{p}_j\right]_- = i\delta^k_j\,,\quad \left[\hat{\theta}^k,\hat{\theta}^j\right]_+ = \frac{1}{2}\delta_{kj}\,. \tag{3.12}$$

One can construct a realization of the algebra (3.12) and relation $\hat{\zeta}^2 = 1$ in the same Hilbert space $\mathcal{R}$ as before. Realization for $\hat{\zeta}$ and $\hat{p}_k$ remains the same and

$$\hat{X}^k = X^k\mathbf{I}\,,\quad \hat{\theta}^k = \frac{1}{2}\Sigma^k\,. \tag{3.13}$$

In the realization the operators of angular momentum $\hat{M}_{\mu\nu}$ and the spin $\hat{s}^k$ have the form

$$\hat{M}_{0j} = \hat{X}_0\hat{p}_j - \hat{X}_j\hat{p}_0 - \frac{i}{2}\frac{\hat{p}_j}{\hat{p}_0} + \frac{\hat{p}_0}{2\hat{\omega}(\hat{\omega}+m)}\epsilon_{jkl}\hat{p}_k\Sigma^l\,,$$
$$\hat{M}_{ij} = \hat{X}_i\hat{p}_j - \hat{X}_j\hat{p}_i - \frac{1}{2}\epsilon_{ijk}\Sigma^k\,,\quad \hat{s}^k = i\epsilon_{kjl}\hat{\psi}^j\hat{\psi}^l = \frac{1}{2}\Sigma^k\,. \tag{3.14}$$

As it is known, the square of the Pauli-Lubanski vector $\hat{W}^\mu = 1/2\epsilon^{\mu\nu\lambda\sigma}\hat{M}_{\mu\nu}\hat{p}_\sigma$ is a Casimir operator for the Poincare algebra. For this realization and in the center mass system

$$\hat{W}^0 = 0\,,\quad \hat{W}^k = m\frac{\hat{p}_0}{\hat{\omega}}\hat{s}^k\,,\quad \hat{W}^2 = -\left(\hat{W}^i\right)^2 = -\frac{3}{4}m^2.$$

The latter confirms that the system in question has spin one half.

Since the Schrödinger equation has the same form (3.7), the quantum mechanics constructed in this gauge is completely equivalent to the standard Dirac theory, namely it is connected with the latter by the same Foldy-Wouthuysen transformation. Moreover, applying the same transformation to the operators (3.14) we get the operators of the angular momentum in the Dirac theory.

$$\mathcal{U}^+\hat{M}_{\mu\nu}\mathcal{U} = \hat{X}_\mu\hat{p}_\nu - \hat{X}_\nu\hat{p}_\mu - \frac{1}{2}\sigma_{\mu\nu}\,, \quad \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]_-\,.$$

Considering the limit $m = 0$, one can remark that all formulas are nonsingular in the mass and admit such a limit. On the classical level, after the gauge fixing, it is possible to use both the variables x^i, p_i, ζ, $\psi^{i\perp}$, ψ^5 or the variables X^i, p_i, ζ, θ^i, the Dirac brackets of the latter do not contain mass at all and expressions of the former via the latter are nonsingular in the mass. The first set of the variables at $m = 0$ splits into two (anti) commuting one with another groups x^i, p_i, $\psi^{i\perp}$, and ψ^5. The Poincare generators are only expressed via the first group of variables and commute with ψ^5. Instead of the Casimir operator W^2, which vanishes at $m = 0$, appears a new one, helicity $\Lambda = \hat{p}^{-1}\hat{p}_k\hat{s}^k$. It turns out that at $m = 0$ the variable ψ^5_a can be omitted from the action (2.1). The quantization of such modified action reproduces the physical sector of the limit of the massive quantum mechanics. As we mentioned above, the Dirac brackets for the variables X^i, p_i, ζ, θ^k do not depend on the mass, that means that realization (3.13) remains in the limit $m = 0$. It is clear that the realization does not depend on the presence of the operator $\hat{\psi}^5$. In the limit we have $\psi^5 = \Lambda$. The Schrödinger equation (3.7) with $m = 0$ gives the Dirac equation with $m = 0$ after the corresponding FW transformation. The total Hilbert space forms now a reducible representation of the Poincare group (right and left neutrinos). It follows from the described structure of the quantum mechanics that in the limit $m = 0$ one does not need the variable ψ^5 in the theory. Indeed, one can take the action (2.1) at $m = 0$ and omit ψ^5 in the beginning. In such a theory, after the same gauge fixing (in particular, $\psi_0 = 0$) we have only the variables x^i, p_i, ζ, $\psi^{i\perp}$ on the constraint surface. Their Dirac brackets and the expressions of the Poincare generators coincide with the corresponding expressions

of the massive theory at $m = 0$. The same realization is available. If one introduces the operator $i\hat{p}^{-1}\hat{p}_k\epsilon_{kjl}\hat{\psi}^{l\perp}\hat{\psi}^{j\perp}$, which is in fact the operator $\hat{\psi}^5$ of the massive case, then the theory literally coincides with the limit of the massive case. In this connection one can remark that the dimensionality of the Hilbert space in the discussed realization does not depend on the presence of the variable ψ^5 at $m = 0$ and coincide with dimensionality of the massive case.

IV. PATH INTEGRAL REPRESENTATION FOR DIRAC PROPAGATOR IN EXTERNAL ELECTROMAGNETIC FIELD

Here we are going to discuss path integral representations for the propagator of relativistic spinning particle in an external electromagnetic field. We will demonstrate that in such representations integrands have the form $\exp iS_{eff}$, where effective actions S_{eff} are very close related with the action (2.1). One ought to say that different kinds of such representations were derived and discussed in papers [17–24]. Below we follow mainly to the work (4.8).

