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PHENOMENOLOGICAL CONSTRAINTS ON PARAMETERS OF THREE NEUTRINO MIXING

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Taking into account previous analyses that constrained some of the parameters involved in three neutrino mixing and investigating a solution to the solar neutrino problem in the scenario of three generation neutrino oscillations in vacuum, we obtain a complete set of phenomenologically acceptable mixing angles and mass differences for the context of three neutrino mixing. We compare this solution with that one coming from two neutrino vacuum oscillation hypothesis to verify that there is a tiny possibility to differentiate them.

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Although direct measurements of neutrino masses and mixing angles are all consistent with zero [1,2], it is well known that some issues, as the solar [3] and the atmospheric [4] neutrino problems, may be an indication that these are non-vanishing parameters once that an economical way of interpreting both experimental results relies on the neutrino oscillation hypothesis [5]. Most of the analyses aiming to conciliate these data with theoretical calculations deal with two generation neutrino oscillations, based on the assumption that there is only one non-vanishing mixing angle and only one convenient non-vanishing squared mass difference generating neutrino oscillations. Nevertheless, there is no fundamental reason for not considering three generation neutrino oscillations in the interpretation of solar or atmospheric neutrino data. Moreover, some experimental evidence has been accumulated for the existence of three light neutrinos [6].

The main difficulty concerning three generation neutrino oscillations is connected with the appearance of too many free oscillating parameters, namely, three mixing angles, one phase and two neutrino squared mass differences. When three neutrino generation oscillation phenomenon is considered in literature such problem is usually overcome arbitrarily fixing some of these free parameters.

In a recent paper [7], using the following parameterization of the mixing matrix [6,8]

$$V = \begin{pmatrix} c_\theta c_\beta & s_\theta c_\beta & s_\beta \\ -s_\theta c_\gamma - c_\theta s_\gamma s_\beta & c_\theta c_\gamma - s_\theta s_\gamma s_\beta & s_\gamma c_\beta \\ s_\theta s_\gamma - c_\theta c_\gamma s_\beta & -c_\theta s_\gamma - s_\theta c_\gamma s_\beta & c_\gamma c_\beta \end{pmatrix}, \tag{1}$$

where β, γ and θ are mixing angles, and considering the little restrictive hierarchy among neutrino masses $m_1 \lesssim m_2 \ll m_3$, investigating τ leptonic decay, pion decay, Z^0 invisible width, τ decay end-point into five pions and assuming world average data for the ratio G_τ/G_μ (the coupling constants of the τ and μ to the charged weak current), we have determined two angles ($11.54^o < \beta < 12.82^o$ and $\gamma < 4.05^o$) and the higher mass ($m_3 \sim 165$ MeV) [7]. An updating of such analysis using the 1994 values of the above mentioned data was performed in Ref. [9]. It was found a smaller value for the higher mass ($m_3 \sim 90$ MeV), still well above the upper limits usually accepted for this mass [2]. Note however that such limits do not apply if we are considering neutrino mixing once that in this case the suppression of the population near the end-point of the tau into multi-pions decay spectrum can be compensated by an enhancement due to resonant mechanisms of hadronization [10].

The purpose of this Brief Report is to complete this set of three neutrino mixing parameters fixing the remaining mixing angle θ and the squared mass difference of the two lightest neutrino eigenstates $\delta m^2 = m_2^2 - m_1^2$, taking

into account the solar neutrino data. We show that considering neutrino oscillations in vacuum, including oscillating terms and, consequently, introducing the several solar neutrino spectra, it is possible to find regions in $\delta m^2 \times \sin^2 2\theta$ parameter space that fit all solar neutrino data in this scenario. Our analysis includes also the updated results of Ref. [9], once that from Fig. 2 of Ref. [7] it is possible to conclude that smaller values of m_3 imply also smaller values for the mixing angle β. And it is well known that the limit case where $\beta \to 0$ leads to the well established case of two generation neutrino oscillations in vacuum [11]. Therefore the values from Ref. [7] can be thought as the opposite phenomenologically acceptable limit case involving three neutrino generations. We will consider both limits as well as intermediate possibilities.

The solar neutrino problem has been confirmed by many experiments. In the following we will consider experimental data from Homestake, Kamiokande and ^{71}Ga experiments [3]. Each of them are sensitive to different types of neutrinos and therefore several different spectra have to be taken into account. We can compare the theoretical solar neutrino flux (ϕ_{th}) calculated from the standard solar model [12] with the observed flux (ϕ_{exp}) measured by each one of the above mentioned experiments. The ratio $R = \phi_{exp}/\phi_{th}$ is given by R(Homestake) $= 0.32 \pm 0.05$, R(Kamiokande) $= 0.51 \pm 0.07$, and for the two experiments based on ^{71}Ga detectors: R(Gallex) $= 0.59 \pm 0.08$ and R(Sage) $= 0.53 \pm 0.10$ [3]. These two last numbers are statistically compatible and we will consider in the present analysis only the Gallex result and not the corresponding weighted average.

