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TWO-LOOP ONE-GLUON EXCHANGE POTENTIAL

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Abstract

Loop corrected perturbative one-gluon exchange potentials must be regularized to be used in phenomenological applications due to the presence of the Landau pole. Such regularization necessarily introduces more parameters or conditions in the theory. We assume that confinement is generated by some non specified non-perturbative mechanism and impose that the perturbative one-gluon exchange produces negligible effects at large distances. The running coupling constant is extracted from the renormalization group β function. It can be used to evaluate long-distance contributions in approaches devised to minimize renormalon ambiguities. Consequences of the particular behavior of the running coupling constant for phenomenological approaches based on linear confining potentials or bag models are discussed.

Perturbative expansions in field theory are asymptotic and must be truncated at some finite order in the coupling constant. This truncation introduces renormalization-scale and scheme dependences. Prescriptions to fix the scale and scheme are based on the apparent rate of convergence of the series, the size of the last term in the truncated series or the sensitivity to changes in the renormalization scale and scheme. There also physical criteria based on the mass scales in a given problem[1, 2]. In such schemes, effects appearing at two-loop order are absorbed by the one-loop running coupling constant.

The calculation of diagrams with such running coupling constant leads to the appearence of the infrared renormalons signalizing that one is using perturbation theory in the infrared region where it is not valid. In ref.[2] it was devised a method based on the operator product expansion to disentangle short-distance from long-distance effects. The short-distance contributions are free of renormalon ambiguities. The long-distance contribution must be combined with other nonperturbative corrections and only this sum is well defined and independent of scale. Without determining the other nonperturbative corrections one can only show that the long-distance correction is

small, suppressed by inverse powers of the large mass scale. To really estimate the size of the long-distance contributions one can try to incorporate nonperturbative effects using some realistic ansatz for the running coupling constant in the infrared region.

It has recently been shown that due to the infrared renormalon ambiguity one can extract either confining or non-confining potentials from the one-gluon exchange by using particular prescriptions to deform the contour away from the renormalon poles in the integration to reconstruct the potential from the inverse Borel transform[3]. Ambiguities in the running coupling constant lead also to ambiguities in the gluon condensate[4]. Physically motivated Ansätze for the running coupling constant and gluon condensate should be used. In the Dyson-Schwinger equation approach one assumes some Ansatz for the running coupling constant in momentum space to approximate the gluon propagator. In quark potential models, one also needs to know the associated potential. In all cases the high-momentum region should coincide with the coupling constant calculated perturbalively from the β function in QCD.

We propose to use heavy quarkonia to define the running coupling constant in the low-momentum region and, at the same time the nonperturbative mechanism responsible for confinement. As it is well known, in heavy quarkonia the nonrelativistic limit of QCD is appropriate and phenomenological potentials are very successful in reproducing the spectra. Furthermore, in the limit of static quarks lattice QCD determines a potential which can be fitted quite well by the perturbative one-gluon exchange plus linear confining potential[5]. It was shown that vacuum effects parametrized by the gluon condensate lead to the linear potential[6]. A gauge invariant nonperturbative calculation of charmonium and bottomonium spectra allows the determination of the gluon condensate value[7].

It is the one-gluon exchange part, calculated in perturbation theory that contains the Landau pole. Theoretically, the better way to eliminate the Landau pole is to start from the β function of QCD and impose that the running coupling constant extrated from it satisfies specific conditions. This approach was used in ref.[8] imposing that the resulting one-gluon exchange potential behaves as determined by perturbation theory at small distances and at large distances gives the linear increasing potential. Such condition determines a relation between the QCD scale parameter $\Lambda_{\overline{MS}}$ and the string tension. However, to reproduce the heavy quarkonia spectra one needs a $\Lambda_{\overline{MS}}$

value of order of 500 MeV which seems to be excluded by other experimental data. Other prescription is to use a potential of the form[9]:

$$V(r) = V_{AF} + ar\,, \tag{1}$$

where

$$V_{AF} = -\frac{16\pi}{25}\frac{1}{rf(r)}[1 + \frac{2\gamma_E + \frac{53}{75}}{f(r)} - \frac{462\ln f(r)}{625f(r)}] \tag{2}$$

with

$$f(r) = \ln[1/(\Lambda_{\overline{MS}}r)^2 + b]\,. \tag{3}$$

The introduction of the parameter b removes the singularity of the asymptotic-freedom potential V_{AF}. This prescription does not allow a unique separation of short- and long-distance effects. Depending on the value of b one has to use different values for the string tension (or condensates that lead to such linear term). Therefore, in this prescription the nonperturbative contribution comes also from the "perturbative" one-gluon exchange. Furthermore, it is not renormalization group invariant.

