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**PATH INTEGRALS AND PSEUDOCLASSICAL  
DESCRIPTION FOR SPINNING PARTICLES IN  
ARBITRARY DIMENSIONS**

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# Path Integrals and Pseudoclassical Description for Spinning Particles in Arbitrary Dimensions

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generalized for the case of the particle with anomalous magnetic moment.

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## Abstract

The propagator of a spinning particle in external Abelian field and in arbitrary dimensions is presented by means of a path integral. The problem has different solutions in even and odd dimensions. In even dimensions the representation is just a generalization of one in four dimensions (it has been known before). In this case a gauge invariant part of the effective action in the path integral has a form of the standard (Berezin-Marinov) pseudoclassical action. In odd dimensions the solution is presented for the first time and, in particular, it turns out that the gauge invariant part of the effective action differs from the standard one. We propose this new action as a candidate to describe spinning particles in odd dimensions. Studying the hamiltonization of the pseudoclassical theory with the new action, we show that the operator quantization leads to adequate minimal quantum theory of spinning particles in odd dimensions. In contrast with the models proposed formerly in this case the new one admits both the operator and the path integral quantization. Finally the consideration is

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## I. INTRODUCTION

Construction of classical and pseudoclassical models of relativistic particles as well as their quantization (first quantization) attracts attention already for a long time due to various reasons. One can mention here both a natural interest to fill in gaps in fundamental theoretical constructions and well-known close relation to the string theory, where the first quantization remains until now the main way to quantum description. It seems that in four dimensions the work in this direction is almost done, namely, there exist wide-acceptable classical and pseudoclassical models (actions) for particles of different spins. Operator quantization of the models lead to the quantum mechanics which are equivalent to one-particle sectors of quantum field theories, whereas the path integral quantization reproduces the corresponding propagators. The main problem in such models is connected with spins description. Usually they introduce Grassmann variables to describe spins, that is why the models are called pseudoclassical. The basic pseudoclassical model for all those constructions in four dimensions is one for the spinning (spin one-half) particle, proposed first by Berezin and Marinov (BM) [1], and then discussed and studied in numerous papers [2]. It was shown [3] that the Dirac propagator in four dimensions can be presented as a path integral (with appropriate gauge fixing conditions) of  $\exp(iS)$ , where the effective action  $S$  is BM action.

If one extends formally BM model to arbitrary space-time dimensions, one can discover that in even dimensions it serves perfectly to describe particles with spin one-half, whereas in odd dimensions difficulties appear. Technically they are connected with the absence of the analog of  $\gamma^5$ -matrix in odd dimensions. Indeed, in the BM action in  $D$  dimensions there are  $D + 1$  dynamical Grassmann variables (see (3.2)) (a Lorentz vector  $\psi^\mu$ ,  $\mu = 0, 1, \dots, D - 1$ , and a Lorentz scalar  $\psi^D$ ) to describe spinning degrees of freedom. The corresponding operators  $\hat{\psi}^\mu$ ,  $\hat{\psi}^D$  obey the Clifford algebra

$$[\hat{\psi}^k, \hat{\psi}^n]_+ = -\frac{1}{2}\eta^{kn}, \quad k, n = 0, 1, \dots, D. \quad (1.1)$$

In even dimensions  $D = 2d$  one can always realize (1.1) by the choice:  $\hat{\psi}^\mu = \frac{i}{2}\gamma^\mu$ ,  $\hat{\psi}^D =$

$\frac{i}{2}\gamma^{D+1}$ , where  $\gamma^\mu$  are  $2^d \times 2^d$   $\gamma$ -matrices in  $D = 2d$  dimensions and  $\gamma^{D+1}$  is an analog of  $\gamma^5$  in four dimensions (see (2.14)). Thus, one gets the minimal quantum theory for spinning particle with  $2^d$  component Dirac equation. In  $D = 2d + 1$  the above mentioned possibility does not exist (there is no  $\gamma^{D+1}$  there) and one has to realize (1.1) by means of  $\gamma$ -matrices from the higher dimensions, for example, by means of  $2^{d+1} \times 2^{d+1}$   $\gamma$ -matrices in  $D + 1$  dimensions. Thus, one obtains a pair of Dirac equations, describing particles with spins  $1/2$  and  $-1/2$ . In odd dimensions these are different particles, related to different irreducible representations of the Lorentz group. Thus, the BM model in odd dimensions does not reproduce the minimal quantum theory in course of the operator quantization. One meets the same kind of difficulties when constructing a path integral representation for spinning particle propagator, generalizing, for example, the approach of the paper [4] to odd dimensions. All that indicates that BM action has to be modified in odd dimensions in order to provide both minimal operator and path integral quantization. One has to remark here that besides of general motivations mentioned above, consideration of the odd dimensional case is important in relation with the corresponding field theory (especially in  $2+1$  dimensions). In recent years the latter theory attracts great attention due to nontrivial topological properties and the possibility of existence of particles with fractional spins and exotic statistics (anyons).

Recently, there were proposed three different types of pseudoclassical actions to describe the massive spinning particles in odd-dimensional space-time, two in [5,6] respectively and the third one in [7]. The first one is classically equivalent to BM action, extended to odd dimensions. It is P- and T- invariant on the classical level and the violation of the symmetry takes place only on quantum level, so that an anomaly is presented. No path integral was written with this action. Moreover, as was remarked by the authors in [6], the P- and T- invariance of this action makes it difficult to understand how the path integral approach can take care of the difference between spins  $1/2$ , and  $-1/2$ . Another action [6] is already P- and T- noninvariant and reproduces the adequate quantum theory in course of quantization

The authors introduced two additional (to those which describe spin) dynamical Grassmann variables trying to avoid well-known difficulties in direct classical treatment of the model (see discussion in [8,9]). However the action is not supersymmetric. It was claimed that the path integral can be constructed, but it was not presented. In the papers [7] a different action was proposed, which is a natural dimensional reduction from the even-dimensional massless (Weyl particles [10,11] in even dimensions) case. The action is supersymmetric, P- and T- noninvariant and can be extended to describe higher spins in odd dimensions [12]. However, as in two previous cases no path integral quantization of the model was given.

In the present paper we have succeeded to construct the path integral representation for spinning particle propagator both in arbitrary even- and odd-dimensional cases. In even dimensions we followed the approach of our paper [4], where the path integral in four dimensions was constructed, using a super-generalization of the Schwinger proper-time representation [13] for the inverse operator, in which the proper time is presented by a pair of even and odd variables. In this case the effective action, which we extract from the path integral, coincides with BM one, extended to arbitrary even-dimensional case. In odd dimensions we have used a different technical trick to write the path integral representation. Namely, in this case one has to use a more complicated super-generalization of the Schwinger representation, where the proper-time has already one even and two Grassmann components. Thus, for the first time, we get a path integral representation for the spinning particle propagator in odd dimensions. Extracting a gauge-invariant part of the effective action from the path integral, we get a new pseudoclassical action to describe spinning particles in odd dimensions. Since the path integral quantization is already done by the construction, we have only to verify that the operator quantization, being applied to the new action, leads to the minimal quantum theory. To this end we analyse the Hamiltonian structure of the theory, deriving all the constraints and Dirac brackets. Then we present an explicit realization of the quantization procedure to get the Dirac equation in odd dimensions. In the end we discuss the peculiarities of the new representations obtained. Finally the consideration is generalized for the case of the particle with anomalous magnetic moment.

