

UNIVERSIDADE DE SÃO PAULO

**INSTITUTO DE FÍSICA
CAIXA POSTAL 66318
05389-970 SÃO PAULO - SP
BRASIL**

PUBLICAÇÕES

IFUSP/P-1242

**THE EFFECTIVE POTENTIAL OF GAUGED NJL
MODEL IN MAGNETIC FIELD**

I. Brevik

Dept. of Applied Mechanics and Norwegian Institute of
Technology
Univer. of Trondheim, Trondheim, Norway

D.M. Gitman

Instituto de Física, Universidade de São Paulo

S.D. Odintsov

Instituto de Física, Universidade de São Paulo and
Universidad del Valle, Cali, Colombia

Outubro/1991

The effective potential of gauged NJL model in magnetic field

I. Brevik

Dept. of Applied Mechanics and Norwegian Institute of Technology

Univer. of Trondheim, Trondheim, Norway

D.M. Gitman*

Instituto de Física, Universidade de São Paulo

Caixa Postal 66318, 05389-970-São Paulo, S.P., Brasil

S.D. Odintsov†

Instituto de Física, Universidade de São Paulo

Caixa Postal 66318, 05389-970-São Paulo, S.P., Brasil;

Universidad del Valle, Cali, Colombia

(October 16, 1996)

Abstract

The formalism, which permits to study the phase structure of gauged NJL-model for arbitrary external fields is developed. The effective potential in the gauged NJL model in the weak magnetic field is found. It is shown that in fixed gauge coupling case the weak magnetic field doesn't influence chiral symmetry breaking condition. The analogy with the situation near black hole is briefly mentioned.

*e-mail: gitman@fma.if.usp.br

†On leave from Tomsk Pedagogical University, 634041 Tomsk, Russia; present e-mail: odintsov@fma.if.usp.br

I. INTRODUCTION

For a long time it is well known that QED may have a non-perturbative strong-coupling phase [1] where the chiral symmetry is broken. The manifestation of this phase maybe found via analysis [2] of the multiple correlated and narrow-peak structures in electron and positron spectra [3]. Schwinger-Dyson (SD) equations approach is usual tool to study such a strong-coupling phase.

The gauged Nambu-Jona-Lasinio (NJL) model represents even more interesting theory where such a non-perturbative strong-coupling chiral symmetry broken phase maybe naturally realized. Moreover, gauged NJL-model maybe used to describe the Standard Model. It may also play the role of low-energy effective theory for QCD. It is very interesting to understand what external conditions may influence the chiral symmetry breaking in such theory. Among such an external conditions the external magnetic field could be very interesting one, as it can be realized in the laboratory experiments.

There has been recently some interest in the study of (non-gauged) NJL model in an external magnetic field [4]. It has been shown that strong magnetic field increases the value of dynamically generated fermionic mass, and in this way, it supports the phase with chiral symmetry breaking. Such a study is closely related with the investigation of effective potential in an external magnetic fields [19] where the possibility of symmetry breaking due to external magnetic field has been shown.

In the present work we discuss the effective potential in the gauged NJL model in an external magnetic field. The explicit evaluation of the effective potential is given in the situation when magnetic field is weak. It is shown that weak magnetic field doesn't influence chiral symmetry breaking.

II. GAUGE HIGGS-YUKAWA MODEL IN EXTERNAL MAGNETIC FIELD

We will start from the $SU(N_c)$ gauge theory with scalars and spinors:

$$L_m = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} m^2 \sigma^2 - \frac{\lambda}{4} \sigma^4 + \sum_{i=1}^{N_f} \bar{\psi}_i i \hat{D} \psi_i - \sum_{i=1}^{n_f} y \sigma \bar{\psi}_i \psi_i. \quad (1)$$

where σ is a single scalar, y is Yukawa coupling and N_f fermions belong to the representation R of group $SU(N_c)$. Moreover, $n_f \ll N_f$, i.e. only n_f fermions have large Yukawa couplings.