As known the propagator of a relativistic spinning particle is the causal Green's function $S^c(x,y)$ of the Dirac equation. For our purpose, it is convenient to deal with the transformed by $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$ function $\tilde{S}^c(x,y) = S^c(x,y)\gamma^5$, which obeys the properly transformed Dirac equation for Green's function

$$\left(\hat{\mathcal{P}}_\nu\tilde{\gamma}^\nu - m\gamma^5\right)\tilde{S}^c(x,y) = \delta^4(x-y), \tag{4.1}$$

where $\hat{\mathcal{P}}_\mu = i\partial_\mu - gA_\mu(x)$ and $\tilde{\gamma}^\mu = \gamma^5\gamma^\mu$. The matrices $\tilde{\gamma}^\nu$, have the same commutation relations as initial ones γ^ν, $[\tilde{\gamma}^\mu, \tilde{\gamma}^\nu]_+ = 2\eta^{\mu\nu}$, so that the tilde sign will be omitted hereafter. For all the γ-matrices we have $[\gamma^m, \gamma^n]_+ = 2\eta^{mn}$, $m, n = \overline{0,3}, 5$; $\eta^{mn} = \mathrm{diag}(1 -1 -1 -1 -1)$.

Similar to Schwinger [25] we present $\tilde{S}^c_{\alpha\beta}(x,y)$ as a matrix element of an operator $\hat{S}^c_{\alpha\beta}$, but, in the coordinate space only,

$$\tilde{S}^c_{\alpha\beta}(x,y) = <x|\hat{S}^c_{\alpha\beta}|y>, \tag{4.2}$$

where the spinor indices are written explicitly for clarity only once and will be omitted hereafter; $|x>$ are eigenvectors for some hermitian operators of coordinates X^μ, the corresponding canonically conjugate operators of momenta are P_μ, so that:

$$X^\mu|x>=x^\mu|x>\,,\quad <x|y>=\delta^4(x-y)\,,\quad \int|x><x|dx=I\,,$$
$$[P_\mu,X^\nu]_-=-i\delta_\mu^\nu,\quad P_\mu|p>=p_\mu|p>\,,\quad <p|p'>=\delta^4(p-p')\,,$$
$$\int|p><p|dp=I\,,\quad <x|P_\mu|y>=-i\partial_\mu\delta^4(x-y)\,,\quad <x|p>=\frac{1}{(2\pi)^2}e^{ipx}\,,$$
$$[\Pi_\mu,\Pi_\nu]_-=-igF_{\mu\nu}(X)\,,\quad \Pi_\mu=-P_\mu-gA_\mu(X)\,. \tag{4.3}$$

The equation (4.1) implies the formal solution for the operator $\hat{S}^c$, $\hat{S}^c=[\Pi_\nu\gamma^\nu-m\gamma^5]^{-1}$. The operator in the square brackets is a pure Fermi one, if one reckons γ-matrices as Fermi operators. In general case the inverse operator to a Fermi operator F can be presented by means of an integral over the super-proper time (λ,χ) of an exponential with an even exponent [24],

$$F^{-1}=\int_0^\infty d\lambda\int e^{i[\lambda(F^2+i\epsilon)+\chi F]}d\chi\,, \tag{4.4}$$

where λ is an even variable and χ is an odd one, the latter anticommutes with F by definition. Here and in what follow integrals over odd variables are understood as Berezin's integrals [1]. The representation (4.4) is an analog of the Schwinger proper-time representation for an inverse operator, convenient in the Fermi case. Using (4.4) and taking into account that $(\Pi_\nu\gamma^\nu-m\gamma^5)^2=\Pi^2-m^2$, one can write for the operator $\hat{S}^c$

$$\hat{S}^c=\int_0^\infty d\lambda\int e^{-i\hat{\mathcal{H}}(\lambda,\chi)}d\chi\,,$$
$$\hat{\mathcal{H}}(\lambda,\chi)=\lambda\left(m^2-\Pi^2+\frac{ig}{2}F_{\alpha\beta}\gamma^\alpha\gamma^\beta\right)+\left(\Pi_\nu\gamma^\nu-m\gamma^5\right)\chi\,.$$

Thus, the Green's function (4.2) takes the form

$$\tilde{S}^c=\tilde{S}^c(x_{out},x_{in})=\int_0^\infty d\lambda\int\langle x_{\rm out}|e^{-i\hat{\mathcal{H}}(\lambda,\chi)}|x_{\rm in}\rangle d\chi\,. \tag{4.5}$$

Now one can present the matrix element entering in the expression (4.5) by means of a path integral. In spite of the operator $\hat{\mathcal{H}}(\lambda,\chi)$ has the γ-matrix structure, it is possible to do as

usual. Namely, first we write $\exp -i\hat{\mathcal{H}}=\left(\exp -i\hat{\mathcal{H}}/N\right)^N$ and then insert $(N-1)$ resolutions of identity $\int|x><x|dx=I$ between all the operators $\exp -i\hat{\mathcal{H}}/N$. Besides, we introduce N additional integrations over λ and χ to transform then the ordinary integrals over these variables into the corresponding path-integrals,

$$\tilde{S}^c=\lim_{N\to\infty}\int_0^\infty d\lambda_0\int d\chi_0 dx_1...dx_{N-1}d\lambda_1...d\lambda_N d\chi_1...d\chi_N$$
$$\times\prod_{k=1}^N\langle x_k|e^{-i\hat{\mathcal{H}}(\lambda_k,\chi_k)\Delta\tau}|x_{k-1}\rangle\delta(\lambda_k-\lambda_{k-1})\delta(\chi_k-\chi_{k-1})\,, \tag{4.6}$$

where $\Delta\tau=1/N$, $x_0=x_{\rm in}$, $x_N=x_{\rm out}$. Bearing in mind the limiting process, one can calculate the matrix elements from (4.6) approximately,

$$\langle x_k|e^{-i\hat{\mathcal{H}}(\lambda_k,\chi_k)\Delta\tau}|x_{k-1}\rangle\approx\langle x_k|1-i\hat{\mathcal{H}}(\lambda_k,\chi_k)\Delta\tau|x_{k-1}\rangle, \tag{4.7}$$