## II. PATH INTEGRAL REPRESENTATION FOR RELATIVISTIC PARTICLE PROPAGATOR IN ARBITRARY DIMENSIONS

### A. Scalar case in $D$ dimensions

Let us first discuss briefly path integral representation for scalar particle propagator in order to make then more transparent for the reader the problems connected with spinning degrees of freedom in arbitrary dimensions. We consider the particle placed into arbitrary external electromagnetic field with potentials  $A_\mu$ , that makes the problem non-trivial and allows one to reveal the features connected with electromagnetic nature of the particle. As it is known, the propagator of the scalar particle, interacting with an external electromagnetic field, is the causal Green function  $D^c(x, y)$  of the Klein-Gordon equation,

$$(\hat{p}^2 - m^2) D^c(x, y) = -\delta^D(x - y), \quad (2.1)$$

where  $\hat{p}_\mu = i\partial_\mu - gA_\mu(x)$ ,  $\mu = 0, \dots, D-1$ , and the Minkowski tensor is  $\eta_{\mu\nu} = \text{diag}(\underbrace{1, -1, \dots, -1}_D)$ . Following Schwinger [13], one can present  $D^c(x, y)$  as a matrix element of an operator  $\hat{D}^c$ ,

$$D^c(x, y) = \langle x | \hat{D}^c | y \rangle, \quad (2.2)$$

where  $|x\rangle$  are eigenvectors for some self-conjugated operators of coordinates  $X^\mu$ ; the corresponding canonical-conjugated operators of momenta are  $P_\mu$ , so that:

$$\begin{aligned} X^\mu |x\rangle &= x^\mu |x\rangle, & \langle x|y\rangle &= \delta^D(x - y), & \int |x\rangle \langle x| dx &= I, \\ [P_\mu, X^\nu]_- &= -i\delta_\mu^\nu, & P_\mu |p\rangle &= p_\mu |p\rangle, & \langle p|p'\rangle &= \delta^D(p - p'), \\ \int |p\rangle \langle p| dp &= I, & \langle x|P_\mu|y\rangle &= -i\partial_\mu \delta^D(x - y), & \langle x|p\rangle &= \frac{1}{(2\pi)^{D/2}} e^{ipx}, \\ [\Pi_\mu, \Pi_\nu]_- &= -igF_{\mu\nu}(X), & \Pi_\mu &= -P_\mu - gA_\mu(X). \end{aligned} \quad (2.3)$$

Equation (2.1) implies  $\hat{D}^c = \hat{F}^{-1}$ ,  $\hat{F} = m^2 - \Pi^2$ . Now one can use the Schwinger proper-time representation for the inverse operator

$$\hat{F}^{-1} = i \int_0^\infty e^{-i\lambda(\hat{F}-i\epsilon)} d\lambda, \quad (2.4)$$

where  $\lambda$  is the proper-time and the infinitesimal quantity  $\epsilon$  has to be put to zero at the end of calculations. Thus, we get for the Green function (2.2)

$$D^c = D^c(x_{out}, x_{in}) = i \int_0^\infty \langle x_{out} | e^{-i\hat{H}(\lambda)} | x_{in} \rangle d\lambda, \quad (2.5)$$

$$\hat{H}(\lambda) = \lambda (m^2 - \Pi^2).$$

Here and in what follows we include the factor  $-i\epsilon$  in  $m^2$ . Now one can present the matrix element entering in the expression (2.5) by means of a path integral. So, as usual, we write  $\exp(-i\hat{H}) = [\exp(-i\hat{H}/N)]^N$  and then insert  $(N-1)$  resolutions of identity  $\int |x\rangle \langle x| dx = I$  between all the operators  $\exp(-i\hat{H}/N)$ . Besides, we introduce  $N$  additional integrations over  $\lambda$  to transform then the ordinary integrals over these variables into the corresponding path-integrals,

$$D^c = i \lim_{N \rightarrow \infty} \int_0^\infty d\lambda_0 \int_{-\infty}^{+\infty} dx_1 \dots dx_{N-1} d\lambda_1 \dots d\lambda_N$$

$$\times \prod_{k=1}^N \langle x_k | e^{-i\hat{H}(\lambda_k)\Delta\tau} | x_{k-1} \rangle \delta(\lambda_k - \lambda_{k-1}), \quad (2.6)$$

where  $\Delta\tau = 1/N$ ,  $x_0 = x_{in}$ ,  $x_N = x_{out}$ . Bearing in mind the limiting process, one can calculate the matrix elements from (2.6) approximately,

$$\langle x_k | e^{-i\hat{H}(\lambda_k)\Delta\tau} | x_{k-1} \rangle \approx \langle x_k | 1 - i\hat{H}(\lambda_k)\Delta\tau | x_{k-1} \rangle, \quad (2.7)$$

using the resolution of identity  $\int |p\rangle \langle p| dp = I$ . In this connection it is important to notice that the operator  $\hat{H}(\lambda)$  has originally the symmetric form in the operators  $X$  and  $P$ . Indeed, the only one term in  $\hat{H}(\lambda)$ , which contains products of these operators is  $[P_\alpha, A^\alpha(X)]_+$ . One can verify that this is maximal symmetrized expression, which can be combined from entering operators (see remark in [14]). Thus, one can write  $\hat{H}(\lambda) = \text{Sym}_{(X,P)} \mathcal{H}(\lambda, X, P)$ , where  $\mathcal{H}(\lambda, x, p)$  is the Weyl symbol of the operator  $\hat{H}(\lambda)$ ,

$$\mathcal{H}(\lambda, x, p) = \lambda (m^2 - \mathcal{P}^2), \quad \mathcal{P}_\mu = -p_\mu - gA_\mu(x).$$

That is a general statement [15], which can be easily checked in that concrete case by direct calculations, that the matrix elements (2.7) are expressed in terms of the Weyl symbols at the middle point  $\bar{x}_k = (x_k + x_{k-1})/2$ . Taking all that into account, one can see that in the limiting process the matrix elements (2.7) can be replaced by the expressions

$$\int \frac{dp_k}{(2\pi)^D} \exp \left\{ i \left[ p_k \frac{x_k - x_{k-1}}{\Delta\tau} - \mathcal{H}(\lambda_k, \bar{x}_k, p_k) \right] \Delta\tau \right\}. \quad (2.8)$$

Using the integral representation for the  $\delta$ -functions, we get for the right side of (2.6)

$$D^c = i \lim_{N \rightarrow \infty} \int_0^\infty d\lambda_0 \int_{-\infty}^{+\infty} dx_1 \dots dx_{N-1} \frac{dp_1}{(2\pi)^D} \dots \frac{dp_N}{(2\pi)^D} d\lambda_1 \dots d\lambda_N$$

$$\times \frac{d\pi_1}{(2\pi)} \dots \frac{d\pi_N}{(2\pi)} \exp \left\{ i \sum_{k=1}^N \left[ p_k \frac{x_k - x_{k-1}}{\Delta\tau} - \mathcal{H}(\lambda_k, \bar{x}_k, p_k) + \pi_k \frac{\lambda_k - \lambda_{k-1}}{\Delta\tau} \right] \Delta\tau \right\}. \quad (2.9)$$

The above expression is, in fact, the definition of the Hamiltonian path integral for the scalar particle propagator,

$$D^c = i \int_0^\infty d\lambda_0 \int_{x_{in}}^{x_{out}} Dx \int_{\lambda_0} D\lambda \int Dp D\pi \exp \left\{ i \int_0^1 [\lambda(\mathcal{P}^2 - m^2) + p\dot{x} + \pi\dot{\lambda}] d\tau \right\}, \quad (2.10)$$

where  $\mathcal{P}_\mu = -p_\mu - gA_\mu(x)$ ,  $\dot{x} = \frac{d}{d\tau}x$ , and so on. The functional integration goes over the trajectories  $x^\mu(\tau), p_\mu(\tau), \lambda(\tau), \pi(\tau)$ , parameterized by some invariant parameter  $\tau \in [0, 1]$  and obeying the boundary conditions  $x(0) = x_{in}$ ,  $x(1) = x_{out}$ ,  $\lambda(0) = \lambda_0$ . To go over to the Lagrangian form of the path integral, one has to perform the integration over the momenta  $p$ . In fact, the result can be achieved by means of the replacement,  $p_\mu \rightarrow -p_\mu - (\dot{x}_\mu/2\lambda) - gA_\mu$ ,  $e = 2\lambda$ . Thus, we get

$$D^c = \frac{i}{2} \int_0^\infty d\epsilon_0 \int_{x_{in}}^{x_{out}} Dx \int_{\epsilon_0} M(\epsilon) D\epsilon \int D\pi$$

$$\times \exp \left\{ i \int_0^1 \left[ -\frac{\dot{x}^2}{2\epsilon} - \epsilon \frac{m^2}{2} - g\dot{x}_\mu A^\mu + \pi\dot{\epsilon} \right] d\tau \right\}, \quad (2.11)$$

where the boundary conditions  $x(0) = x_{in}$ ,  $x(1) = x_{out}$ ,  $\epsilon(0) = \epsilon_0$  are supposed and the measure  $M(\epsilon)$  has the form

$$M(\epsilon) = \int Dp \exp \left\{ \frac{i}{2} \int_0^1 \epsilon p^2 d\tau \right\}. \quad (2.12)$$

A discussion of the role of the measure (2.12) one can find in [4].