We will briefly describe now the modified $1/N$ approximation [5] to study the theory (1) at high energies.

i) As usually in $1/N$ - approximation the gauge coupling constant is supposed to be small

$$\frac{g^2 N_c}{4\pi} \ll 1.$$

ii) The number of fermions should be comparable with N_c

$$N_f \sim N_c.$$

iii) We are working in the leading order of $1/N$ expansion, i.e. scalar loop contributions should be negligible

$$\left| \frac{\lambda}{y^2} \right| \leq N_c.$$

Working in frames of such modified $1/N$ approximation [5], one can write the standard one-loop RG equations for the coupling constants,

$$\begin{aligned} \frac{dg(t)}{dt} &= -\frac{b}{(4\pi)^2} g^3(t), \\ \frac{dy(t)}{dt} &= \frac{y(t)}{(4\pi)^2} [a y^2(t) - c g^2(t)], \\ \frac{d\lambda(t)}{dt} &= \frac{u y^2(t)}{(4\pi)^2} [\lambda(t) - y^2(t)]. \end{aligned} \quad (2)$$

Here $b = (11N_c - 4T(R)N_f)/3$, $c = 6 C_2(R)$, $a = u/4 = 2 n_f N_c$. For example, in fundamental representation $T(R) = \frac{1}{2}$, $C_2(R) = \frac{N_c^2 - 1}{2N_c}$. As usually, renormalization group parameter $t = \ln \frac{\mu}{\mu_0}$.

Let us consider now the situation when the theory (1) interacts with the external magnetic field [8]. It means that spinor derivative in (1) contains the external magnetic field

connection. Then, one can calculate the effective potential $V(\sigma)$ for the constant scalar field σ . Working in the modified $1/N_c$ - approximation and treating the external magnetic field exactly we get:

$$V = \frac{1}{2} m^2 \sigma^2 + \frac{\lambda}{4} \sigma^4 + \frac{a}{(4\pi)^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^2} e^{-s M_F^2} \coth(eHs) eH, \quad (3)$$

where e is electric charge, H is magnetic field $M_F = y\sigma$ plays the role of fermionic mass, Λ is cut-off. Note that effective potential (3) is written in regularized form but not in renormalized form.

In order to write the effective potential in renormalized form we will make the expansion in powers of H (derivative expansion) following the technique of ref [19] (taking account also of renormalization conditions)

$$V = \frac{1}{2} m^2 \sigma^2 + \frac{\lambda}{4} \sigma^4 - \frac{a M_F^4}{2(4\pi)^2} \left[\ln \frac{M_F^2}{\mu^2} - \frac{3}{2} \right] - \frac{ae^2 H^2}{24(\pi)^2} \left[\ln \frac{M_F^2}{\mu^2} \right], \quad (4)$$

where we suppose that $M_F^2 > eH$. Here, third term gives the standard Coleman-Weinberg one loop potential [7], and last term in (4) is Schwinger's effective potential.

One can also study the case when the external magnetic field being weak is bigger than scalar: $eH > M_F^2$, $eH < 1$. Then, one can get (compare with [9])

$$V = \frac{1}{2} m^2 \sigma^2 + \frac{\lambda}{4} \sigma^4 - \frac{a M_F^4}{2(4\pi)^2} \left[\ln \frac{eH}{\mu^2} - \frac{3}{2} \right] - \frac{ae^2 H^2}{24(\pi)^2} \left[\ln \frac{eH}{\mu^2} \right]. \quad (5)$$

Generally speaking, one should also add to effective potential (4,5) the classical potential of magnetic field

$$V_{cl}^H = \frac{1}{4} F_{\mu\nu}^2 = \frac{1}{2} H^2.$$

Similarly, one can consider other regimes for external magnetic field.

The analysis of RG equations for gauge, Yukawa and scalar coupling constants has been given in [5]. It has been shown that theory maybe non-trivial and stable in above approximation (gauge coupling and Yukawa coupling constants will be asymptotically free). On

the same time, there will be non-trivial solution for scalar coupling constants [5] only when $c > b$. Note that if one considers the coupling constants space then the solution of [5] lie on the line between the Gaussian fixed point and the fixed point of [10].