Since the linear term can originate from the gluon condensate, a genuine nonperturbative quantity, we think a much better prescription is to use the running coupling constant defined by the β function with the condition that the one-gluon exchange potential be constant for large distances. This is a natural way to unambiguously connect short-distance effects with perturbative effects and long-distance with nonperturbative physics only. This can be implement with the following conditions:

$$\beta(y) = \beta_0 y^2 + \beta_1 y^3 + \mathcal{O}(y^4) \qquad \text{for} \quad y \to 0\,, \tag{4}$$

and

$$\beta(y) = -y + c + \mathcal{O}(1/y) \qquad \text{for} \quad y \to \infty\,, \tag{5}$$

From the solution of the renormalization group equation for the effective coupling constant α as a function of the scale $x = \ln(\Lambda r)$:

$$x = \frac{1}{\beta_0 \alpha} + \frac{\beta_1 \ln \alpha}{\beta_0^2} - \int_0^\alpha dy(\frac{1}{\beta(y)} - \frac{1}{\beta_0 y^2} + \frac{\beta_1}{\beta_0^2 y})\,, \tag{6}$$

one sees that for $r \to \infty$ the asymptotic behaviour (5) of the β-function implies that the (perturbative) part of the potential

$$V(r) = -\frac{4}{3}\frac{\alpha(r)}{r} \to \text{constant}\,, \tag{7}$$

indeed becomes constant at large distances.

We choose the constant c in (5) so that the simplest ansatz for the β function satisfying both (4) and (5) :

$$\beta(y) = \begin{cases} \beta_0 y^2 + \beta_1 y^3 & \text{for } y < y_m \\ -y + c & \text{for } y > y_m \end{cases}$$

is sufficiently smooth. Requiring first and second derivatives at y_m to match determines c and y_m in terms of the scheme independent and perturbatively known coefficients β_0, β_1 :

$$\begin{aligned} c &= y_m + \beta_0 y_m^2 + \beta_1 y_m^3 \\ 3\beta_1 y_m^2 &+ 2\beta_0 y_m + 1 = 0 \end{aligned} \tag{8}$$

This gives $y_m = 0.228$, which is the value of α_s at the scale ~3-5 GeV, where a 2-loop calculation can still be trusted. If one uses this perturbative potential to fit the lattice result[5] (in dimensionless units) a string tension of $2.7\Lambda^2$ or a gluon condensate $\phi^2 = 16\Lambda^4$ are needed (fig.1). Comparing to the very successful Cornell model[10] one sees that while the approach of ref.[9] affords a string tension 30% smaller than that of Cornell, the above renormalization group invariant definition needs a 14% larger string tension. It has however the advantage of reproducing the Cornell potential in the region of 0.3 to 0.8 fm, the important region for the heavy quarkonia spectra. Besides it has the right small-distance behavior, leading to more reliable calculations of decay annihilation rates.

If one uses the above definition for the running coupling constant in the infrared region one is able to estimate large-distance contributions in the calculation of physical quantities based on mass scales of the processes studied[2]. It has consequences also in hyperfine splitting calculations. The splittings depend on the derivative of the potential and since it becomes constant for large distances there are no long-distance spin effects as expected from phenomenology. This allows the use of the running coupling constant to calculate for instance the nucleon-delta splitting in the bag model, where all the nonperturbative effects are on the vacuum pressure.

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Fig.1 - Nonperturbative potentials that must be added to the renormalization group invariant perturbative potential (dashed line) so that the full potential coincides with the lattice result (solid line) in dimensionless units. Dots are generated from a linear term with a string tension of value $2.7\Lambda^2$. Crosses are generated from the gluon condensate of value $\phi^2 = 16\Lambda^4$.