## B. Spinor case in even $D = 2d$ dimensions

As known, the propagator of a relativistic spinning particle is the causal Green's function  $S^c(x, y)$  of the Dirac equation. In  $D$  dimensions the equation for this function has the form

$$(\hat{P}_\mu \gamma^\mu - m) S^c(x, y) = -\delta^D(x - y), \quad (2.13)$$

where  $\hat{P}_\mu = i\partial_\mu - gA_\mu(x)$ ,  $\mu = 0, \dots, D-1$ , and  $\gamma^\mu$  are  $\gamma$ -matrices in  $D$  dimensions,  $[\gamma^\mu, \gamma^\nu]_+ = 2\eta^{\mu\nu}$ . Thus, in fact, we meet here the problem how to deal with an inverse operator (to the Dirac one  $\hat{P}_\mu \gamma^\mu - m$ ) which has a complicated  $\gamma$ -matrix structure. As it is known [16], in even dimensions a matrix representation of the Clifford algebra with dimensionality  $\dim \gamma^\mu = 2^{D/2} = 2^d$  always exists. In other words  $\gamma^\mu$  are  $2^d \times 2^d$  matrices. In such dimensions one can introduce another matrix,  $\gamma^{D+1}$ , which anticommutes with all  $\gamma^\mu$  (analog of  $\gamma^5$  in four dimensions),

$$\gamma^{D+1} = r\gamma^0\gamma^1\dots\gamma^{D-1}, \quad r = \begin{cases} 1, & \text{if } d \text{ is even} \\ i, & \text{if } d \text{ is odd} \end{cases}, \quad (2.14)$$

$$[\gamma^{D+1}, \gamma^\mu]_+ = 0, \quad (\gamma^{D+1})^2 = -1.$$

The existence of the matrix  $\gamma^{D+1}$  in even dimensions allows one to pass to the Dirac operator which is homogeneous in  $\gamma$ -matrices. Indeed, let us rewrite the equation (2.13) in terms of the transformed by  $\gamma^{D+1}$  propagator  $\tilde{S}^c(x, y)$ ,

$$\tilde{S}^c(x, y) = S^c(x, y)\gamma^{D+1}, \quad (\hat{P}_\mu \tilde{\gamma}^\mu - m\gamma^{D+1}) \tilde{S}^c(x, y) = \delta^D(x - y), \quad (2.15)$$

where  $\tilde{\gamma}^\mu = \gamma^{D+1}\gamma^\mu$ . The matrices  $\tilde{\gamma}^\mu$  have the same commutation relations as initial ones  $\gamma^\mu$ ,  $[\tilde{\gamma}^\mu, \tilde{\gamma}^\nu]_+ = 2\eta^{\mu\nu}$ , and anticommute with the matrix  $\gamma^{D+1}$ . The set of  $D+1$   $\gamma$ -matrices  $\tilde{\gamma}^\nu$  and  $\gamma^{D+1}$  form a representation of the Clifford algebra in odd  $2d+1$  dimensions. Let us denote such matrices via  $\Gamma^n$ ,

$$\Gamma^n = \begin{cases} \tilde{\gamma}^\mu, & n = \mu = 0, \dots, D-1 \\ \gamma^{D+1}, & n = D \end{cases}, \quad (2.16)$$

$$[\Gamma^k, \Gamma^n]_+ = 2\eta^{kn}, \quad \eta_{kn} = \text{diag}(\underbrace{1, -1, \dots, -1}_{D+1}), \quad k, n = 0, \dots, D.$$

In terms of these matrices the equation (2.15) takes the form

$$\hat{P}_n \Gamma^n \tilde{S}^c(x, y) = \delta^D(x - y), \quad \hat{P}_\mu = i\partial_\mu - gA_\mu(x), \quad \hat{P}_D = -m. \quad (2.17)$$

Now again, similar to (2.2), we present  $\tilde{S}^c(x, y)$  as a matrix element of an operator  $\hat{S}^c$  (the coordinate representation (2.3)),

$$\tilde{S}_{ab}^c(x, y) = \langle x | \hat{S}_{ab}^c | y \rangle, \quad a, b = 1, 2, \dots, 2^d, \quad (2.18)$$

where the spinor indices  $a, b$  are written here explicitly for clarity and will be omitted hereafter. The equation (2.17) implies  $\hat{S}^c = \hat{F}^{-1}$ ,  $\hat{F} = \Pi_n \Gamma^n$ , where  $\Pi_\mu$  are defined in (2.3), and  $\Pi_D = -m$ . The operator  $\hat{F}$  is homogeneous in  $\gamma$ -matrices, we can consider it as a pure Fermi one, if one reckons  $\gamma$ -matrices as Fermi-operators. Now, instead of the Schwinger proper-time representation (2.4) which is convenient for Bose-type operators one can use a different representation by means of an integral over the super-proper time ( $\lambda, \chi$ ) of an exponential with an even exponent. Namely, one can write<sup>1</sup>

$$\hat{S}^c = \frac{\hat{F}}{\hat{F}^2} = \int_0^\infty d\lambda \int e^{i\lambda(\hat{F}^2 + i\epsilon) + \chi\hat{F}} d\chi, \quad (2.19)$$

$$\hat{F}^2 = \Pi^2 - m^2 - \frac{ig}{2} F_{\mu\nu} \Gamma^\mu \Gamma^\nu,$$

where  $\lambda$  is an even variable and  $\chi$  is an odd one, the latter anticommutes with  $\hat{F}$  (with  $\gamma$ -matrices) by the definition. Here and in what follows integrals over odd variables are understood as Berezin's integrals [17]. The representation (2.19) is an analog of the Schwinger proper-time representation for the inverse operator convenient in the Fermi case. Such representation was introduced for the first time in the paper [4]. Thus, the Green function (2.18) takes the form

$$\tilde{S}^c = \tilde{S}^c(x_{out}, x_{in}) = \int_0^\infty d\lambda \int \langle x_{out} | e^{-i\hat{H}(\lambda, \chi)} | x_{in} \rangle d\chi, \quad (2.20)$$

$$\hat{H}(\lambda, \chi) = \lambda \left( m^2 - \Pi^2 + \frac{ig}{2} F_{\mu\nu} \Gamma^\mu \Gamma^\nu \right) + \Pi_n \Gamma^n \chi.$$

<sup>1</sup>Here and in what follows  $\Pi^2 = \Pi_\mu \Pi^\mu$  and so on.

Now one can present the matrix element entering in the expression (2.20) by means of a path integral. In spite of the fact that the operator  $\hat{H}(\lambda, \chi)$  has  $\gamma$ -matrix structure, it is possible to proceed as in scalar case. An analog of the formula (2.6) has the form

$$\begin{aligned} \tilde{S}^c &= \lim_{N \rightarrow \infty} \int_0^{\infty} d\lambda_0 \int d\chi_0 \int_{-\infty}^{+\infty} dx_1 \dots dx_{N-1} d\lambda_1 \dots d\lambda_N \int d\chi_1 \dots d\chi_N \\ &\times \prod_{k=1}^N \langle x_k | e^{-i\hat{H}(\lambda_k, \chi_k) \Delta\tau} | x_{k-1} \rangle \delta(\lambda_k - \lambda_{k-1}) \delta(\chi_k - \chi_{k-1}). \end{aligned} \quad (2.21)$$