III. GAUGED NJL MODEL

Let us turn now to the $SU(N_c)$ gauged NJL-model with four-fermion coupling constant G . The corresponding Lagrangian maybe written as follows:

$$L = -\frac{1}{4}G_{\mu\nu}^2 + i\sum_{i=1}^{N_f}\bar{\Psi}_i\hat{D}\Psi_i + G\sum_{i=1}^{n_f}(\bar{\Psi}_i\Psi_i)^2. \quad (6)$$

As usually the way to study such a model consists in the rewriting of Lagrangian (6) in terms of equivalent theory with an auxiliary scalar field σ (fermionic condensate). The latter theory should be identified with the Higgs-Yukawa model.

As it has been shown in [11] RG approach maybe applied in order to construct such an identification. In this approach one can put a set of boundary conditions for the effective couplings of the gauge Higgs-Yukawa model at $t_\Lambda = \ln \frac{\Lambda}{\mu_0}$ (where Λ is UV cut-off) in order to prove the equivalence of gauge Higgs-Yukawa model and gauged NJL-model. More exactly, one proves [5] that sequence of gauge Higgs-Yukawa theories parametrized by Λ is equivalent to the corresponding sequence of gauged NJL models. Moreover, this equivalence is kept even at $\Lambda \rightarrow \infty$. Because gauge Higgs-Yukawa theory is renormalizable, the gauged NJL model maybe also called renormalizable in this sense [5].

We will consider below only fixed gauge coupling case, $b \rightarrow +0$. That case reminds us so-called asymptotically finite theories [12]. The general case for arbitrary b can be also done. However the corresponding expressions can not be written in explicit form so they are not very instructive.

The solution of first RG Eq. (2) is the same in gauge Higgs-Yukawa theory, or in gauged NJL model:

$$\eta(t) \equiv \frac{g^2(t)}{g_0^2} \equiv \frac{\alpha(t)}{\alpha_0} = \left(1 + \frac{b}{2\pi} \alpha_0 t\right)^{-1}. \quad (7)$$

For fixed gauge coupling limit $b \rightarrow +0$ we get

$$\eta^{-c/b}(t)_{b \rightarrow 0} \rightarrow \exp\left(\frac{\alpha}{\alpha_c} t\right) = \left(\frac{\mu}{\mu_0}\right)^{\alpha/\alpha_c}, \quad (8)$$

where $\alpha \equiv \alpha_0$ is the initial value for gauge coupling and

$$\alpha_c^{-1} \equiv \frac{c}{2\pi} = \frac{3C_2(R)}{\pi}. \quad (9)$$

For scalar and Yukawa couplings we should solve RG eqs. (2) in general case. Then, the compositeness conditions [11] should be used in order to obtain running Yukawa and scalar couplings in gauged NJL model for $b \rightarrow +0$. The result looks as following [5]

$$\begin{aligned} y^2(t) &= \frac{(4\pi)^2}{2a} \frac{\alpha}{\alpha_c} \left[1 - \left(\frac{\mu}{\Lambda}\right)^{\frac{a}{\alpha_c}}\right]^{-1} \equiv y_\Lambda^2(t), \\ \frac{\lambda(t)}{y^4(t)} &= \frac{2a}{(4\pi)^2} \frac{\alpha_c}{\alpha} \left[1 - \left(\frac{\mu}{\Lambda}\right)^{\frac{2a}{\alpha_c}}\right] \equiv \frac{\lambda_\Lambda(t)}{y_\Lambda^4(t)}, \end{aligned} \quad (10)$$

where $t < t_\Lambda$ and condition $c > b$ is automatically satisfied because c is a positive constant. Note that in the limit $\Lambda \rightarrow \infty$, running coupling constants (10) tend to the fixed points of ref. [10].

In addition, we should use also a compositeness condition for the mass. It leads to the following running mass [5]:

$$m^2(t) = \frac{2a}{(4\pi)^2} \left(\frac{\Lambda^2}{\mu^2}\right)^W y_\Lambda^2(t) \mu^2 \left[\frac{1}{g_4(\Lambda)} - \frac{1}{W}\right], \quad (11)$$

where $W \equiv 1 - \frac{\alpha}{2\alpha_c}$ and $g_4(\Lambda)$ is dimensionless constant defined by

$$G \equiv \left(\frac{(4\pi)^2 g_4(\Lambda)}{a \Lambda^2}\right). \quad (12)$$

Note that in case of an external gravitational field one has extra compositeness conditions, for example, for scalar-gravitational coupling [9] (for non-gauged NLJ-model, see also [13]).

