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Inelasticity Distributions in High-Energy p-Nucleus Collisions

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Abstract

Inelasticity distributions in high-energy p-nucleus collisions are computed in the framework of the Interacting Gluon Model, with the impact-parameter fluctuation included. A proper account of the peripheral events by this fluctuation has shown to be vital for the overall agreement with several reported data. The energy dependence is found to be weak.

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Inelasticity is one of the basic quantities describing high-energy hadronic and nuclear collisions. Thus, since early times its study has deserved a special attention both of the experimentalists and of the theoreticians. Yet, experimental data are rather scarce and the theoretical understanding of several aspects such as its average value, its distribution and the energy dependence, is far from being satisfactory.

One of the main characteristics of the high-energy hadronic or nuclear collisions is the existence of a large event-by-event fluctuation, exhibited in several observed quantities. Thus, in a given experimental setup and even under the same initial condition of colliding objects, events with different final state configurations take place. Such a fluctuation has either a quantum mechanical or statistical origin or even simply associated with the impact parameter. A convenient model which takes the quantum mechanical fluctuation in the initial stage of the collision into account, and thus may provide us with the inelasticity distribution, is the Interacting Gluon Model [1] (IGM). This model is based on an idea

[2] that in high-energy collisions valence quarks weakly interact so that they almost pass thorough, whereas gluons interact strongly, producing an indefinite number of mini-fireballs which eventually form a unique large central fireball (all possible $q\bar{q}$ sea quarks are, in this model, "converted" to equivalent gluons).

However, the main drawback of the original version was the lack of an appropriate account of the impact-parameter fluctuation. In a previous work [3] (hereafter called I), we have improved it, by including this effect. Conceptually, this fluctuation is evidently necessary in any realistic description of hadronic or nuclear collisions, but we have shown that it also modifies the observables in a significant amount. In I, our interest was to study the effects of the initial-condition fluctuations in hydrodynamical models and so fixed our attention mainly on the rapidity and pseudo-rapidity distributions of the produced particles, by considering p-p collisions, where, seemingly, the fluctuation effects manifest themselves more conspicuously. In the present note, we shall focus our attention upon the inelasticity and extend the previous calculations also to p-nucleus collisions.

The impact parameter $\vec{b}$ defines, in the first place, the *probability density of occurrence of a reaction* (apart from the normalization) $F(\vec{b}) = 1 - |S(\vec{b})|^2$, where we assume that the inelastic processes occur due to the gluon-gluon fusion. So the eikonal function is written as

$$|S(\vec{b})|^2 = \exp\{-C\int d\vec{b}'\int d\vec{b}''\, D_p(\vec{b}')\, D_A(\vec{b}'')\, f(\vec{b}+\vec{b}'-\vec{b}'')\}, \qquad (1)$$

where $D_p(\vec{b})$ is the gluon thickness function of proton, $D_A(\vec{b})$ is the one for the nucleus A and C is an energy-dependent parameter to be determined by the normalization condition $\int F_{pp}(\vec{b})\,d\vec{b} = \sigma_{pp}^{inel}(\sqrt{s})$ for pp collision. Notice that, because of this, the pA cross-section $\sigma_{pA}^{inel}(\sqrt{s}) = \int F_{pA}(\vec{b})\,d\vec{b}$ may be calculated by using (1), once pp cross-section is fixed. We have taken

$$\sigma_{pp}^{inel} = 56\,(\sqrt{s})^{-1.12} + 18.16\,(\sqrt{s})^{0.16} \qquad (2)$$

as an input [4]. The function $f(\vec{b})$ in (1) gives account of the finite *effective* gluon interaction range (with the screening effect taken into account) and is subject to the constraint