The matrix elements in (2.21) can be replaced by the expressions

$$\int \frac{dp_k}{(2\pi)^D} \exp \left\{ i \left[ p_k \frac{x_k - x_{k-1}}{\Delta\tau} - \mathcal{H}(\lambda_k, \chi_k, \bar{x}_k, p_k) \right] \Delta\tau \right\}, \quad (2.22)$$

where  $\mathcal{H}(\lambda, \chi, x, p)$  is the Weyl symbol of the operator  $\hat{H}(\lambda, \chi)$  in the sector of coordinates and momenta,

$$\mathcal{H}(\lambda, \chi, x, p) = \lambda \left( m^2 - \mathcal{P}^2 + \frac{ig}{2} F_{\mu\nu} \Gamma^\mu \Gamma^\nu \right) + \mathcal{P}_n \Gamma^n \chi,$$

and  $\mathcal{P}_\mu = -p_\mu - gA_\mu(x)$ ,  $\mathcal{P}_D = -m$ . The multipliers (2.22) are noncommutative due to the  $\gamma$ -matrix structure and are situated in (2.21) so that the numbers  $k$  increase from the right to the left. For the two  $\delta$ -functions, accompanying each matrix element (2.22) in the expression (2.21), we use the integral representations

$$\delta(\lambda_k - \lambda_{k-1}) \delta(\chi_k - \chi_{k-1}) = \frac{i}{2\pi} \int e^{i[\pi_k(\lambda_k - \lambda_{k-1}) + \nu_k(\chi_k - \chi_{k-1})]} d\pi_k d\nu_k,$$

where  $\nu_k$  are odd variables. Then we attribute formally the index  $k$ , to  $\gamma$ -matrices, entering into (2.22), and then we attribute to all quantities the "time"  $\tau_k$ , according to the index  $k$  they have,  $\tau_k = k\Delta\tau$ , so that  $\tau \in [0, 1]$ . Introducing the T-product which acts on  $\gamma$ -matrices it is possible to gather all the expressions, entering in (2.21), in one exponent and deal then with the  $\gamma$ -matrices like with odd variables. Thus, we get for the right side of (2.21)

$$\begin{aligned} \tilde{S}^c &= T \int_0^{\infty} d\lambda_0 \int d\chi_0 \int_{x_{in}}^{x_{out}} Dx \int Dp \int_{\lambda_0} D\lambda \int_{\chi_0} D\chi \int D\pi \int D\nu \\ &\times \exp \left\{ i \int_0^1 \left[ \lambda \left( \mathcal{P}^2 - m^2 - \frac{ig}{2} F_{\mu\nu} \Gamma^\mu \Gamma^\nu \right) + \chi \mathcal{P}_n \Gamma^n + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi} \right] d\tau \right\}, \end{aligned} \quad (2.23)$$

where  $x(\tau)$ ,  $p(\tau)$ ,  $\lambda(\tau)$ ,  $\pi(\tau)$ , are even and  $\chi(\tau)$ ,  $\nu(\tau)$  are odd trajectories, obeying the boundary conditions  $x(0) = x_{in}$ ,  $x(1) = x_{out}$ ,  $\lambda(0) = \lambda_0$ ,  $\chi(0) = \chi_0$ . The operation of T-ordering acts on the  $\gamma$ -matrices which are supposed formally to depend on time  $\tau$ . The expression (2.23) can be transformed then as follows:

$$\begin{aligned} \tilde{S}^c &= \int_0^{\infty} d\lambda_0 \int d\chi_0 \int_{x_{in}}^{x_{out}} Dx \int Dp \int_{\lambda_0} D\lambda \int_{\chi_0} D\chi \int D\pi \int D\nu \exp \left\{ i \int_0^1 \left[ \lambda \left( \mathcal{P}^2 - m^2 \right. \right. \right. \\ &\left. \left. - \frac{ig}{2} F_{\mu\nu} \frac{\delta_l}{\delta\rho_\mu} \frac{\delta_l}{\delta\rho_\nu} \right) + \chi \mathcal{P}_n \frac{\delta_l}{\delta\rho_n} + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi} \right] d\tau \right\} T \exp \int_0^1 \rho_n(\tau) \Gamma^n d\tau \Big|_{\rho=0}, \end{aligned}$$

where odd sources  $\rho_n(\tau)$  are introduced. They anticommute with the  $\gamma$ -matrices by definition. One can present the quantity  $T \exp \int_0^1 \rho_n(\tau) \Gamma^n d\tau$  via a path integral over odd trajectories [4,18],

$$\begin{aligned} T \exp \int_0^1 \rho_n(\tau) \Gamma^n d\tau &= \exp \left( i \Gamma^n \frac{\partial_l}{\partial \theta^n} \right) \int_{\psi(0)+\psi(1)=\theta} \exp \left[ \int_0^1 \left( \psi_n \dot{\psi}^n - 2i \rho_n \psi^n \right) d\tau \right. \\ &\left. + \psi_n(1) \psi^n(0) \right] \mathcal{D}\psi \Big|_{\theta=0}, \quad \mathcal{D}\psi = D\psi \left[ \int_{\psi(0)+\psi(1)=0} D\psi \exp \left\{ \int_0^1 \psi_n \dot{\psi}^n d\tau \right\} \right]^{-1}, \end{aligned} \quad (2.24)$$

where  $\theta^n$  are odd variables, anticommuting with  $\gamma$ -matrices, and  $\psi^n(\tau)$  are odd trajectories of integration, obeying the boundary conditions, which are pointed out below the signs of the integration. Using (2.24) we get the Hamiltonian path integral representation for the Green function in question:

$$\begin{aligned} \tilde{S}^c &= \exp \left( i \Gamma^n \frac{\partial_l}{\partial \theta^n} \right) \int_0^{\infty} d\lambda_0 \int d\chi_0 \int_{\lambda_0} D\lambda \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int Dp \int D\pi \int D\nu \\ &\times \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi \exp \left\{ i \int_0^1 \left[ \lambda \left( \mathcal{P}^2 - m^2 + 2ig F_{\mu\nu} \psi^\mu \psi^\nu \right) + 2i \mathcal{P}_n \psi^n \chi \right. \right. \\ &\left. \left. - i \psi_n \dot{\psi}^n + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi} \right] d\tau + \psi_n(1) \psi^n(0) \right\} \Big|_{\theta=0}. \end{aligned} \quad (2.25)$$

Integrating over momenta, we get the path integral in the Lagrangian form,

$$\begin{aligned} \tilde{S}^c &= \exp \left( i \Gamma^n \frac{\partial_l}{\partial \theta^n} \right) \int_0^{\infty} de_0 \int d\chi_0 \int_{e_0} M(e) De \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int D\pi \int D\nu \\ &\times \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi \exp \left\{ i \int_0^1 \left[ -\frac{\dot{x}^2}{2e} - \frac{e}{2} m^2 - g\dot{x}A + ieg F_{\mu\nu} \psi^\mu \psi^\nu \right. \right. \\ &\left. \left. + i \left( \frac{\dot{x}_\mu \psi^\mu}{e} - m\psi^D \right) \chi - i\psi_n \dot{\psi}^n + \pi\dot{e} + \nu\dot{\chi} \right] d\tau + \psi_n(1) \psi^n(0) \right\} \Big|_{\theta=0}, \end{aligned} \quad (2.26)$$

where the measure  $M(e)$  is defined by the eq. (2.12).

### C. Spinor case in odd $D = 2d + 1$ dimensions

In odd dimensions a possibility to construct the matrix  $\gamma^{D+1}$  (2.14) does not exist. Hence, the trick which was used to make the Dirac operator homogeneous in  $\gamma$ -matrices does not work here. Nevertheless, the problem of the path integral construction may be solved in a different way.

As it is known, in odd dimensions  $D = 2d + 1$  there exist two exact non-equivalent irreducible representations of the Clifford algebra with the dimensionality  $2^{[D/2]} = 2^d$ . Let us mark these representations by the index  $s = \pm$ . Thus, we have two non-equivalent sets of  $\gamma$ -matrices which we are going to denote as  $\Gamma_{(s)}^n$ ,  $n = 0, 1, \dots, 2d$  (remark that now we use Latin indices  $n, k$ , and so on, as Lorentz ones). Such matrices can be constructed, e.g. from the corresponding matrices in  $D = 2d$  dimensions as follows:

$$\Gamma_{(s)}^n = \begin{cases} \gamma^\mu, & n = \mu = 0, \dots, D-1 \\ s\gamma^{D+1}, & n = D \end{cases}, \quad (2.27)$$

$$[\Gamma_{(s)}^k, \Gamma_{(s)}^n]_+ = 2\eta^{kn}, \quad \eta_{kn} = \text{diag}(\underbrace{1, -1, \dots, -1}_{D+1}), \quad k, n = 0, \dots, D.$$

In odd dimensions there exists also a duality relation which is important for our purposes

$$\Gamma_{(s)}^n = \frac{sr}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \Gamma_{(s)k_1} \dots \Gamma_{(s)k_{2d}}, \quad r = \begin{cases} 1, & \text{if } d \text{ is even} \\ i, & \text{if } d \text{ is odd} \end{cases}. \quad (2.28)$$

Here  $\epsilon^{nk_1 \dots k_{2d}}$  is the Levi-Civita tensor density in  $D$  dimensions.