using the resolutions of identity $\int|p><p|dp$. In this connection it is important to notice that the operator $\hat{\mathcal{H}}(\lambda_k,\chi_k)$ has originally the symmetric form in the operators $\hat{x}$ and $\hat{p}$. Indeed, the only one term in $\hat{\mathcal{H}}(\lambda_k,\chi_k)$, which contains products of these operators is $[P_\alpha,A^\alpha(X)]_+$. One can verify that this is maximal symmetrized expression, which can be combined from entering operators (see remark in [26]). Thus, one can write $\hat{\mathcal{H}}(\lambda,\chi)={\rm Sym}_{(\hat{x},\hat{p})}\,\mathcal{H}(\lambda,\chi,\hat{x},\hat{p})$, where $\mathcal{H}(\lambda,\chi,x,p)$ is the Weyl symbol of the operator $\hat{\mathcal{H}}(\lambda,\chi)$ in the sector of coordinates and momenta, $\mathcal{H}(\lambda,\chi,x,p)=\lambda\left(m^2-\mathcal{P}^2+\frac{ig}{2}F_{\alpha\beta}\gamma^\alpha\gamma^\beta\right)+(\mathcal{P}_\nu\gamma^\nu-m\gamma^5)\chi$, and $\mathcal{P}_\nu=-p_\nu-gA_\nu(x)$. That is a general statement [27], which can be easily checked in that concrete case by direct calculations, that the matrix elements (4.7) are expressed in terms of the Weyl symbols in the middle point $\overline{x}_k=(x_k+x_{k-1})/2$. Taking all that into account, one can see that in the limiting process the matrix elements (4.7) can be replaced by the expressions

$$\int\frac{dp_k}{(2\pi)^4}\exp i\left[p_k\frac{x_k-x_{k-1}}{\Delta\tau}-\mathcal{H}(\lambda_k,\chi_k,\overline{x}_k,p_k)\right]\Delta\tau\,, \tag{4.8}$$

which are noncommutative due to the γ-matrix structure and are situated in (4.6) so that the numbers k increase from the right to the left. For the two δ-functions, accompanying each matrix element (4.7) in the expression (4.6), we use the integral representations

$$\delta(\lambda_k - \lambda_{k-1})\delta(\chi_k - \chi_{k-1}) = \frac{i}{2\pi}\int e^{i[\pi_k(\lambda_k-\lambda_{k-1})+\nu_k(\chi_k-\chi_{k-1})]}d\pi_k d\nu_k,$$

where ν_k are odd variables. Then we attribute formally to γ-matrices, entering into (4.8), index k, and then we attribute to all quantities the "time" τ_k, according the index k they have, $\tau_k = k\Delta\tau$, so that $\tau \in [0,1]$. Introducing the T-product, which acts on γ-matrices, it is possible to gather all the expressions, entering in (4.6), in one exponent and deal then with the γ-matrices like with odd variables. Thus, we get for the right side of (4.6)

$$\tilde{S}^c = \mathrm{T}\int_0^\infty d\lambda_0 \int d\chi_0 \int_{x_{in}}^{x_{out}} Dx \int Dp \int_{\lambda_0} D\lambda \int_{\chi_0} D\chi \int D\pi \int D\nu \tag{4.9}$$
$$\times \exp\left\{i\int_0^1 \left[\lambda\left(\mathcal{P}^2 - m^2 - \frac{ig}{2}F_{\alpha\beta}\gamma^\alpha\gamma^\beta\right) + \left(m\gamma^5 - \mathcal{P}_\nu\gamma^\nu\right)\chi + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi}\right] d\tau\right\},$$

where x, p, λ, π , are even and χ, ν are odd trajectories, obeying the boundary conditions $x(0) = x_{\rm in}$, $x(1) = x_{\rm out}$, $\lambda(0) = \lambda_0$, $\chi(0) = \chi_0$. The operation of T-ordering acts on the γ-matrices, which suppose formally to depend on time τ. The expression (4.9) can be reduced to:

$$\tilde{S}^c = \int_0^\infty d\lambda_0 \int d\chi_0 \int_{x_{in}}^{x_{out}} Dx \int Dp \int_{\lambda_0} D\lambda \int_{\chi_0} D\chi \int D\pi \int D\nu \exp\left\{i\int_0^1 \left[\lambda\left(\mathcal{P}^2 - m^2\right.\right.\right.$$
$$\left.\left.\left. - \frac{ig}{2}F_{\alpha\beta}\frac{\delta_l}{\delta\rho_\alpha}\frac{\delta_l}{\delta\rho_\beta}\right) + \left(m\frac{\delta_l}{\delta\rho_5} - \mathcal{P}_\nu\frac{\delta_l}{\delta\rho_\nu}\right)\chi + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi}\right] d\tau\right\} \mathrm{T}\exp\int_0^1 \rho_n(\tau)\gamma^n d\tau\Big|_{\rho=0},$$

where five odd sources $\rho_n(\tau)$ are introduced, which anticommute with the γ-matrices by definition. One can present the quantity $\mathrm{T}\exp\int_0^1 \rho_n(\tau)\gamma^n d\tau$ via a path integral over odd trajectories [24],

$$\mathrm{T}\exp\int_0^1 \rho_n(\tau)\gamma^n d\tau = \exp\left(i\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_{\psi(0)+\psi(1)=\theta} \exp\left[\int_0^1 \left(\psi_n\dot{\psi}^n - 2i\rho_n\psi^n\right) d\tau\right.$$
$$\left. + \psi_n(1)\psi^n(0)\right]\mathcal{D}\psi\big|_{\theta=0}, \quad \mathcal{D}\psi = D\psi\left[\int_{\psi(0)+\psi(1)=0} D\psi \exp\left\{\int_0^1 \psi_n\dot{\psi}^n d\tau\right\}\right]^{-1}, \tag{4.10}$$

where θ^n are odd variables, anticommuting with γ-matrices and $\psi^n(\tau)$ are odd trajectories of integration, obeying the boundary conditions, which are pointed out below the signs of integration. Using (4.10) we get the hamiltonian path integral representation for the Green's function in question:

$$\tilde{S}^c = \exp\left(i\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty d\lambda_0 \int d\chi_0 \int_{\lambda_0} D\lambda \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int Dp \int D\pi \int D\nu$$
$$\times \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi \exp\left\{i\int_0^1 \left[\lambda\left(\mathcal{P}^2 - m^2 + 2igeF_{\alpha\beta}\psi^\alpha\psi^\beta\right) + 2i\left(\mathcal{P}_\alpha\psi^\alpha - m\psi^5\right)\chi\right.\right.$$
$$\left.\left. - i\psi_n\dot{\psi}^n + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi}\right] d\tau + \psi_n(1)\psi^n(0)\right\}\Big|_{\theta=0},$$