To conclude, we obtained the description of the gauged NJL model (in fixed gauge coupling case) via correspondent running coupling constants. These running couplings were obtained using the equivalence with the gauge Higgs-Yukawa model.

IV. RG IMPROVED EFFECTIVE POTENTIAL AND DYNAMICAL CHIRAL SYMMETRY BREAKING

We will be interested here in the calculation of the effective potential for gauged NJL-model in an external magnetic field. The analysis of such an effective potential gives the answer to the question about the possibility of dynamical symmetry breaking.

Usually for the evaluation of the effective potential in gauged NJL model one can use Schwinger-Dyson equation [14]. However this equation, which presumably gives the non-perturbative results is well understood only in the situation of no external fields. In some very special circumstances (for example, weak coupled massless QED in an external weak magnetic field [15], or weak gravitational field [16]) one can generalize SD equation taking into account also background fields. However, in general situations (arbitrary external gauge or gravitational fields) that doesn't seem to be possible [16].

However, in the model under consideration one may use RG technique because we have RG formulation of the gauged NJL-model. It has been shown [5] that in flat space the RG improved effective potential coincides with the one which was obtained via ladder SD equation [14]. Hence we expect that our results will be equivalent to results one can obtain from ladder SD equation, which is not formulated yet in background gauge fields.

The technique to study RG improved effective potential (effective Lagrangian) is quite well-known in situation where background fields are not presented [17]. It is also known how to generalize it (for the case of an external gravitational field, see [18]). So we will not give any details of this RG improvement technique (see [17,18]).

Using the fact that effective potential satisfies the RG equation, one can solve this equation by the method of characteristics. Then

$$V(g, y, \lambda, e, m^2, \xi, \sigma, \mu) = V(\bar{g}(t), \bar{y}(t), \bar{\lambda}(t), \bar{e}(t), \bar{m}^2(t), \bar{\sigma}(t), \mu e^t). \quad (13)$$

Here, the effective coupling constants $\bar{g}(t), \dots, \bar{m}^2(t)$ are defined by eqs. (10,11) at scale μe^t , $\sigma(t)$ is written in [5]. The choice for RG parameter t will be discussed below. As it

stands the relation (12) is too general to be used in any real calculation. In other words, the boundary condition to define V at $t = 0$ should be added. The one-loop effective potential (4,5) is convenient to be used as such boundary condition.

As first case, we will consider the effective potential (4) as boundary condition. Then RG parameter t is defined in the same way as in [5] (from the condition of the vanishing of the logarithmic term). The final answer will be given by RG improved effective potential written in [5] (at finite cut-off or at cut-off tends to infinity, see (6,13) or (6,16) of [5]) (for correction of misprint in [5], see [9]) plus H-dependent term. Note that H dependent term appears in RG improved potential in the combination

$$\frac{1}{2}H^2 - H^2 \frac{a\bar{e}^2(t)}{24\pi^2} \left(\ln \frac{\bar{M}_F^2(t)}{\mu^2 e^{2t}} \right).$$

Then,

$$\frac{(4\pi)^2}{2a} \frac{V}{\mu^4} = \frac{1}{2} \left[\frac{1}{g_{4R}(\mu)} - \frac{1}{g_{4R}^*} \right] \frac{y_*^2 \sigma^2(\mu)}{\mu^2} + \frac{\alpha_c}{4\alpha} \left(1 + \frac{3\alpha}{2\alpha_c} \right) \left[\frac{y_* \sigma(\mu)}{\mu} \right]^{4/2-W} + \frac{1}{2} \frac{H^2 (4\pi)^2}{\mu^4} \frac{1}{2a},$$

where $g_4^* \equiv W$, y_* denotes $y_\Lambda(t)$ (10) at $\Lambda \rightarrow \infty$. We wrote explicitly renormalized value of effective potential. For $H = 0$ it coincides with the result of refs. [5,14]. Hence, one can see that in such approach there is no effect of magnetic field to chiral symmetry breaking, which may occur in the case without external field.