The propagator  $S^c(x, y)$  obeys the Dirac equation in the dimensions under consideration

$$(\hat{\mathcal{P}}_n \Gamma_{(s)}^n - m) S^c(x, y) = -\delta^D(x - y), \quad (2.29)$$

where  $\hat{\mathcal{P}}_n = i\partial_n - gA_n(x)$ . Thus, we get for the operator  $\hat{S}^c$  entering in (2.18),  $\hat{S}^c = -\hat{F}^{-1}$ ,  $\hat{F} = \Pi_n \Gamma_{(s)}^n - m$ , where all the  $\Pi_n$  are defined by the equations (2.3). In the case under consideration it is convenient to present the inverse operator in the following form

$$\hat{S}^c = \frac{\hat{F}_{(+)}^{-1}}{-\hat{F}_{(+)} \hat{F}^{-1}} = s \frac{\hat{A}}{\hat{B}}, \quad \hat{F}_{(+)} = \Pi_n \Gamma_{(s)}^n + m, \quad (2.30)$$

$$\hat{A} = \frac{r}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \Pi_n \Gamma_{(s)k_1} \dots \Gamma_{(s)k_{2d}} + sm,$$

$$\hat{B} = m^2 - \Pi^2 + \frac{ig}{2} F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n.$$

The form of the operator  $\hat{A}$  in (2.30) was obtained from the operator  $\hat{F}_{(+)}$  by means of the duality relation (2.28). Now both operators  $\hat{A}$  and  $\hat{B}$  are even in  $\gamma$ -matrices, so we can treat them as Bose-type operators. For their ratio we are going to use a new kind of integral representation which is a combination of the Schwinger type (2.4) representation for  $\hat{B}^{-1}$  and additional representation of the operator  $\hat{A}$  by means of a Gaussian integral over two Grassmannian variables  $\chi_1$  and  $\chi_2$  with the involution property  $(\chi_1)^+ = \chi_2$ . Namely, one can write

$$\hat{S}^c = s \int_0^\infty d\lambda \int e^{-i[\lambda \hat{B} + \chi \hat{A}]} d\chi, \quad \chi = \chi_1 \chi_2, \quad d\chi = d\chi_1 d\chi_2. \quad (2.31)$$

Thus, we get for the propagator

$$S^c = s \int_0^\infty d\lambda \int \langle x_{\text{out}} | e^{-i\hat{\mathcal{H}}(\lambda, \chi)} | x_{\text{in}} \rangle d\chi, \quad (2.32)$$

$$\hat{\mathcal{H}}(\lambda, \chi) = \lambda \left( m^2 - \Pi^2 + \frac{ig}{2} F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n \right) + \chi \left( \frac{r}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \Pi_n \Gamma_{(s)k_1} \dots \Gamma_{(s)k_{2d}} + sm \right).$$

Starting from this point one can proceed similarly to the even-dimensional case to construct a path integral for the right side of (2.32). The Hamiltonian form of such path integral is

$$S^c = s \exp \left( i \Gamma_{(s)}^n \frac{\partial_t}{\partial \theta^n} \right) \int_0^\infty d\lambda_0 \int d\chi_0 \int_{\lambda_0} D\lambda \int_{\chi_0} D\chi \int_{x_{\text{in}}}^{x_{\text{out}}} Dx \int Dp \int D\pi \int D\nu \quad (2.33)$$

$$\times \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi \exp \left\{ i \int_0^1 \left[ \lambda (\mathcal{P}^2 - m^2 + 2ig F_{kn} \psi^k \psi^n) - \chi (sm \right. \right.$$

$$\left. \left. + r \frac{(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \mathcal{P}_n \psi_{k_1} \dots \psi_{k_{2d}} \right) - i\psi_n \dot{\psi}^n + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi} \right] d\tau + \psi_n(1)\psi^n(0) \Big|_{\theta=0},$$

where  $x(\tau)$ ,  $p(\tau)$ ,  $\lambda(\tau)$ ,  $\pi(\tau)$  are even and  $\psi(\tau)$ ,  $\chi_1(\tau)$ ,  $\chi_2(\tau)$ ,  $\nu_1(\tau)$ ,  $\nu_2(\tau)$  are odd trajectories, obeying the boundary conditions  $x(0) = x_{\text{in}}$ ,  $x(1) = x_{\text{out}}$ ,  $\lambda(0) = \lambda_0$ ,  $\chi(0) = \chi_0$ ,  $\psi(0) + \psi(1) = \theta$ , and the notations are used

$$\chi = \chi_1 \chi_2, \quad \nu \dot{\chi} = \nu_1 \dot{\chi}_1 + \nu_2 \dot{\chi}_2, \quad d\chi = d\chi_1 d\chi_2, \quad D\chi = D\chi_1 D\chi_2, \quad D\nu = D\nu_1 D\nu_2.$$

Integrating over momenta, we get a path integral in the Lagrangian form,

$$S^c = \frac{s}{2} \exp \left( i \Gamma_{(s)}^n \frac{\partial \ell}{\partial \theta^n} \right) \int_0^\infty de_0 \int d\chi_0 \int_{e_0} M(e) De \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int D\pi \int D\nu \quad (2.34)$$

$$\times \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi \exp \left\{ i \int_0^1 \left[ -\frac{\dot{x}^2}{2e} - \frac{e}{2} m^2 - g \dot{x}_n A^n + ieg F_{kn} \psi^k \psi^n \right. \right.$$

$$\left. \left. - \chi \left( sm + \frac{r}{e} \frac{(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \dot{x}_n \psi_{k_1} \dots \psi_{k_{2d}} \right) - i \psi_n \dot{\psi}^n + \pi \dot{e} + \nu \dot{\chi} \right] d\tau + \psi_n(1) \psi^n(0) \right\} \Big|_{\theta=0},$$

where the measure  $M(e)$  is defined by the eq. (2.12) and  $e(0) = e_0$ .

One can also get a different form of the path integral for the Dirac propagator in odd dimensions. To this end, instead of (2.30), one has to write

$$\hat{S}^c = \frac{\hat{F}}{-\hat{F}\hat{F}} = s \frac{\hat{A}}{\hat{B}}, \quad (2.35)$$

$$\hat{A} = \frac{r}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \Pi_n \Gamma_{(s)k_1} \dots \Gamma_{(s)k_{2d}} - sm,$$

$$\hat{B} = m^2 - \Pi^2 + \frac{ig}{2} F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n - \frac{2srm}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \Pi_n \Gamma_{(s)k_1} \dots \Gamma_{(s)k_{2d}},$$

and then proceed as before. Thus, we get one more form of the Lagrangian path integral

$$S^c = \frac{s}{2} \exp \left( i \Gamma_{(s)}^n \frac{\partial \ell}{\partial \theta^n} \right) \int_0^\infty de_0 \int d\chi_0 \int_{e_0} M(e) De \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int D\pi \int D\nu \quad (2.36)$$

$$\times \int_{\psi(0)+\psi(1)=\theta} \mathcal{D}\psi \exp \left\{ i \int_0^1 \left[ -\frac{\dot{x}^2}{2e} - \frac{e}{2} m^2 - g \dot{x}_n A^n(x) + ieg F_{kn}(x) \psi^k \psi^n + sm \chi \right. \right.$$

$$\left. \left. - \left( \frac{\chi}{e} - sm \right) \frac{r(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \dot{x}_n \psi_{k_1} \dots \psi_{k_{2d}} - i \psi_n \dot{\psi}^n + \pi \dot{e} + \nu \dot{\chi} \right] d\tau + \psi_n(1) \psi^n(0) \right\} \Big|_{\theta=0}.$$

### III. PSEUDOCCLASSICAL DESCRIPTION OF SPINNING PARTICLES IN ARBITRARY DIMENSIONS

The path integral representations for particles propagators have also an important heuristic value. They give a possibility to guess the form of actions to describe the particles classically or pseudoclassically if we believe that such representations should have the form  $\int \exp(iS) D\varphi$ . Here  $\varphi$  is a set of variables and  $S$  a classical action. Indeed, let us take the simplest example of the scalar particle. Here the path integral representation of the propagator has the form (2.11). The exponent in the integrand of the path integral can be

treated as a Lagrangian action of the relativistic spinless particle. This exponent consists of two parts. The first one

$$S = - \int_0^1 \left[ -\frac{\dot{x}^2}{2e} - e \frac{m^2}{2} - g \dot{x}_\mu A^\mu \right] d\tau \quad (3.1)$$

is well-known gauge-invariant (reparametrization-invariant) action of the relativistic spinless particle. The corresponding gauge transformations read  $\delta x = \dot{x} \xi$ ,  $\delta e = \frac{d}{d\tau}(e\xi)$ . The second term in the exponent can be treated as a gauge-fixing term which corresponds to the gauge condition  $\dot{e} = 0$ . Quantization of the action (3.1) leads to the corresponding quantum theory of a scalar particle [20] which is equivalent to one-particle sector of the scalar quantum field theory. Thus, we have a closed circle: propagator (which is a representative of the one-particle sector of the scalar quantum field theory) - path integral for it - classical action of a point-like particle - quantization - one-particle sector of the scalar quantum field theory.