Integrating over momenta, we get a path integral in the lagrangian form,

$$\tilde{S}^c = \exp\left(i\gamma^n\frac{\partial_\ell}{\partial\theta^n}\right)\int_0^\infty de_0 \int d\chi_0 \int_{e_0} De \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int D\pi \int D\nu \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi$$
$$\times M(e)\exp\left\{i\int_0^1 \left[-\frac{\dot{x}^2}{2e} - \frac{e}{2}m^2 - g\dot{x}A(x) + iegF_{\mu\nu}(x)\psi^\mu\psi^\nu + i\left(\frac{\dot{x}_\mu\psi^\mu}{e} - m\psi^5\right)\chi\right.\right.$$
$$\left.\left. - i\psi_n\dot{\psi}^n + \pi\dot{e} + \nu\dot{\chi}\right] d\tau + \psi_n(1)\psi^n(0)\right\}\Big|_{\theta=0}, \tag{4.11}$$

where

$$M(e) = \int Dp \exp\left\{\frac{i}{2}\int_0^1 ep^2 d\tau\right\}. \tag{4.12}$$

A discussion of the role of the measure (4.12) one can find in [24].

The exponent in the integrand (4.11) can be considered as an effective and nondegenerate lagrangian action of a spinning particle in an external field. It consists of two principal parts. The first one, which unites two summand with the derivatives of e and χ, can be treated as a gauge fixing term and corresponds to the gauge conditions $\dot{e} = \dot{\chi} = 0$. The rest part of the effective action, in fact, coincides with the gauge invariant action (2.1) of a spinning particle.

V. SPINOR STRUCTURE OF DIRAC PROPAGATOR

In this section we are going to demonstrate that Dirac propagator can only be expressed through a bosonic path integral over coordinates; the integrand of this path integral differs from the corresponding expression in scalar case by a spin factor, which spinor structure is completely described. This problem attracted attention of researchers already for a long time. So, Feynman, who had written first his path integral for probability amplitude in nonrelativistic quantum mechanics [28] and then a path integral for causal Green's function of Klein-Gordon equation (propagator of a scalar particle) [29], had also been attempting to write a representation for Dirac propagator via a bosonic path integral [30]. After Berezin had introduced his integral over grassmannian variables, it turned out to be possible to present this propagator via both bosonic and grassmannian path integrals, as that was demonstrated in the previous section. Nevertheless, attempts to write Dirac propagator via a bosonic path integral only where continued. So, Polyakov [31] assumed that the propagator of free Dirac electron in $D = 3$ Euclidean space-time can be presented by means of a bosonic path integral, similar to scalar particle, modified by a so called spinor factor. This idea was developed in [32] to write spinor factor for Dirac fermions, interacting with a non-Abelian gauge field in D dimensional Euclidean space-time. Unfortunately, in that representation the spinor factor itself was presented via some additional bosonic path integrals, and its γ-matrix structure was not defined explicitly. As was shown in [33] the problem can be solved directly on the way of doing all grassmannian integrations in the expression (4.11). Let us consider here this solution. So, we start with the representation (4.11) for Dirac propagator, in which we first integrate over π and ν, and then use arisen δ-functions to remove the functional integration over e and χ,

$$\tilde{S}^c = -\exp\left\{i\gamma^n \frac{\partial_\ell}{\partial\theta^n}\right\} \int_0^\infty de_0 \int_{x_{in}}^{x_{out}} Dx \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi\; M(e_0) \int_0^1 \left(\frac{\dot{x}_\mu \psi^\mu}{e_0} - m\psi^5\right) d\tau \quad (5.1)$$
$$\times \exp\left\{ i\int_0^1 \left[-\frac{\dot{x}^2}{2e_0} - \frac{e_0}{2}m^2 - g\dot{x}A(x) + ige_0 F_{\mu\nu}(x)\psi^\mu\psi^\nu - i\psi_n\dot{\psi}^n\right] d\tau + \psi_n(1)\psi^n(0)\right\}\Bigg|_{\theta=0} .$$

Then, it is convenient to replace the integration over ψ by one over related odd velocities ω,

$$\psi(\tau) = \frac{1}{2}\int_0^1 \varepsilon(\tau-\tau')\omega(\tau')d\tau' + \frac{1}{2}\theta\,, \quad \omega(\tau) = \dot{\psi}(\tau)\,, \; \varepsilon(\tau) = \text{sign}\,\tau\,. \quad (5.2)$$

There are not more any restrictions on ω; because of (5.2) the boundary conditions for ψ are obeyed automatically. The corresponding Jacobian does not depend on variables and cancels with the same one from the measure (4.12). Thus[4] ,

---

[4]Here and further, we are using condensed notations, e.g. $\omega\varepsilon\omega = \int_0^1 d\tau d\tau' \omega(\tau)\varepsilon(\tau-\tau')\,\omega(\tau')$ and so on.

$$\tilde{S}^c = -\frac{1}{2}\exp\left\{i\gamma^n \frac{\partial_\ell}{\partial\theta^n}\right\} \int_0^\infty de_0 \int_{x_{in}}^{x_{out}} Dx \int \mathcal{D}\omega\; M(e_0)\left[\frac{\dot{x}_\mu}{e_0}(\varepsilon\omega^\mu + \theta^\mu) - m\left(\varepsilon\omega^5 + \theta^5\right)\right]$$
$$\times \exp\left\{ i\left[-\frac{\dot{x}^2}{2e_0} - \frac{e_0}{2}m^2 - g\dot{x}A(x) - \frac{ie_0 g}{4}(\omega^\mu\varepsilon - \theta^\mu) F_{\mu\nu}(x)(\varepsilon\omega^\nu + \theta^\nu) + \frac{i}{2}\omega_n\varepsilon\omega^n\right]\right\}\Bigg|_{\theta=0} ,$$

where

$$\mathcal{D}\omega = D\omega\left[\int D\omega \exp\left\{-\frac{1}{2}\omega^n\varepsilon\omega_n\right\}\right]^{-1} . \quad (5.3)$$

