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Abstract

We compute the mass correction at finite temperature and with external boundary conditions in an interacting scalar field theory. Some remarks are made about the validity of the result at one-loop order.

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Finite Temperature Quantum Field Theory (FTQFT) has an outstanding interest, whose range goes from condensed matter physics to QCD and cosmological problems. On the other hand, field theories defined on manifolds not homeomorphics to R^D displays a lot of appealing features, such as dynamical mass generation, vacuum instabilities and symmetry breaking or restoration [1]. This is the case also in the FTQFT. In both cases, the mathematical background underlying these (quantum) effects is the same – the nontrivial topological properties of the spacetime in which the theory is defined.

The Casimir effect [2] also illustrates the effect of *global constraints* on the dynamical properties of the theory. Besides, there is some controversy on the thermal effects in Casimir energy calculations (see for instance [3]).

In this letter, we generalize the computation of the dynamical mass generated by the imposition of Dirichlet *boundary conditions* (BC) on a pair of hyperplanes in a scalar field theory with a $\lambda\phi^4$ self-interaction [4, 5, 6], including finite temperature effects [1]. Although the theory is not renormalizable for $D \neq 4$, we perform the computation for generic D (at the one-loop order the infinities can be safely controlled). Moreover, for $D = 4$, renormalization properties are not affected by the thermal and boundary corrections [8].

In fact, external BC (Dirichlet, periodic, Neumann or any other) affect not only Feynman diagrams associated with energy calculations, but all diagrams: diagrams with and without boundary conditions can have finite numerical differences. So, other physical quantities (such as masses, coupling constants, anomalous magnetic moments [9], pair production probabilities,

[1] During the completion of this work, we became aware of a similar calculation in $D = 4$ for the same model, but with periodic conditions in both imaginary-time and spatial coordinates [7] (in [7], the behavior of the coupling constant was also investigated).

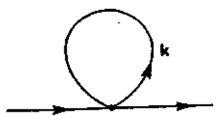
etc.) could, in principle, be affected by these external BC. Hence, as argued by Casimir, Wick normal ordering postulate is no longer valid for a field in these situations, and the diagrams usually discarded by the Wick prescription could contribute for the value of these quantities.

We conclude with some remarks about the validity of the results obtained, and we point out the needed for a resummation of the perturbative expansion into an effective expansion.

1. The bare Euclidean action is

$$S[\phi_0] = \int d^Dx\Big[-\frac{1}{2}\phi_0\Box\,\phi_0 + \frac{1}{2}m_0^2\phi_0^2 + \frac{\lambda_0}{4!}\phi_0^4\Big]. \qquad (1)$$

The parallel plates are located at $x_D = 0$ and $x_D = L$, where we have the following BC: $\phi_0(y, x_D = 0) = \phi(y, x_D = L) = 0$. Also, adopting the imaginary-time version of FTQFT, the field is periodic in the imaginary (Euclidean) time x_0, with period $\beta := \frac{1}{T}$ ($\hbar = k_B = c = 1$). The lowest order contribution in λ to the topological mass comes from the tadpole diagram (without propagators in the external legs)



which represents the 1PI self-energy part to order λ. Due to the formal analogy between the FTQFT and the Casimir effect [5, 6], the Feynman rules which describe the effect of the plates are similar to those that incorporate the effects of the temperature [10, 11]. The analytical expression of the diagram is then

$$\Sigma^{(1)} = \frac{1}{2}(-\lambda)\mu^{D-\omega}\sum_{n=1}^{\infty}\sum_{l=-\infty}^{\infty}\frac{1}{\beta L}\int\frac{d^{\omega-2}\bar{k}}{(2\pi)^{\omega-2}}\Big[\bar{k}^2+(\frac{\pi n}{L})^2+(\frac{2\pi l}{\beta})^2+m_R^2\Big]^{-1}, \qquad (2)$$