Let us now turn to the case of spinning particle, which is certainly of the main interest for us, using the experience in the simple spinless case. Namely, looking on the path integrals (2.26) or (2.34) we can guess the form of the actions for spinning particles in even and odd dimensions. Hence, the exponent in the integrand of the right side of (2.26) can be treated as a pseudoclassical action of the spinning particle in even dimensions. Separating the gauge-fixing terms and boundary terms, we get a gauge-invariant pseudoclassical action

$$S = \int_0^1 \left[ -\frac{z^2}{2e} - \frac{e}{2} m^2 - g \dot{x}_\mu A^\mu + ieg F_{\mu\nu} \psi^\mu \psi^\nu - im \psi^D \chi - i \psi_n \dot{\psi}^n \right] d\tau,$$

$$z^\mu = \dot{x}^\mu - i \psi^\mu \chi. \quad (3.2)$$

There are two type of gauge transformations in the theory with the action (3.2): reparametrizations,

$$\delta x^\mu = \dot{x}^\mu \xi, \quad \delta e = \frac{d}{d\tau}(e\xi), \quad \delta \psi^n = \dot{\psi}^n \xi, \quad \delta \chi = \frac{d}{d\tau}(\chi \xi), \quad (3.3)$$

and supertransformations,

$$\delta x^\mu = i \psi^\mu \epsilon, \quad \delta e = i \chi \epsilon, \quad \delta \chi = \dot{\epsilon}, \quad \delta \psi^\mu = \frac{z^\mu}{2e} \epsilon, \quad \delta \psi^D = \frac{m}{2} \epsilon, \quad (3.4)$$

where  $\xi$  is even and  $\epsilon$  is odd  $\tau$ -dependent parameters.

The action (3.2) is a trivial generalization of the BM action to  $D$  dimensions. The quantization of the action (3.2) in even dimensions can be done [19] completely similar to the four-dimensional case [20]. It reproduces the quantum theory of spinning particle (in particular, the Dirac equation), which is equivalent to the one-particle sector of the spinor field quantum theory, with the propagator  $S^c$  (2.13).

In odd dimensions, the path integral (2.34) prompts us the following pseudoclassical action to describe spinning particles in such dimensions:

$$S = \int_0^1 \left[ -\frac{z^2}{2e} - \frac{e}{2} m^2 - g \dot{x}_n A^n + i e g F_{kn} \psi^k \psi^n - \kappa m \chi - i \psi_n \dot{\psi}^n \right] d\tau = \int_0^1 L d\tau, \\ z^n = \dot{x}^n + r \frac{(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \psi_{k_1} \dots \psi_{k_{2d}} \chi. \quad (3.5)$$

We suppose that  $\kappa$  is an even constant, which will be discussed below and  $\chi = \chi_1 \chi_2$ , with  $\chi_1$  and  $\chi_2$  being Grassmannian variables, obeying the involution properties:  $\chi_1^{\dagger} = \chi_2$ . Interpreting the variable  $\chi$  in such a way, we can discover that the action (3.5) is gauge-invariant (reparametrization- and supergauge-invariant). The corresponding gauge transformations have the form: reparametrizations

$$\delta x^n = \dot{x}^n \xi, \quad \delta e = \frac{d}{d\tau}(e\xi), \quad \delta \psi^n = \dot{\psi}^n \xi, \\ \delta \chi_1 = \dot{\chi}_1 \xi + \frac{1}{2} \chi_1 \dot{\xi}, \quad \delta \chi_2 = \dot{\chi}_2 \xi + \frac{1}{2} \chi_2 \dot{\xi}, \quad (3.6)$$

and two sets of nonlocal (in time) supertransformations,

$$\delta x^n = i \epsilon^{nk_1 \dots k_{2d}} \psi_{k_1} \dots \psi_{k_{2d}} U, \quad \delta \psi^n = -\frac{d}{e} \epsilon^{nk_1 k_2 \dots k_{2d}} z_{k_1} \psi_{k_2} \dots \psi_{k_{2d}} U, \\ \delta e = 0, \quad \delta \chi_1 = \theta_1, \quad \delta \chi_2 = \theta_2, \quad U = \frac{ir(2i)^{2d}}{(2d)!} \int_0^\tau [\chi_1 \theta_2 - \chi_2 \theta_1] d\tau, \quad (3.7)$$

where  $\xi$  is even and  $\theta_{1,2}$  are odd  $\tau$ -dependent parameters.

As was already mentioned in the Introduction, formerly there were proposed three different kinds of action to describe spinning particles in odd dimensions [5-7]. It has been shown that an adequate quantum theory arises in course of those actions quantization (at

least the Dirac equation appears). However it was not demonstrated how one can construct the path integral for the propagator by means of those actions (path integral quantization was not done). We already have proved that our new action (3.5) allows one to write the corresponding path integral and now we are going to check that the direct (operator) quantization leads to the corresponding quantum theory. To this end, as usual, we need to analyse the Hamiltonian structure of the theory with the action (3.5).

Introducing the canonical momenta

$$p_n = \frac{\partial L}{\partial \dot{x}^n} = -\frac{z_n}{e} - g A_n(x), \quad P_n = \frac{\partial_r L}{\partial \dot{\psi}^n} = -i \psi_n, \\ P_e = \frac{\partial L}{\partial \dot{e}} = 0, \quad P_{\chi_{1,2}} = \frac{\partial_r L}{\partial \dot{\chi}_{1,2}} = 0, \quad (3.8)$$

one can see that there exist primary constraints  $\Phi^{(1)} = 0$ ,

$$\Phi_{1,2}^{(1)} = P_{\chi_{1,2}}, \quad \Phi_3^{(1)} = P_e, \quad \Phi_{4n}^{(1)} = P_n + i \psi_n. \quad (3.9)$$

We construct the total Hamiltonian according to the standard procedure [21,22] (we use the notations of the book [22]), and get  $H^{(1)} = H + \lambda \Phi^{(1)}$ , with

$$H = -\frac{e}{2} (\mathcal{P}^2 - m^2 + 2i e g F_{kn} \psi^k \psi^n) + \chi \left( r \frac{(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \mathcal{P}_n \psi_{k_1} \dots \psi_{k_{2d}} + \kappa m \right),$$

where  $\mathcal{P} = -p_n - g A_n(x)$ . From the consistency conditions (Dirac procedure) we find a set of independent secondary constraints  $\Phi^{(2)} = 0$ ,

$$\Phi_1^{(2)} = \mathcal{P}^2 - m^2 + 2i e g F_{kn} \psi^k \psi^n, \quad \Phi_2^{(2)} = r \frac{(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \mathcal{P}_n \psi_{k_1} \dots \psi_{k_{2d}} + \kappa m. \quad (3.10)$$

One can go over from the initial set of constraints  $(\Phi^{(1)}, \Phi^{(2)})$  to the equivalent one  $(\check{\Phi}^{(1)}, \check{\Phi}^{(2)})$ , where  $\check{\Phi}^{(2)} = \Phi^{(2)}(\psi \rightarrow \psi + \frac{i}{2} \Phi_4^{(1)})$ . The new set of constraints can be explicitly divided in a set of first-class constraints, which is  $(\Phi_{1,2,3}^{(1)}, \check{\Phi}^{(2)})$  and in a set of second-class constraints, which is  $\Phi_{4n}^{(1)}$ .