One can prove for a function $f(\theta)$ in the Grassmann algebra the following identity holds

$$\exp\left\{i\gamma^n\frac{\partial_\ell}{\partial\theta^n}\right\} f(\theta)\Big|_{\theta=0} = f\left(\frac{\partial_\ell}{\partial\zeta}\right)\exp\left\{i\zeta_n\gamma^n\right\}\Big|_{\zeta=0}$$
$$= \sum_{k=0}^{4}\sum_{n_1\cdots n_k} f_{n_1\cdots n_k}\frac{\partial_\ell}{\partial\zeta_{n_1}}\cdots\frac{\partial_\ell}{\partial\zeta_{n_k}}\sum_{l=0}^{4}\frac{i^l}{l!}(\zeta_n\gamma^n)^l\Big|_{\zeta=0} , \quad (5.4)$$

where ζ_n are some odd variables. Using (5.4), we get

$$\tilde{S}^c = -\frac{1}{2}\int_0^\infty de_0 \int_{x_{in}}^{x_{out}} Dx\; M(e_0)\left[\frac{\dot{x}_\mu}{e_0}\left(\varepsilon\frac{\delta_\ell}{\delta\rho_\mu} + \frac{\partial_\ell}{\partial\zeta_\mu}\right) - m\left(\varepsilon\frac{\delta}{\delta\rho_5} + i\gamma^5\right)\right] \quad (5.5)$$
$$\times \exp\left\{ i\left[-\frac{\dot{x}^2}{2e_0} - \frac{e_0}{2}m^2 - g\dot{x}A(x) + \frac{ie_0 g}{4}F_{\mu\nu}(x)\frac{\partial_\ell}{\partial\zeta_\mu}\frac{\partial_\ell}{\partial\zeta_\nu}\right]\right\} R\left[x,\rho,\frac{\partial_\ell}{\partial\zeta}\right]\exp\left\{i\zeta_\mu\gamma^\mu\right\}\Big|_{\rho,\zeta=0}$$

with

$$R\left[x,\rho,\frac{\partial_\ell}{\partial\zeta}\right] = \int \mathcal{D}\omega \exp\left\{-\frac{1}{2}\omega^n T_{nk}(x|g)\omega^k + I_n\omega^n\right\}, \quad (5.6)$$
$$I_\mu = \rho_\mu - \frac{e_0 g}{2}\frac{\partial_\ell}{\partial\zeta_\nu}F_{\nu\mu}(x)\varepsilon,\; I_5 = \rho_5,$$

$$T_{nk}(x|g) = \begin{pmatrix} \Lambda_{\mu\nu}(x|g) & 0 \\ 0 & -\varepsilon \end{pmatrix}, \quad \Lambda_{\mu\nu}(x|g) = \eta_{\mu\nu}\varepsilon - \frac{e_0}{2}\varepsilon g F_{\mu\nu}(x)\varepsilon\,. \quad (5.7)$$

where $\rho_n(\tau)$ are odd sources for $\omega^n(\tau)$. Integral in (5.6) is gaussian one. It can be easily done [1], remembering its original definition,

$$R\left[x,\rho,\frac{\partial_\ell}{\partial\zeta}\right] = \left[\frac{\text{Det}T(x|g)}{\text{Det}T(x|0)}\right]^{1/2}\exp\left\{-\frac{1}{2}I_n\left[T^{-1}(x|g)\right]^{nk} I_k\right\} , \quad (5.8)$$

The ratio $\text{Det}T(x|g)/\text{Det}T(x|0)$ in (5.8) can be replaced by $\text{Det}\Lambda(x|g)/\text{Det}\Lambda(x|0)$, due to the structure (5.7) of the matrix $T(x|g)$, and the latter can be presented in a convenient form, which allows one to avoid problems with calculations of determinants of matrices with continuous indices,

$$\frac{\mathrm{Det}\Lambda(x|g)}{\mathrm{Det}\Lambda(x|0)} = \exp\left\{-e_0\int_0^g dg'\ \mathrm{Tr}\ \mathcal{G}(x|g')F(x)\right\},\ \ \mathcal{G}^{\mu\nu}(x|g) = \frac{1}{2}\varepsilon\left[\Lambda^{-1}(x|g)\right]^{\mu\nu}\varepsilon\ . \tag{5.9}$$

Substituting (5.9) into (5.5), and performing functional differentiations with respect to ρ_μ, we get

$$\tilde{S}^c = -\frac{1}{2}\int_0^\infty de_0\int_{x_{in}}^{x_{out}} Dx\ M(e_0)\left[\frac{\dot{x}^\mu}{e_0}K_{\mu\nu}(x)\frac{\partial_\ell}{\partial\zeta_\nu} - im\gamma^5\right]\exp\left\{i\left[-\frac{\dot{x}^2}{2e_0} - \frac{e_0}{2}m^2\right.\right.$$
$$\left.\left.-g\dot{x}A(x) + \frac{ie_0}{2}\int_0^g dg'\ \mathrm{Tr}\ \mathcal{G}(x|g')F(x) + \frac{ie_0 g}{4}\left(F(x)K(x)\right)_{\mu\nu}\frac{\partial_\ell}{\partial\zeta_\mu}\frac{\partial_\ell}{\partial\zeta_\nu}\right]\right\}\exp\left\{i\zeta_\mu\gamma^\mu\right\}\Big|_{\zeta=0},$$
$$K_{\mu\nu}(x) = \eta_{\mu\nu} + e_0 g\left(\mathcal{G}(x|g)F(x)\right)_{\mu\nu}\ .$$