Next, we analyse RG improved effective potential for the function (5) chosen as boundary condition. With the condition of vanishing of logarithmic terms in the effective potential (5) we find t as follows:

$$e^t = \left(\frac{eH}{\mu^2} \right)^{1/2}. \quad (14)$$

Then RG improved effective potential (13) at finite cut-off is

$$\frac{(4\pi)^2}{2a} \frac{V}{\mu^4} = \frac{x^2}{2} \left(\frac{\Lambda^2}{\mu^2} \right)^W \left[\frac{1}{g_4(\Lambda)} - \frac{1}{W} \right] + \frac{x^4}{4} \left(\frac{eH}{\mu^2} \right)^{\frac{1}{2}} \left[\frac{3}{2} + \frac{\alpha_c}{\alpha} - \frac{\alpha_c}{\alpha} \left(\frac{\sqrt{eH}}{\Lambda} \right)^{\frac{2\alpha_c}{\alpha}} \right] + \frac{1}{2} H^2 \frac{(4\pi)^2}{2a\mu^4}, \quad (15)$$

where $x = \frac{y_\Lambda(\mu)\sigma_\Lambda(\mu)}{\mu}$.

At the limit $\Lambda \rightarrow \infty$ we obtain

$$\frac{(4\pi)^2}{2a} \frac{V}{\mu^4} = \frac{x_*^2}{2} \left[\frac{1}{g_{4R}(\mu)} - \frac{1}{g_{4R}^*} \right] + \frac{x_*^4}{4} \left(\frac{eH}{\mu^2} \right)^{\frac{3-\alpha_c}{\alpha_c}} \left[\frac{3}{2} + \frac{\alpha_c}{\alpha} \right] + \frac{(4\pi)^2 H^2}{4a\mu^4}. \quad (16)$$

The main qualitative result of this calculation is that, background magnetic field in the above described approach doesn't change no-external field condition of chiral symmetry breaking:

$$\frac{1}{g_{4R}(\mu)} - \frac{1}{g_{4R}^*} < 0. \quad (17)$$

Of course, the above result is caused only by the fact that we treated external magnetic field as expansion over powers of H . One can proceed using the exact result (8). Then there are no problems, in principle, to calculate the effective potential explicitly. However the final result will be too complicated, (it cannot be presented with analytical expressions) and not very instructive.

V. DISCUSSION

In summary, we found the effective potential for gauged NJL model in external magnetic field. It is shown that structure of effective potential is changed if compare with the case of no magnetic field. However the weak magnetic field doesn't influence chiral symmetry breaking condition. The technique developed in this work opens the way to the study of gauged NJL model at external gauge fields without using of Schwinger-Dyson ladder equation.

It is interesting to note that one can consider gauged NJL model in external gravitational and magnetic fields (see [6] for non gauged case). The situation of physical interest maybe magnetically charged black hole. Then, $R = 0$ but $R_{\mu\nu\alpha\beta}$ (or $R_{\mu\nu}$) maybe not zero.

In applying again derivative expansion also over powers of $R_{\mu\nu\alpha\beta}$ we will find the following additional term

$$\left(\beta + \beta_1 \ln \frac{M_F^2}{\mu^2} \right) R_{\mu\nu\alpha\beta}^2$$

in the potential (4). Proceeding as above we will again see that external gravitational field which describes the black hole doesn't change the condition of chiral symmetry breaking. Of course, strong gravitational fields for gravitational fields of other configurations (say $R \neq 0$) change drastically the phase structure of gauged NJL model [9].

Finally, one can immediately extend this paper to the study of chiral symmetry breaking in the strong magnetic field (making use of (3) for big enough H) or in the external constant electric field where particle creation occurs [8].

Acknowledgments - The work by SDO has been supported by Colciencias (Colombia) and FAPESP (São Paulo, Brasil), the work by Gitman has been supported by CNPq (Brasil).