Let us consider the Dirac quantization, where the second-class constraints define the Dirac brackets and therefore the commutation relations. The first-class constraints, being

applied to the state vectors, define physical states. Thus, we get for essential operators and nonzero commutation relations:

$$[\hat{x}^\mu, \hat{p}_\nu] = i\{x^\mu, p_\nu\}_{D(\Phi_4^{(1)})} = i\delta_\nu^\mu, \quad [\hat{\psi}^k, \hat{\psi}^n]_+ = i\{\psi^k, \psi^n\}_{D(\Phi_4^{(1)})} = -\frac{1}{2}\eta^{kn}. \quad (3.11)$$

According to the scheme of quantization selected, operators of the second-class constraints are identically zero, whereas the operators of the first-class constraints have to annul physical state vectors. Taking that into account, one may construct a realization of the commutation relations (3.11) in a Hilbert space  $\mathcal{R}$  whose elements  $\mathbf{f} \in \mathcal{R}$  are  $2^d$ -component columns dependent only on  $x$ , such that

$$\hat{x}^n = x^n \mathbf{I}, \quad \hat{p}_n = -i\partial_n \mathbf{I}, \quad \hat{\psi}^n = \frac{i}{2}\Gamma_{(s)}^n, \quad (3.12)$$

where  $\mathbf{I}$  is  $2^d \times 2^d$  unit matrix, and  $\Gamma_{(s)}^n$ , are  $\gamma$ -matrices in  $D = 2d + 1$  dimensions, see (2.27).

Besides of that, we have the following equations for the physical state vectors

$$\hat{\Phi}_1^{(2)} \mathbf{f}(x) = 0, \quad \hat{\Phi}_2^{(2)} \mathbf{f}(x) = 0, \quad (3.13)$$

where  $\hat{\Phi}^{(2)}$  are operators, which correspond to the constraints (3.10). Taken into account the duality relation (2.28), one can write the second equation (3.13) in the form

$$(\hat{\mathcal{P}}_n \Gamma_{(s)}^n + s\kappa m) \mathbf{f}(x) = 0, \quad (3.14)$$

where  $\hat{\mathcal{P}}_n = i\partial_n - gA_n(x)$ . Thus, we get the Dirac equation in course of the operator quantization if we put  $\kappa = -s$  in course of the quantization. By this choice of  $\kappa$  the first equation (3.13) is simply a consequence of the Dirac equation. It is the squared Dirac equation.

#### IV. CONSIDERATION FOR SPINNING PARTICLES WITH ANOMALOUS MAGNETIC MOMENT

One can generalize the construction of path integrals to the case of the particles with an anomalous magnetic moment (AMM). In four dimensions the corresponding pseudoclassical

models were discussed in [30-33]. The path integral representation for the propagator was constructed in [34]. We may use our approach starting with the generalized by Pauli [35] Dirac equation and considering it in  $D$  dimensions:

$$\left[ \hat{\mathcal{P}}_\nu \gamma^\nu - \left( m + \frac{\mu}{2} \sigma^{\alpha\beta} F_{\alpha\beta} \right) \right] \Psi(x) = 0. \quad (4.1)$$

Here  $\sigma^{\alpha\beta} = \frac{i}{2}[\gamma^\alpha, \gamma^\beta]_-$ , and  $\mu$  stands for AMM. As before we have to consider two cases of even and odd dimensions separately. In even dimension  $D = 2d$  one has to modify the equation (2.15) to the following form

$$\left[ \hat{\mathcal{P}}_\mu \tilde{\gamma}^\mu - \gamma^{D+1} \left( m + i\frac{\mu}{2} F_{\alpha\beta} \tilde{\gamma}^\alpha \tilde{\gamma}^\beta \right) \right] \tilde{S}^c(x, y) = \delta^D(x - y), \quad (4.2)$$

Proceeding as before, we get the following generalization of the path integral representation (2.26)

$$\begin{aligned} \tilde{S}^c = & \exp\left(i\Gamma_{(s)}^n \frac{\partial \ell}{\partial \theta^n}\right) \int_0^\infty de_0 \int d\chi_0 \int_{e_0} M(e) De \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int D\pi \int D\nu \\ & \times \int_{\psi(0)+\psi(1)=\theta} D\psi \exp\left\{i \int_0^1 \left[ -\frac{\dot{z}^2}{2e} - \frac{e}{2} M^2 + ieg F_{\mu\nu} \psi^\mu \psi^\nu - \dot{x}^\alpha (gA_\alpha + 4i\mu\psi^D F_{\alpha\beta} \psi^\beta) \right. \right. \\ & \left. \left. + i \left( \frac{\dot{x}_\mu \psi^\mu}{e} - M^* \psi^D \right) \chi - i\psi_n \dot{\psi}^n + \pi \dot{e} + \nu \dot{\chi} \right] d\tau + \psi_n(1)\psi^n(0) \right\} \Big|_{\theta=0}, \end{aligned} \quad (4.3)$$

where  $M = m - 2i\mu F_{\alpha\beta} \psi^\alpha \psi^\beta$ . Thus, a pseudoclassical action for spinning particle with AMM has the form in even dimensions

$$\begin{aligned} S = & \int_0^1 \left[ -\frac{z^2}{2e} - \frac{e}{2} M^2 - \dot{x}^\alpha (gA_\alpha + 4i\mu\psi^D F_{\alpha\beta} \psi^\beta) \right. \\ & \left. + ieg F_{\mu\nu} \psi^\mu \psi^\nu - iM^* \psi^D \chi - i\psi_n \dot{\psi}^n \right] d\tau, \quad z^\mu = \dot{x}^\mu - i\psi^\mu \chi. \end{aligned} \quad (4.4)$$

In odd dimensions  $D = 2d + 1$  we have to modify the equation (2.29), introducing Pauli term,

$$\left[ \hat{\mathcal{P}}_n \Gamma_{(s)}^n - \left( m + i\frac{\mu}{2} F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n \right) \right] S^c(x, y) = -\delta^D(x - y). \quad (4.5)$$

Thus, we get for the operator  $\hat{S}^c$  entering in (2.18),

$$\hat{S}^c = -\hat{F}^{-1}, \quad \hat{F} = \Pi_n \Gamma_{(s)}^n - \left( m + i\frac{\mu}{2} F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n \right),$$

where all  $\Pi_n$  are defined by eq. (2.3). Using duality relation (2.28) we can present  $\hat{S}^c$  in the following form

$$\begin{aligned}\hat{S}^c &= \frac{\hat{F}_{(+)} - \hat{F}_{(-)}}{-\hat{F}_{(+)}\hat{F}_{(-)}} = s \frac{\hat{A}}{\hat{B}}, \quad \hat{F}_{(+)} = \Pi_n \Gamma_{(s)}^n + \left( m + i \frac{\mu}{2} F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n \right), \\ \hat{A} &= \frac{r}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \Pi_n \Gamma_{(s)k_1} \dots \Gamma_{(s)k_{2d}} + s \left( m + i \frac{\mu}{2} F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n \right), \\ \hat{B} &= m^2 - \Pi^2 + i \left( m\mu + \frac{g}{2} \right) F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n - \frac{\mu^2}{4} \left( F_{kn} \Gamma_{(s)}^k \Gamma_{(s)}^n \right)^2 \\ &\quad - i \frac{s\mu r}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} [F_{kn}, \Pi^n]_{+} \Gamma_{(s)k_1} \dots \Gamma_{(s)k_{2d}}.\end{aligned}\quad (4.6)$$

Then the representation (2.32) takes place, where  $\hat{\mathcal{H}}(\lambda, \chi) = \lambda \hat{B} + \chi \hat{A}$ , with  $\hat{A}$  and  $\hat{B}$  defined by (4.6). Proceeding similarly to the case without AMM we get the Hamiltonian form of the path integral for  $S^c$ ,

$$\begin{aligned}S^c &= s \exp \left( i \Gamma_{(s)}^n \frac{\partial_l}{\partial \theta^n} \right) \int_0^\infty d\lambda_0 \int d\chi_0 \int_{\lambda_0} D\lambda \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int Dp \int D\pi \int D\nu \\ &\times \int_{\psi(0)+\psi(1)=\theta} D\psi \exp \left\{ i \int_0^1 \left[ \lambda (\mathcal{P}^2 - M^2 + 2ig F_{kn} \psi^k \psi^n \right. \right. \\ &\quad \left. \left. + \frac{s\mu r (2i)^D}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} F_{kn} \mathcal{P}^n \psi_{k_1} \dots \psi_{k_{2d}} \right) - \chi \left( sM + r \frac{(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \mathcal{P}_n \psi_{k_1} \dots \psi_{k_{2d}} \right) \right. \\ &\quad \left. - i\psi_n \dot{\psi}^n + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi} \right] d\tau + \psi_n(1)\psi^n(0) \Big|_{\theta=0}.\end{aligned}\quad (4.7)$$