Now the differentiation over ζ can be fulfilled explicitly, using (5.4). Thus, finally, remembering that $\tilde{S}^c = S^c\gamma^5$, and the γ-matrices in the last expression have to be replaced by $\gamma^5\gamma^\mu$ if we are interesting in the propagator S^c, one gets

$$S^c = \frac{i}{2}\int_0^\infty de_0\int_{x_{in}}^{x_{out}} Dx\ M(e_0)\Phi(x,e_0)\exp\left\{i\left[-\frac{\dot{x}^2}{2e_0} - \frac{e_0}{2}m^2 - g\dot{x}A(x)\right]\right\}\ , \tag{5.10}$$

$$\Phi(x,e_0) = \left[m + (2e_0)^{-1}\dot{x}K(x)\left(2 - ge_0F(x)K(x)\right)\gamma - im\frac{e_0 g}{4}\left(F(x)K(x)\right)_{\mu\nu}\sigma^{\mu\nu}\right.$$
$$\left.-i\frac{g}{4}\left(\dot{x}K(x)\gamma\right)\left(F(x)K(x)\right)_{\mu\nu}\sigma^{\mu\nu} + m\frac{e_0^2 g^2}{16}\left(F(x)K(x)\right)^*_{\mu\nu}\left(F(x)K(x)\right)^{\mu\nu}\gamma^5\right]$$
$$\times\exp\left\{-\frac{e_0}{2}\int_0^g dg'\ \mathrm{Tr}\ \mathcal{G}(x|g')F(x)\right\}\ , \tag{5.11}$$

where $\left(F(x)K(x)\right)^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\left(F(x)K(x)\right)^{\alpha\beta}$, and $\epsilon_{\mu\nu\alpha\beta}$ is Levi-Civita symbol, normalized by $\epsilon_{0123} = -1$.

Thus, we get a representation for Dirac propagator as a path integral over bosonic trajectories of a functional, which spinor structure is found explicitly, namely, its decomposition in all independent γ-structures is given. The functional $\Phi(x,e_0)$ can be called spin factor, and namely it distinguishes Dirac propagator from the scalar one. One needs to stress that spin factor is gauge invariant, because of its dependence of $F_{\mu\nu}(x)$ only. In the same manner one can describe the isospinor structure of relativistic particle propagators [33].

VI. SPINNING PARTICLE WITH ANOMALOUS MAGNETIC MOMENTUM

It is possible to get a generalization of the action (2.1) of a spinning particle in the presence of an anomalous magnetic momentum [34]. The relativistic quantum theory of a spinning particle, which has an anomalous magnetic momentum μ, was formulated by Pauli [35]. In this case he generalized the Dirac equation to the following form:

$$\left(\hat{\mathcal{P}}_\nu\gamma^\nu - m - \frac{\mu}{2}\sigma^{\alpha\beta}F_{\alpha\beta}\right)\Psi(x) = 0,\ \ \hat{\mathcal{P}}_\nu = i\partial_\nu - gA_\nu(x)\ . \tag{6.1}$$

An analog of the action (2.1), which after quantization reproduces the Dirac-Pauli theory can be written in the following form:

$$S = \int_0^1\left[-\frac{\dot{x}^2}{2e} - e\frac{M^2}{2} - \dot{x}^\alpha\left(gA_\alpha + 4i\mu\psi^5F_{\alpha\beta}\psi^\beta\right) + igeF_{\alpha\beta}\psi^\alpha\psi^\beta\right.$$
$$\left.+i\left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - M^*\psi^5\right)\chi - i\psi_n\dot{\psi}^n\right]d\tau\ , \tag{6.2}$$

where $M = m - 2i\mu F_{\alpha\beta}\psi^\alpha\psi^\beta$, and $M^* = m + 2i\mu F_{\alpha\beta}\psi^\alpha\psi^\beta$. The symmetry (2.2) and (2.3) remains for the action (6.2). One can derive equations of motion and check that they really describe a classical particle with anomalous magnetic momentum μ. Doing the Dirac quantization of the theory we arrive to the equation (6.1) as a condition on the physical state vectors [36].

In the same manner as in Sect.IV one can find a path integral representation for the transformed by γ^5 causal Green's function $\tilde{S}^c = \tilde{S}^c(x_{in}, y_{out})$, of the Dirac-Pauli equation (6.1),

$$\tilde{S}^c = \exp\left(i\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty de_0\int\int d\chi_0\int_{e_0} De\int_{\chi_0} D\chi\int_{x_{in}}^{x_{out}} Dx\int D\pi\int D\nu\int_{\psi(0)+\psi(1)=\theta}\mathcal{D}\psi$$
$$\times M(e)\exp\left\{i\int_0^1\left[-\frac{\dot{x}^2}{2e} - e\frac{M^2}{2} - \dot{x}^\alpha\left(gA_\alpha + 4i\mu\psi^5F_{\alpha\beta}\psi^\beta\right) + igeF_{\alpha\beta}\psi^\alpha\psi^\beta\right.\right.$$
$$\left.\left.+\ i\left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - M^*\psi^5\right)\chi - i\psi_n\dot{\psi}^n + \pi\dot{e} + \nu\dot{\chi}\right]d\tau + \psi_n(1)\psi^n(0)\right\}\Big|_{\theta=0}\ , \tag{6.3}$$

where $M^* = m + 2i\mu F_{\alpha\beta}\psi^\alpha\psi^\beta$, and the measure $\mathcal{M}(e)$ has the form (5.11).

One can treat the exponent in the integrand (6.3) as an effective and nondegenerate lagrangian action of spinning particle with an anomalous magnetic momentum; the gauge invariant part of this action coincides with (6.2).

VII. DISCUSSION

As we have seen from the discussion presented in Sect.III, the quantum mechanics constructed from the Berezin-Marinov action (2.1) admits the limit $m = 0$. As a result one gets the quantum theory of massless particle, which is described by the Dirac equation with $m = 0$, without any additional restrictions on the four-spinor $\Psi(x)$,

$$i\partial_\mu\gamma^\mu\Psi(x) = 0 \,. \tag{7.1}$$

How is was said, the variable ψ^5 can be omitted from the action (2.1) at $m = 0$. The quantization of such a modified action reproduces the physical sector in the limit $m = 0$ of the massive quantum mechanics. Unfortunately, such a quantum theory, describes massless spin one half particle with the all possible values of helicity (right and left neutrinos). As it is known, the right (left) neutrino is described by a four-spinor, which obeys, besides the Dirac equation (7.1), the Weyl condition as well,

$$\left(\gamma^5 - \alpha\right)\Psi(x) = 0\,, \quad \alpha = 1\,(-1)\,, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3\,. \tag{7.2}$$

There were several attempts [37,38] to modify the action (2.1) at $m = 0$ so that in course of quantization one can get a quantum mechanics with wave functions obeying both equations (7.1) and (7.2) at the same time. Unfortunately, all they do not solve the problem (see a discussion in [39]).