where $\bar{k} = (k_1, ..., k_{D-1})$, m_R is the renormalized mass, and μ is a mass scale ($\sim L^{-1}$) introduced to keep $\Sigma^{(1)}$ with the right ($\sim L^{-2}$) dimension. We are using the dimensional regularization method, $\int\frac{d^{D-2}k}{(2\pi)^{D-2}} \Longrightarrow \mu^{D-\omega}\int\frac{d^{\omega-2}k}{(2\pi)^{\omega-2}}$, and an analytical continuation to the neighbourhood of $\omega = D$ is to be performed after the identification and elimination of the polar part. The mass and wave function counterterms are defined by imposing the limit

$$\lim_{\omega\to D} Z_\phi[p^2 + Z_\phi^{-1}(m_R^2 + \delta m^2) - \Sigma^{(1)}] \qquad (3)$$

be finite [12] ($Z_\phi = 1 + \delta Z_\phi$). At this order, there is no coupling constant renormalization.

With the aid of the relation

$$d^Dk = \frac{2\pi^{D/2}}{\Gamma(D/2)}|k|^{D-1}d|k|, \qquad (4)$$

the integration of the continuous part leads directly to

$$\Sigma^{(1)} = -\frac{\lambda\pi^{\omega/2-1}\mu^{D-\omega}}{2(2\pi)^{\omega-2}\beta L}\Gamma(2-\omega/2)\sum_{n=1}^{\infty}\sum_{l=-\infty}^{\infty}\Big[(\frac{\pi n}{L})^2 + (\frac{2\pi l}{\beta})^2 + m_R^2\Big]^{\omega/2-2}, \qquad (5)$$

which we write as (see the appendix)

$$\begin{aligned}\Sigma^{(1)} &= -\frac{\lambda\pi^{\omega/2-1}\mu^{D-\omega}}{2(2\pi)^{\omega-2}\beta L}\Gamma(2-\omega/2)\\ &\quad\times\Big[E_1^{m_R^2}(2-\omega/2;\pi^2/L^2) + 2E_2^{m_R^2}(2-\omega/2;\pi^2/L^2;4\pi^2/\beta^2)\Big].\end{aligned} \qquad (6)$$

Using the analytical continuation of the inhomogeneous Epstein function, collected in the appendix, equations (A2) and (A3), we obtain ($\nu = \frac{m_R L}{\pi}$)

$$\begin{aligned}\Sigma^{(1)} &= -\frac{\lambda\pi^{\omega/2-1}\mu^{D-\omega}}{2(2\pi)^{\omega-2}}\Big\{-\frac{m_R^{\omega-3}}{4\sqrt{\pi}L}\Gamma(3/2-\omega/2)+\frac{m_R^{\omega-2}}{4\pi}\Gamma(1-\omega/2)\\ &+\frac{m_R^{\omega-2}}{\pi}\sum_{n=1}^{\infty}\frac{K_{\omega/2-1}(2m_R Ln)}{(m_R Ln)^{\omega/2-1}}+4\frac{\sqrt{\pi}(L\beta)^{3/2-\omega/2}}{(2\pi)^{5/2-\omega/2}L}\\ &\times\sum_{n,l=1}^{\infty}\Big(\frac{l^2}{n^2+\nu^2}\Big)^{3/4-\omega/4}K_{\omega/2-3/2}\Big(\frac{\beta l\pi}{L}\sqrt{n^2+\nu^2}\Big)\Big\}. \qquad (7)\end{aligned}$$

The polar part of $\Sigma^{(1)}$ is given by $\Gamma(1-\omega/2)$ (poles in $\omega = 2,4,6,...$; D even), and $\Gamma(3/2-\omega/2)$ (poles in $\omega = 3,5,7...$; D odd). For D even, the relevant part of $\Sigma^{(1)}$ is

$$\bar{\Sigma}^{(1)} = -\frac{\lambda\pi^{\omega/2-1}\mu^{D-\omega}}{2(2\pi)^{\omega-2}}\frac{m_R^{\omega-2}}{4\pi}\Gamma(1-\omega/2). \qquad (8)$$