In the following we write the fraction of the total solar neutrino flux to be detected in each of these experiments after neutrinos having oscillated in vacuum with survival probability $P_{\nu_e\to\nu_e}(E)$. For Homestake (H) and Gallex (G):

$$R(J) = \sum_X a_X^J \int_{E>E_{thre}^J} f^X(E) P_{\nu_e\to\nu_e}(E) dE, \tag{2}$$

where a_X^J is a coefficient indicating the sensitivity of each experiment $J = H, G$ to specific neutrinos produced in nuclear reactions. The normalized to the unit energy spectrum of neutrinos produced in reaction X and the threshold energy for each one of these experiments are denoted by $f^X(E_i)$ and E_{thre}^J, respectively, and given by Ref. [12].

Due to the high energy threshold kept in Kamiokande experiment, approximately only the highest energy neutrinos produced in ^8B nuclear reaction will be detected in this experiment. Nevertheless, neutrino flux measured by Kamiokande facilities is not merely the electron neutrino one since detector electrons will interact with other neutrino flavors via neutral currents. For energies involved in the solar neutrino experiments, the ν_e-electron scattering cross section $\sigma(E)$ is about seven times larger than ν_μ-electron and ν_τ-electron cross sections. Furthermore, this cross-section varies linearly with energy [13], hence, for Kamiokande (K) we have:

$$R(K) = \frac{1}{C} \int_{E>E_{thre}^J} \sigma(E) f^{^8B}(E) \left[\frac{1}{7} + \frac{6}{7} P_{\nu_e\to\nu_e}(E)\right] dE. \tag{3}$$

where C is a normalization constant obtained by the above integral when $P_{\nu_e\to\nu_e}(E) = 1$.

Note that Eqs. (2) and (3) are valid if neutrinos oscillate in both two or three generation scenarios since the only difference between these two possibilities is concentrated on the input survival probability $P_{\nu_e\to\nu_e}(E)$. Here we use the mixing matrix parameterization shown in Eq. (1) to obtain the probability of finding a neutrino ν_e after a length r where it oscillated in a context of three generations, if at the origin it was a pure ν_e. $P_{\nu_e\to\nu_e}(E) = |\langle\nu_e(r)|\nu_e(0)\rangle|^2$, or explicitly [5]:

$$P_{\nu_e\to\nu_e}(E) = 1 - 2c_\beta^2 s_\beta^2 \left(1 - \cos\frac{2\pi r}{L}\right) - 2c_\theta^2 s_\theta^2 c_\beta^4 \left(1 - \cos\frac{2\pi r}{L_{12}}\right), \tag{4}$$

where the wavelengths $L_{ij} = 2\pi/(E_i - E_j)$, that is, $L_{12} = 4\pi p/\delta m^2$ and we have considered a heavy non-relativistic third neutrino such that $L_{13} \approx L_{23} \approx L \approx 2\pi/m_3$.

As we said before, it can be seen from Eq. (4) that the limit case where $\beta \to 0$ corresponds to simple two neutrino oscillations. Furthermore, for the mass range we are considering here, the shorter wavelength $L \approx 3.8 \times 10^{-8}$ eV^{-1} is much smaller than the typical distance $r \approx 7.5 \times 10^{17}$ eV^{-1} traveled by the neutrinos from the Sun to the Earth and, therefore, we can average out the cosine term involving L in Eq. (4).

Finally, we can introduce Eq. (4) for the electron neutrino survival probability after vacuum oscillations into Eqs. (2) and Eq. (3) and compare the results with the ratios $R(J)$ for each of the relevant experiments ($J = H, K, G$). From this procedure we can find the parameter region where oscillation effects make the theoretical values of the solar neutrino survival probability compatible with the experimental solar neutrino data.

In Fig. 1 we show this compatibility region in $\delta m^2 \times \sin^2 2\theta$ parameter space for the three solar neutrino experiments at 68% (Fig.1a), 90% (Fig.1b) and 95% C.L. (Fig.1c). In each of these figures two cases are shown. The solid contours indicate the case where the mixing angle β assumes its maximum phenomenologically acceptable value around 12^o [7], which corresponds to the maximum value for $m_3 \approx 165$ MeV. The dashed contours are obtained assuming the other limit case where $\beta = 0$, which reduces the oscillation phenomenon to a simple two neutrino generation one. Note that the intermediate case where $m_3 \approx 90$ MeV [9], and consequently $0^o \lesssim \beta \lesssim 12^o$, lies in between these two shown areas, being a continuous and not large deformation of them. The results obtained in this figure show that the values of these parameters have to lie on $\sin^2 2\theta > 0.65$ and $3 \times 10^{-11}\,\mathrm{eV}^2 \lesssim \delta m^2 \lesssim 10^{-10}\,\mathrm{eV}^2$, for 95% C.L. case and smaller regions for other cases. It can be noticed also that the two generation limit at 95% shown in Fig.1c was also found in Ref. [11].