In the paper [39] we have proposed a new pseudoclassical action to describe the Weyl particles,

$$S = \int_0^1 \left[-\frac{1}{2e}\left(\dot{x}^\mu - i\psi^\mu\chi + i\epsilon^{\mu\nu\rho\varsigma}b_\nu\psi_\rho\psi_\varsigma + \frac{\alpha}{2}b^\mu\right)^2 - i\psi_\mu\dot{\psi}^\mu\right]d\tau\,, \tag{7.3}$$

where x^μ, e, ψ^μ, χ have the same meaning as in (2.1), the variables b^μ form an even four-vector, and α is an even constant. The action admits both quasi-canonical quantization (with fixation of the gauge freedom, which corresponds to two types of gauge transformations of existing three ones) and the Dirac quantization. Both of them lead to the theory of the Weyl particles in the above mentioned sense.

Recently [40] we have also proposed a new pseudoclassical model for a massive spin one half particle in 2+1 dimensions, interacting with an external Abelian gauge field. Such a model has an important meaning not only for the deeper understanding of the quantum theory of relativistic particles, but also because of a close connection with the theory of interacting anyons, which attracts in recent years great attention. As it is known, attempts to extend the pseudoclassical description of the spinning particle to an arbitrary odd-dimensional case had met some problems, which are connected with the absence of an analog of γ^5-matrix in odd-dimensions. For instance, in 2+1 dimensions the direct generalization of the Berezin-Marinov action (standard action) does not reproduce a minimal quantum theory of a spinning particle, which has to provide only one value of the spin projection (1/2 or $-1/2$). In papers [41,42] they have proposed two modifications of the standard action to get such a minimal theory, but these models can not be considered as an satisfactory solution of the problem. The action [41], in fact, is classically equivalent to the standard action and does not provide required quantum properties in course of canonical and path-integral quantization. Moreover, it is P- and T-invariant, so that an anomaly is present. Another one [42] does not obey gauge supersymmetries and therefore loses the main attractive features in such kind of models, which allows one to treat them as prototypes of superstrings or some modes in superstring theory.

The action, we have proposed, to describe a Dirac particle in $2+1$ dimension has the form

$$S = \int_0^1 \left[-\frac{z^2}{2e} - e\frac{m^2}{2} - g\dot{x}^\mu A_\mu + igeF_{\mu\nu}\psi^\mu\psi^\nu - im\psi^3\chi - \frac{1}{2}sm\kappa - i\psi_a\dot{\psi}^a\right]d\tau$$

$$\equiv \int_0^1 L d\tau \ , \quad s = \pm \ ; \quad z^\mu = \dot{x}^\mu - \imath\psi^\mu\chi + i\varepsilon^{\mu\nu\lambda}\psi_\nu\psi_\lambda\kappa \ ; \tag{7.4}$$

the Latin indices a, b, c, ... run over 0, 1, 2, 3, whereas the Greek (Lorentz) ones μ, ν, ... run over 0, 1, 2; x^μ, e, κ are even and ψ^a, χ are odd variables; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the strength tensor and g is the $U(1)$-charge of the particle, interacting with an external gauge field $A_\mu(x)$, which can have the Maxwell or (and) Chern-Simons nature; $\varepsilon^{\mu\nu\lambda}$ is the totally antisymmetric tensor density of Levi-Civita in $2+1$ dimensions normalized by $\varepsilon^{012} = +1$; $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$, $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1)$.

It obeys three gauge symmetries-one reparametrization symmetry and two supergauge symmetry. It is P- and T-noninvariant in full accordance with the expected properties of the minimal theory in $2+1$ dimensions, which has to describe only one value of the spin projection. Dirac quantization (without explicit gauge fixing on the classical level) and quasicanonical quantization with fixation of the gauge freedom, which corresponds to two types of gauge transformations of the three existing, leads to the quantum theory of spin 1/2 Dirac particle in $2+1$ dimensions. Technically, the Dirac equation in $2+1$ dimensions arises in both schemes of quantization in different ways, but both quantum theories appear to be equivalent. It is interesting that the model (7.4) of can also be derived in course of a dimensional reduction from the model (7.3) of the Weyl particle in $3+1$ dimensions.

In the conclusion one ought to say that lately models of superparticles and particles with higher spins were constructed by analogy with the case of the spin one half relativistic particle. For example, a direct generalization of the action (2.1) for particles with arbitrary spin $N/2$ in $3+1$ dimensions was proposed in [43,44]. It can be written in the form

$$S = \int_0^1 \left[-\frac{1}{2e}\left(\dot{x}^\mu - i\psi_a^\mu\chi_a\right)^2 - \frac{e}{2}m^2 - im\psi_a^5\chi_a + \frac{1}{2}f_{ab}\left(i\left[\psi_{an},\ \psi_b^n\right]_- + \kappa_{ab}\right) - i\psi_{an}\dot{\psi}_a^n \right] d\tau \ ,$$

where x^μ, e and f_{ab} are even and ψ_a^n, χ_a are odd variables (f_{ab} is antisymmetric), dependent on a parameter $\tau \in [0,1]$, $\mu = \overline{0,3}$; $a, b = \overline{1,N}$; $n = (\mu, 5) = \overline{0,3}, 5$; $\eta_{\mu\nu} = \mathrm{diag}(1-1-1-1)$; $\eta_{mn} = \mathrm{diag}(1-1-1-1-1)$. The summand $\frac{1}{2}\kappa_{ab}\int_0^1 f_{ab}d\tau$, with even coefficients κ_{ab} plays the role of a Chern-Simons term and can be added only in case $N = 2$ without breaking

of the rotational gauge symmetry [37]. Thus, $\kappa_{ab} = \kappa\epsilon_{ab}\delta_{N,2}$ with an even constant κ and two dimensional Levi-Civita symbol ϵ_{ab}. Dirac quantization of theories with this action, particularly, for $N = 2$ was considered in [45,46,37] and canonical one in the case $N = 2$ in [16]. However, the problem to construct classical or pseudoclassical models for particles with any spins in arbitrary dimensions is not solved completely presently.