REFERENCES

- [1] T. Maskawa and H. Nakajima, *Progr. Theor. Phys.* **52** (1974) 1326; R. Fukuda and T. Kugo, *Nucl. Phys.* **B117** (1976) 250; W. A. Bardeen, C. N. Leung and S. T. Love, *Nucl. Phys.* **B273** (1986) 649
- [2] L. S. Celenza, V. K. Mishra, C. M. Shakin and K. F. Lin, *Phys. Rev. Lett.* **57** (1986) 55; Y. J. Ng and Y. Kikuchi; *Phys. Rev.* **D36** (1987) 2876; D.G. Caldi and A. Chodos, *Phys. Rev.* **36** (1987) 2876
- [3] T. Cowan et al., *Phys. Rev. Lett.* **56** (1986) 444
- [4] S. P. Klevansky and R.H. Lemmer, *Phys. Rev.* **D39** (1989) 3478; H. Suganuma and T. Tatsumi, *Ann. Phys.* **208** (1991) 470; S. Schramm, B. Müller and A. J. Schramm, *Mod. Phys. Lett.* **A8** (1992) 973; D. K. Hong, Y. Kim and S.-J. Sin, hep-th/9603157
- [5] M. Harada, Y. Kikukawa, T. Kugo and H. Nakano, *Progr. Theor. Phys.* **92** (1994) 1161
- [6] D. M. Gitman, S. D. Odintsov and Yu. I. Shilinov, *Phys. Rev.* **D54** (1996) 2968
- [7] S. Coleman and E. Weinberg, *Phys. Rev.* **D7** (1973) 1888;
- [8] E. S. Fradkin, D. M. Gitman and Sh. M. Shvartsman, *Quantum Electrodynamics with Unstable Vacuum*, (Springer-Verlag, Berlin 1991)
- [9] B. Geyer and S. D. Odintsov, *Phys. Lett.* **B376** (1996) 260; *Phys. Rev.* **D53** (1996) 7321
- [10] B. Pendleton and G. Ross, *Phys. Lett.* **B98** (1981) 291
- [11] W. Bardeen, C. Hill and M. Lindner, *Phys. Rev.* **D41** (1990) 1647
- [12] S. D. Odintsov and I. L. Shapiro, *Mod. Phys. Lett.* **A14** (1989) 1479; *JETP Lett.* **49** (1989) 125
- [13] C. T. Hill and D. S. Salopek, *Ann. Phys.* **213** (1992) 21; T. Muta and S. D. Odintsov, *Mod. Phys. Lett.* **A6** (1991) 3641
- [14] W. A. Bardeen and S. T. Love, *Phys. Rev.* **D45** (1992) 4672; K.-I. Kondo, M. Tanabashi and K. Yamawaki, *Progr. Theor. Phys.* **89** (1993) 1249
- [15] C. N. Leung, Y. J. Ng and A. W. Ackley, hep-th 9512114; *Phys. Rev.* **D54** (1996) 4181
- [16] S. D. Odintsov and Yu. I. Shilinov, *Class. Quant. Grav.* **8** (1990) 887
- [17] S. Coleman and E. Weinberg, *Phys. Rev.* **D7** (1973) 1888; M. Sher, *Phys. Repts.* **179** (1989) 274; C. Ford, D. R. T. Jones, P.-W. Stevenson and M. B. Einhorn, *Nucl. Phys.* **B395** (1993) 17; M. Bando, T. Kugo, N. Maekawa and H. Nakano, *Progr. Theor. Phys.* **90** (1993) 405; J. A. Casas, J. R. Espinosa and M. Quiros, *Phys. Lett.* **B342** (1995) 171; B. Kastening, *Phys. Rev.* **D54** (1996) 3965
- [18] I. L. Buchbinder and S. D. Odintsov, *Class. Quant. Grav.* **2** (1985) 721; E. Elizalde and S. D. Odintsov, *Phys. Lett.* **B303** (1993) 240; *Z.Phys.* **C64** (1994) 699
- [19] A. Salam and J. Strathdee, *Nucl. Phys.* **B90** (1975) 203; A. Linde, *Phys. Lett.* **B62** (1976) 435