We observe also from this figure that the solutions involving two and three neutrino oscillations in vacuum do not completely overlap. Interesting enough, two neutrino solution is not entirely nested in the region indicating a three neutrino solution. This is because transitions from the first to the third family, which are not present in two neutrino oscillations, can push the electron neutrino survival probability below the minimum value allowed by the experimental data. Detailed experimental investigation of these non overlapped regions would be, in principle, a possible way to distinguish two and three neutrino vacuum oscillation solutions to the solar neutrino problem. Nevertheless, both theoretical and experimental errors present in the data used in this analysis, make this distinction very difficult. Future solar neutrino experiments can improve this situation.

We conclude this paper with the following. Taking into account previous analysis that constrained some of the three neutrino mixing and mass parameters such that $11.54^o < \beta < 12.82^o$, $\gamma < 4.05^o$ and $m_3 < 165$ MeV, we obtained limits of variations for the remaining oscillating parameters by imposing a solution to the solar neutrino problem in a context of three neutrino vacuum oscillations. We obtained $\sin^2 2\theta > 0.65$ and $3 \times 10^{-11}\,\mathrm{eV}^2 \lesssim \delta m^2 \lesssim 10^{-10}\,\mathrm{eV}^2$, at 95% C.L. and smaller regions for other cases. Our analysis includes also other values for the mass parameter m_3 and consequently for the mixing angle β, which can be derived from other phenomenological analyses. We presented the limit case where the oscillation phenomenon occurs in only two neutrino generations to make evident some differences between two and three generation vacuum oscillating solutions to the solar neutrino problem.

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I. FIGURES

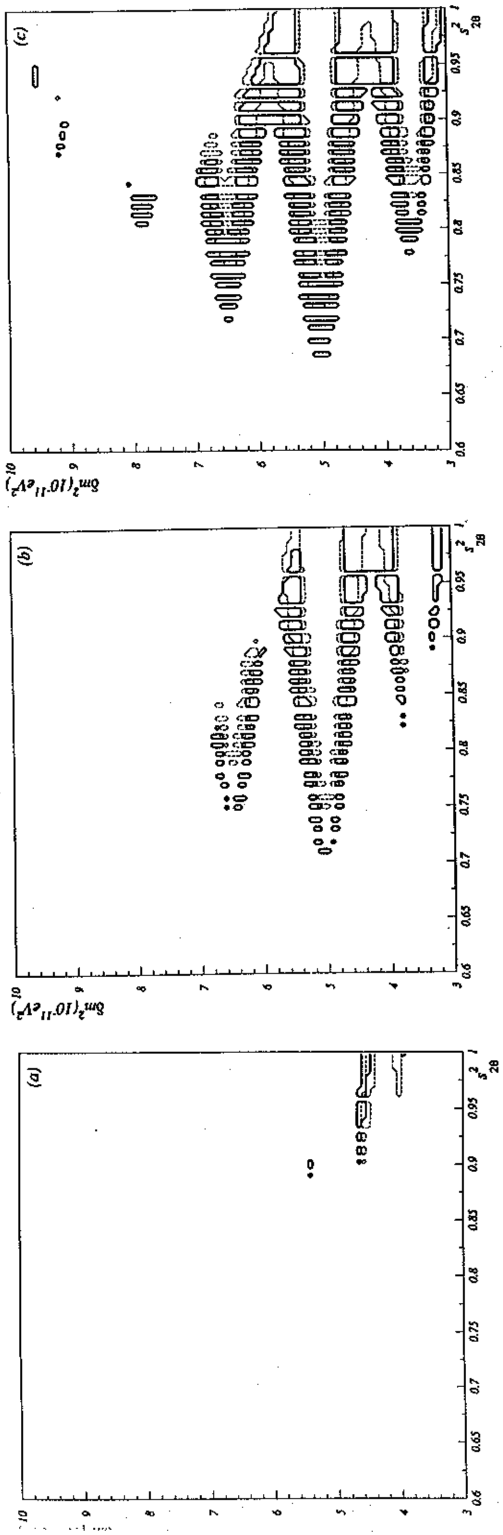

FIG. 1. Here we show the compatibility region in the plane $\delta m^2 \times \sin^2 2\theta$ for all experiments at 68% (a), 90% (b) and 95% (c) C.L. The dashed contours correspond to the two generation case and the solid contours to the three generation solution.