Here the following notations are used

$$\chi = \chi_1 \chi_2, \quad \nu \dot{\chi} = \nu_1 \dot{\chi}_1 + \nu_2 \dot{\chi}_2, \quad d\chi = d\chi_1 d\chi_2, \quad D\chi = D\chi_1 D\chi_2, \quad D\nu = D\nu_1 D\nu_2.$$

Integrating over momenta, we get a path integral in the Lagrangian form,

$$\begin{aligned}S^c &= \frac{s}{2} \exp \left( i \Gamma_{(s)}^n \frac{\partial_l}{\partial \theta^n} \right) \int_0^\infty de_0 \int d\chi_0 \int_{e_0} M(e) De \int_{\chi_0} D\chi \int_{x_{in}}^{x_{out}} Dx \int D\pi \int D\nu \\ &\times \int_{\psi(0)+\psi(1)=\theta} D\psi \exp \left\{ i \int_0^1 \left[ -\frac{\dot{x}^2}{2e} - \frac{e}{2} M^2 + ieg F_{kn} \psi^k \psi^n - \dot{x}^n (gA_n \right. \right. \\ &\quad \left. \left. + \frac{s\mu r (2i)^D}{2(2d)!} F_{nk} \epsilon^{kk_1 \dots k_{2d}} \psi_{k_1} \dots \psi_{k_{2d}} \right) - \chi \left( sM + \frac{r (2i)^{2d}}{e (2d)!} \epsilon^{nk_1 \dots k_{2d}} \dot{x}_n \psi_{k_1} \dots \psi_{k_{2d}} \right) \right. \\ &\quad \left. - i\psi_n \dot{\psi}^n + \pi\dot{e} + \nu\dot{\chi} \right] d\tau + \psi_n(1)\psi^n(0) \Big|_{\theta=0},\end{aligned}\quad (4.8)$$

Thus, a pseudoclassical action for spinning particle with AMM has the following form in odd dimensions

$$\begin{aligned}S &= \int_0^1 \left[ -\frac{z^2}{2e} - \frac{e}{2} M^2 - \dot{x}^n \left( gA_n + \frac{s\mu r (2i)^D}{2(2d)!} F_{nk} \epsilon^{kk_1 \dots k_{2d}} \psi_{k_1} \dots \psi_{k_{2d}} \right) \right. \\ &\quad \left. + ieg F_{kn} \psi^k \psi^n - sM\chi - i\psi_n \dot{\psi}^n \right] d\tau, \quad z^n = \dot{x}^n + r \frac{(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \psi_{k_1} \dots \psi_{k_{2d}} \chi.\end{aligned}\quad (4.9)$$

Quantizing both actions (4.4) and (4.9) one reproduces the Dirac Pauli equation (4.1) similarly to the case without AMM.

## V. DISCUSSION

One of the new features in the pseudoclassical model in odd dimensions (3.5) consists in a new interpretation of the even variable  $\chi$  as a composite bifermionic type variable. One ought to say that only from the point of view of the operator quantization, one may treat  $\chi$  as an unique bosonic variable. Indeed, one can believe that  $\chi$  is simply an ordinary Lagrange multiplier as it occurred always before. Doing the Dirac procedure, bearing in mind this interpretation, one gets finally the same first class constraint (3.10) and the same Dirac brackets. Thus, the result of the Dirac quantization will be the same. However, as was demonstrated above, the new interpretation is necessary for the path integral construction, or for path integral quantization. Besides of that it provides the desirable supersymmetry of the pseudoclassical action. Indeed, treating  $\chi$  as an unique even variable, we have the following symmetries of the action (3.5): reparametrizations

$$\delta x^n = \dot{x}^n \xi, \quad \delta e = \frac{d}{d\tau}(e\xi), \quad \delta \psi^n = \dot{\psi}^n \xi, \quad \delta \chi = \frac{d}{d\tau}(\chi\xi), \quad (5.1)$$

and gauge transformations,

$$\begin{aligned}\delta x^n &= i \epsilon^{nk_1 \dots k_{2d}} \psi_{k_1} \dots \psi_{k_{2d}} \beta, \quad \delta \psi^n = -\frac{d}{e} \epsilon^{nk_1 k_2 \dots k_{2d}} z_{k_1} \psi_{k_2} \dots \psi_{k_{2d}} \beta, \\ \delta e &= 0, \quad \delta \chi = -\frac{i(2d)!}{r(2i)^{2d}} \dot{\beta},\end{aligned}\quad (5.2)$$

where both  $\xi$  and  $\beta$  are even  $\tau$ -dependent parameters. Thus, in this case no supersymmetry is present. One ought to say that the presence of the nonlocal supersymmetry (3.7) in the model (3.5) is a characteristic feature of the pseudoclassical theory only, since it was proved

in [22] that for singular theories with bosonic variables any gauge transformations are local in time. Treating  $\chi$  as a composite bifermionic type variable, we meet also for the first time a situation when the action and Hamiltonian are quadratic in Lagrange multipliers.

Another question is how to interpret the constant  $\kappa$  in the action (3.5). This question is directly related with the well-known problem of classical inconsistency of some kind of constraints in pseudoclassical mechanics. Indeed, if one treats  $\kappa$  as an ordinary complex parameter, then from the classical point of view the constraint equation  $\Phi_2^{(2)} = 0$  is inconsistent. Such a difficulty appears not for the first time in the pseudoclassical mechanics, (see for example [23]). Here the following point of view is possible. One can believe that in classical theory  $\kappa$  is an even, bifermionic type element of the Berezin algebra,  $\kappa^2 = 0$ . Then the above-mentioned constraint equation appears to be consistent in the classical theory. At the same time, as was pointed out in [9], one has to admit a possibility to change the nature of the parameters in course of transition from the pseudoclassical mechanics to the quantum theory (why we admit such a possibility for the dynamical variables?). Namely, in quantum theory the parameter  $\kappa$  appears to be a real number, whose possible values are defined by the quantum dynamics. For example, the path integral quantization of the action (3.5) demands  $\kappa \rightarrow s$ , whereas the operator quantizations demands  $\kappa \rightarrow -s$ , where  $s = \pm 1$  defined an irreducible representation of the Clifford algebra, see (2.27). To get the same quantization for  $\kappa$  both in path integral quantization and operator quantization (at given and fixed choice of the irreducible representations for  $\gamma$  matrices) one has to consider another action, which can be extracted from the alternative path integral representation (2.36). Such an action has the form

$$S = \int_0^1 \left[ -\frac{z^2}{2e} - \frac{e}{2} m^2 - g \dot{x}_n A^n + i e g F_{kn} \psi^k \psi^n + \kappa m \chi - i \psi_n \dot{\psi}^n \right] d\tau = \int_0^1 L d\tau ,$$

$$z^n = \dot{x}^n + r \frac{(2i)^{2d}}{(2d)!} \epsilon^{nk_1 \dots k_{2d}} \psi_{k_1} \dots \psi_{k_{2d}} (\chi - \kappa e m) , \quad (5.3)$$

with the same variables and parameters.

One may also note that path integral representations for particles propagators have not only a pure theoretical interest. It makes possible to calculate effectively these propagators

in various configurations of external fields (see for example [24]). Such propagators are necessary composite parts for calculations in quantum field theory with non-perturbative backgrounds [25]. That is why the representations for the propagators obtained can be useful in quantum field theory in higher dimensions, which attract attention already for a long time since the Kaluza-Klein ideas.

The presented path integral representations may be useful in the so called spin factor problem, which was opened first by Polyakov [26]. He assumed that the propagator of a free Dirac electron in  $D = 3$  Euclidean space-time can be presented by means of a bosonic path integral similar to the scalar particle case, modified by a so called spin factor. This idea was developed by several authors, see for example [27], in particular to derive the spin factor for spinning particles interacting with external fields. Surprisingly, it was shown in [28] that all the Grassmannian integrations in the representation (2.34) of Dirac propagator in an arbitrary external field in four dimensions can be done, so that an expression for the spin factor was derived as a given functional of the bosonic trajectory. Having such an expression for the spin factor one can use it to calculate the propagator in some particular cases of external fields [29]. This way of calculation automatically provides the explicit  $\gamma$ -matrix structure of the propagators, that facilitate a lot concrete calculations with the propagators. The new path integral representation obtained by us in the present paper allows one to get the expression for the spin factor in the same manner already in arbitrary dimensions. The corresponding details will be published soon.

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