The author thanks Brazilian foundations CNPq for support.

REFERENCES

[1] F.A. Berezin, *The Method of Second Quantization* (Nauka, Moscow, 1965); *Introduction to Algebra and Analysis with Anticommuting Variables* (Moscow State University Press, Moscow, 1983); *Introduction to Superanalysis* (D. Reidel, Dordrecht, 1987)

[2] F.A. Berezin, M.S. Marinov, Pisma Zh. Eksp. Theor. Fiz. **21** (1975) 678 [JETP Lett. **21** (1975) 320]; Ann. Phys. (N.Y.) **104** (1977) 336

[3] R. Casalbuoni, Nuovo Cimento **A33** (1976) 115; 389

[4] A. Barducci, R. Casalbuoni and L. Lusanna, Nuovo Cimento **A35** (1976) 377

[5] L. Brink, S. Deser, B. Zumino, P. di Vechia and P. Howe, Phys. Lett. **B64** (1976) 435

[6] L. Brink, P. di Vechia and P. Howe, Nucl. Phys. **B118** (1977) 76

[7] A.P. Balachandran, P. Salomonson, B. Skagerstam and J. Winnberg, Phys. Rev. **D15**, (1977) 2308

[8] W.K.H. Panofsky and M. Phillips, *Classical Electricity and Magnetism* , (Addison-Wesley, 1962)

[9] P.A.M. Dirac, *Lectures on Quantum Mechanics* (New York: Yeshiva University, 1964)

[10] D.M. Gitman and I.V. Tyutin, *Quantization of Fields with Constraints* (Springer-Verlag, 1990)

[11] D.M. Gitman and I.V. Tyutin, Pis'ma Zh.Eksp.Teor.Fiz. **51**, 3 (1990) 188; Class. Quantum Grav. **7** (1990) 2131

[12] S.P. Gavrilov and D.M. Gitman, Class. Quantum Grav. **9** (1992) 1

[13] L.L. Foldy and S.A. Wouthuysen, Phys. Rev. **78** (1950) 29

[14] M.H.L. Pryce, Proc. R. Soc. A **195** (1948) 62

[15] G.V. Grigoryan and R.P. Grigoryan, Sov. J. Nucl. Phys. **53** (1991) 1062

[16] D.M Gitman, A.E. Gonçalves and I.V. Tyutin, Preprint IFUSP/P-1087 (1993); Int.J.Mod.Phys. A **10** (1995) 701

[17] E.S. Fradkin, *Green's Function Method in Quantized Field Theory and Quantum Statistics*, Proc. PhIAN **29** (Nauka, Moscow, 1965) [English transl.:Cons. Bureau, N.Y.1967]

[18] B.M. Barbashov, JETP **48** (1965) 607

[19] I.A. Batalin and E.S. Fradkin, Teor. Mat. Fiz. **5** (1970) 190

[20] M. Henneaux, C. Teitelboim, Ann. Phys. **143** (1982) 127

[21] N.V. Borisov and P.P. Kulish, Teor. Math. Fiz. **51** (1982) 335

[22] V.Ya. Fainberg and A.V. Marshakov, Nucl. Phys. **B306** (1988) 659; Proc. PhIAN **201** (1990) 139 (Nauka, Moscow, 1991)

[23] E.S. Fradkin, D.M. Gitman and Sh.M. Shvartsman, *Quantum Electrodynamics with Unstable Vacuum*, (Springer-Verlag, Berlin, 1991); Europhys. Lett. **15** (1991) 241

[24] E.S. Fradkin and D.M. Gitman, Phys. Rev. **D44** (1991) 3230

[25] J. Schwinger, Phys.Rev **82** (1951) 664

[26] B.S. De Witt, Rev. Mod. Phys. **29** (1957) 337

[27] F.A. Berezin, Uspekhi Fiz. Nauk. **132** (1980) 497; F.A. Berezin and M.A. Shubin, *Schrödinger Equation* (Moscow State University, Moscow, 1983)

[28] R.P. Feynman, Rev. Mod. Phys. **20** (1948) 367

[29] R.P. Feynman, Phys. Rev. **80** (1950) 440

[30] R.P. Feynman, Phys. Rev. **84** (1951) 108

[31] A.M. Polyakov, Mod. Phys. Lett. **A3** (1988) 325

[32] G.M. Korchemsky, Int. J. Mod. Phys. **A7** (1992) 339

[33] D.M. Gitman and Sh.M. Shvartsman, Phys. Lett. **B 318** (1993) 122; Errata, Phys. Lett. **B 331** (1994) 449

[34] D.M. Gitman and A.V. Saa, Modern Phys. Lett. **A8** (1993) 463

[35] W. Pauli, Rev. Mod. Phys **13** (1941) 203

[36] D.M. Gitman and A.V. Saa, Class. Quant. Grav. **10** (1993) 1447

[37] P. Howe, S. Penati, M. Pernici and P. Townsend, Class. Quant. Grav. **6** (1989) 1125

[38] A. Barducci, R. Casalbuoni, D. Dominici and L. Lusanna, Phys. Lett. **B100** (1981) 126

[39] D.M Gitman, A.E. Gonçalves and I.V. Tyutin, Preprint IFUSP/P-1107 (1994); Phys. Rev. **D50** (1994) 5439

[40] D.M. Gitman, A.E. Gonçalves and I.V. Tyutin, Preprint IFUSP/P-1144, Marco/1995.

[41] M.S. Plyushchay, Mod. Phys. Lett. **A8**, 937 (1993).

[42] J.L. Cortes, M.S. Plyushchay and L. Velazquez, Phys. Lett. B **306**, 34 (1993).

[43] P. P. Srivastava, Nuovo Cim. Lett. **19** (1977) 239

[44] V. D. Gershun and V. I. Tkach, Pis'ma Zh. Eksp. Teor. Fiz. **29** (1979) 320

[45] A. Barducci and L. Lusanna, Nuovo Cim. **A77** (1983) 39; J. Phys. **A16** (1983) 1993

[46] J. Gomis, M. Novell and K. Rafanelli, Phys. Rev. **D34** (1986) 1072