Expanding $\bar{\Sigma}^{(1)}$ around the simple pole in $\omega = D$, we obtain ($n = D/2-1$, $\epsilon = \frac{D-\omega}{2}$)

$$\begin{aligned}\bar{\Sigma}^{(1)} &= -\frac{\lambda\pi^{\omega/2-1}}{2(2\pi)^{\omega-2}}\frac{m_R^{\omega-2}}{4\pi}[1+\frac{(D-\omega)}{2}\ln\mu^2+O(\epsilon^2)]\\ &\times\frac{(-1)^{D/2-1}}{(D/2-1)!}\Big[\frac{2}{D-\omega}+\psi(D/2)+O(\epsilon)\Big]\\ &= -\frac{(-1)^{D/2-1}}{(D/2-1)!}\frac{\lambda\pi^{\omega/2-1}}{2(2\pi)^{\omega-2}}\frac{m_R^{\omega-2}}{4\pi}\\ &\times\Big[\frac{2}{D-\omega}+\ln\mu^2+\psi(D/2)+O(\epsilon)\Big]. \qquad (9)\end{aligned}$$

We can obtain from (3) the mass counterterm to the order λ. Thus,

$$\delta m^2 = -\frac{(-1)^{D/2-1}}{(D/2-1)!}\frac{\lambda\pi^{D/2-2}}{4(2\pi)^{D-2}}m_R^{D-2}(D-\omega)^{-1}. \qquad (10)$$

There is no field scale renormalization to this order. The result (10) agrees with those of [13] for $D = 4$.

The renormalized self-energy to the order λ is given by

$$\begin{aligned}\Sigma_R^{(1)} &= -\frac{\lambda\pi^{D/2-1}}{2(2\pi)^{D-2}}\Big\{-\frac{m_R^{D-3}}{4\sqrt{\pi}L}\Gamma(3/2-D/2)\\ &+\frac{(-1)^{D/2-1}}{(D/2-1)!}\frac{m_R^{D-2}}{4\pi}[\ln\mu^2+\psi(D/2)]+\frac{m_R^{D-2}}{\pi}\sum_{n=1}^{\infty}\frac{K_{D/2-1}(2m_R Ln)}{(m_R Ln)^{D/2-1}}\\ &+4\frac{\sqrt{\pi}(L\beta)^{3/2-D/2}}{(2\pi)^{5/2-D/2}L}\sum_{n,l=1}^{\infty}\Big(\frac{l^2}{n^2+\nu^2}\Big)^{\frac{3-D}{4}}K_{D/2-3/2}\Big(\frac{\beta l\pi}{L}\sqrt{n^2+\nu^2}\Big)\Big\}. \qquad (11)\end{aligned}$$

Taking the limit $m_R \to 0$ (for $D > 3$), we obtain

$$\begin{aligned}\Sigma_R^{(1)}|_{m_R\to 0} &= -\frac{\lambda\pi^{D/2-2}}{4(2\pi)^{D-2}L^{D-2}}\zeta_R(D-2)\Gamma(D/2-1)\\ &-2\frac{\lambda\pi^{D/2-1/2}\beta^{3/2-D/2}}{(2\pi)^{D/2+1/2}L^{D/2-1/2}}\sum_{n,l=1}^{\infty}\Big(\frac{l}{n}\Big)^{3/2-D/2}K_{D/2-3/2}\Big(\frac{\beta nl\pi}{L}\Big). \qquad (12)\end{aligned}$$

Here we have used the expansion

$$\Big(\frac{x}{2}\Big)^{\nu}K_\nu(x) = \frac{1}{2}\Gamma(\nu)+O(x^2), \qquad (\nu > 0, \quad x \ll 1) \qquad (13)$$

and identified the Riemann zeta function

$$\zeta_R(d) = \sum_{n=1}^{\infty} n^{-d}. \qquad (14)$$

According to definition (3), the square of the topological mass is given by the negative of this value. Hence,

$$m_T^2(\beta, L) = \frac{\lambda \pi^{D/2-2}}{4(2\pi)^{D-2}L^{D-2}}\zeta_R(D-2)\Gamma(D/2-1)$$
$$+2\frac{\lambda\pi^{D/2-1/2}\beta^{3/2-D/2}}{(2\pi)^{D/2+1/2}L^{D/2-1/2}}\sum_{n,l=1}^{\infty}(\frac{l}{n})^{3/2-D/2}K_{D/2-3/2}(\frac{\beta n l \pi}{L}). \quad (15)$$

In the zero temperature limit, $\beta \to \infty$, we can use the approximation

$$K_\nu(x) \simeq \left(\frac{\pi}{2x}\right)^{1/2} e^{-x}, \quad (x >> 1) \quad (16)$$

to obtain

$$m_T^2(L, T=0) = \frac{\lambda\pi^{D/2-2}}{4(2\pi)^{D-2}L^{D-2}}\zeta_R(D-2)\Gamma(D/2-1). \quad (17)$$

For the relevant case of $D=4$, we get from (15) and (17),

$$m_T^2(L,T) = \frac{\lambda}{96L^2} + \frac{\lambda}{2\pi L}\sqrt{\frac{T}{L}}\sum_{n,l=1}^{\infty}(\frac{n}{2l})^{1/2}K_{1/2}(\frac{nl\pi}{TL}). \quad (18)$$

The first term agrees with the result of [5, 6]. The second term contains the thermal correction to the topological mass.

2. To check the above result, we take the limit $L \to \infty$. In this limit, the first term of (18) trivially disappears. For the second term, we must consider that

$$\frac{1}{L}\sum_{n=1}^{\infty} f\left((\frac{\pi n}{L})^2\right) \Longrightarrow \int \frac{dk}{2\pi} f(k^2). \quad (19)$$

Thus, in the limit $L \to \infty$:

$$\begin{aligned} m_T^2(L\to\infty, T) &= \frac{\lambda}{2\pi}\sum_{l=1}^{\infty}\left(\frac{T}{2\pi l}\right)^{1/2}\int\frac{dk}{2\pi}(k^2)^{1/4}K_{1/2}(\frac{l\sqrt{k^2}}{T}) \\ &= \frac{\lambda T^2}{2\sqrt{2}\pi^2\Gamma(1/2)}\zeta_R(2)\int_0^\infty dz\, z^{1/2}\, K_{-1/2}(z), \quad (20) \end{aligned}$$

where in the second line we used (4), the reflection formula $K_\nu(z) = K_{-\nu}(z)$, and defined $z = \frac{l|k|}{T}$. Then, with the aid of the formula [14]

$$\int dz\, z^\nu\, K_{\nu-1}(z) = -z^\nu\, K_\nu(z), \quad (21)$$

and the expansions (13) and (16), we obtain

$$m_T^2(T, L\to\infty) = \frac{\lambda}{24}T^2, \quad (22)$$

which is just the (leading) result for the thermal mass according to [11]. Thus, our result, eq. (15), reduces to the well known cases when $T=0$, eq.(17) for $D=4$, and when $L\to\infty$, eq.(22).

3. We observe that the result (15) is also valid for D even. In this case, however, the counterterm is not given by (10). In fact, it is L-dependent: the model is not renormalizable for $D \neq 4$, anyway.

CONCLUSIONS

We can see from (15) or (18) that the combined effects of the thermal and topological (one-loop) corrections is to increase the mass. In a theory with a spontaneously broken symmetry, this result indicates the possibility of a symmetry restauration when T increases and/or L decreases. This conclusion agrees with those of the reference [7]. Nevertheless, the result (12) is the first term of an expansion for m_R very small; the virtue of this is to isolate the purely topological and thermal corrections (the generalization for $m_R \neq 0$ is straightforward). The real problem is for $m_R^2 < 0$. In this case, the summations in (2) are ill-defined, and the expression for $\Sigma^{(1)}$ has real poles (not of UV nature). This is related to the instability of the effective potential found in [11].

On the other hand, a well known problem in high-temperature field theory are the hard thermal loops, for which the relevant infrared cutoff in loop diagrams is the thermal mass. Thus, an infinite subset of diagrams formally of higher order in the perturbative expansion can contribute to a given lower order. This signals a breakdown of the perturbative expansion at some order of the coupling constant (note that the result (22) is $\sim \lambda T^2$). The improved solution is to resum the perturbative series into an effective expansion, systematically including all effects to leading order in the coupling constant [15]. In FTQFT, this program has been worked-out up to the three loop order and beyond [16]. We observe that a similar problem can also occur in the case of compactified space (for both periodic or Dirichlet boundary condition): analogously to eq. (22), eq.(18) shows that the purely topological mass is ($\sim \lambda L^{-2}$). Thus, in principle, we expect problems for loops with "soft" external momenta (p_0 and $|\vec{p}|$ of order $\lambda^{1/2}L^{-1}$) and "hard" internal momenta (k_0 and $|\vec{k}|$ of order L^{-1}), for $L^{-1} >> \lambda^{1/2}L^{-1} >> m_0$.

There is also the interesting (and novel) question of breakdown of the perturbative expansion due to both thermal and topological effects. In this case, the resummed effective expansion can involve two parameters, β and L. These questions are under study, and will be the subject of a separate letter.

APPENDIX

Here, we give the analytical continuations of the modified inhomogeneous Epstein functions used in the main text. For details, see [17, 18]. This function is defined by

$$E_N^{c^2}(s;a_1,a_2,...,a_N) := \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\cdots\sum_{n_N=1}^{\infty}\frac{1}{[a_1n_1^2+...+a_Nn_N^2+c^2]^s}, \qquad \text{(A.1)}$$

where $a_1,...a_N, c^2 > 0$, N is an integer, and $\mathcal{R}s$ is large enough. The above series converge only in a definite range of s. The analytical continuation to a meromorphic function in the complex s plane is, for $N=1$,

$$\begin{aligned} E_1^{\nu^2}(s,1) &= -\frac{1}{2\nu^{2s}}+\frac{\sqrt{\pi}\,\Gamma(s-\frac{1}{2})}{2\Gamma(s)\nu^{2s-1}} \\ &+\frac{2\sqrt{\pi}}{\Gamma(s)}\sum_{n=1}^{\infty}\left(\frac{n\pi}{\nu}\right)^{s-\frac{1}{2}}K_{s-\frac{1}{2}}(2\pi n\nu), \end{aligned} \qquad \text{(A.2)}$$

where $K_\alpha(x)$ is the modified Bessel function of the second kind. Similarly, for $N=2$

$$\begin{aligned} E_2^{c^2}(s;a_1,a_2) = &-\frac{1}{2}E_1^{c^2}(s;a_1)+\frac{1}{2}\sqrt{\frac{\pi}{a_2}}\frac{\Gamma(s-1/2)}{\Gamma(s)}E_1^{c^2}(s-1/2;a_1)+\frac{2\pi^s}{\Gamma(s)a_2^{s/2+1/4}} \\ &\times\sum_{n_1,n_2=1}^{\infty}n_2^{s-1/2}(a_1n_1^2+c^2)^{1/4-s/2}K_{1/2-s}\left(\frac{2\pi n_2}{\sqrt{a_2}}\sqrt{a_1n_1^2+c^2}\right). \end{aligned} \qquad \text{(A.3)}$$

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