$\int f(\vec{b})\, d\vec{b} = 1$. The simplest choice of $f(\vec{b})$ would be $\delta(\vec{b})$, which represents a point interaction, but we preferred to parametrize it as a Gaussian with a range $\approx 0.8\ fm$, which is more consistent with the finite range of the strong interaction and also describes better the data. For $D_p(\vec{b})$ we take here a Gaussian distribution. Thus we have eventually

$$D_p(\vec{b}) = f(\vec{b}) = (a/\pi)\, e^{-ab^2}, \tag{3}$$

with $a = 3/(2\,R_p^2)$, where $R_P \approx 0.8\ fm$ is the proton radius. For $D_A(\vec{b})$, we take the z-integral of a Woods-Saxon distribution

$$D_A(\vec{b}) = \int_{-\infty}^{+\infty} \rho_A(\vec{b}, z)\, dz = \int_{-\infty}^{+\infty} \frac{\rho_0}{1 + \exp\left[(r - R_0)/d\right]} dz\,, \tag{4}$$

where $R_0 = r_0 A^{1/3}$, $r_0 = 1.2$ fm, $d = 0.54$ fm and $\rho_A(\vec{r})$ is normalized to A. Thus we get

$$F_{pA}(\vec{b}) = 1 - \exp\{-C\, h(\vec{b})\}\,, \tag{5}$$

with

$$h(\vec{b}) = a \int_0^{\infty} db'\, b'\, D_A(\vec{b}')\, I_0(a\, b\, b')\, e^{-a(b^2+b'^2)/2}\,, \tag{6}$$

where I_0 is a modified Bessel function.

Secondly, the impact parameter determines *how the energy and momentum of the fire-ball fluctuate*, because as the impact parameter increases the average fire-ball mass becomes smaller. In I, we have incorporated this effect by writing the gluon momentum distribution functions as

$$G_p(x, \vec{b}) = D_p(\vec{b})\,/x\,, \qquad G_A(y, \vec{b}) = D_A(\vec{b})\,/y\,, \tag{7}$$

where x and y are the Feynman variables of gluons in p and A, respectively, in the equal-velocity (e.v.) frame. With this notation, the density of gluon pairs that fuse contributing to the final fireball may be expressed as

$$\begin{aligned} w(x, y; \vec{b}) &= \int d\vec{b}' \int d\vec{b}''\, G_p(x, \vec{b}'')\, G_A(y, \vec{b}'')\, \sigma_{gg}(x, y)\, f(\vec{b} + \vec{b}' - \vec{b}'')\, \theta\left(xy - M_{\min}^2/s\right) \\ &= h(\vec{b}) w(x, y)\,, \end{aligned} \tag{8}$$

with

$$w(x, y) = \left[\sigma_{gg}(x, y)\,/xy\right]\, \theta\left(xy - M_{\min}^2/s\right), \tag{9}$$

where $M_{\min} = 2m_\pi$ and the gluon-gluon cross-section is parametrized as [6]

$$\sigma_{gg}(x, y) = \alpha\,/\,(xys)\,, \tag{10}$$

with $\alpha = 21.35$, which has been fixed by using the pp inelasticity data [5]. Observe that in (8), its impact-parameter dependence is factorized out.

Now, we shall give a brief account of how to obtain the probability density $\chi(E, P; \vec{b})$ of collision at an impact parameter $\vec{b}$, forming a fireball with an energy E and a momentum P. We assume that the colliding proton and nucleus form a central fireball, via gluon exchanges depositing in it momenta $x(\vec{b})\sqrt{s}/2$ and $-y(\vec{b})\sqrt{s}/2$, respectively. Let n_i be the number of gluon pairs that carry momenta $x_i\sqrt{s}/2$ and $-y_i\sqrt{s}/2$. Thus,

$$\sum_i n_i x_i = x(\vec{b}) \qquad \text{and} \qquad \sum_i n_i y_i = y(\vec{b})\,. \tag{11}$$

In what follows, we will omit the explicit $\vec{b}$ dependence of x and y in order not to overload the notation. The energy and momentum of the central fire ball in the e.v. frame of the incident particles are given by

$$E = (x + y)\sqrt{s}/2, \qquad P = (x - y)\sqrt{s}/2 \tag{12}$$

and its invariant mass M and rapidity Y are respectively

$$M = \sqrt{sxy} \equiv \kappa\sqrt{s} \quad \text{and} \quad Y = (1/2)\ln(x/y)\,. \tag{13}$$

With these notations, we can follow the prescription given in [1] and write the relative probability of forming a fireball with a specific energy and momentum as

$$\Gamma(x, y; \vec{b}) \simeq \exp\{-\mathbf{\tilde{X}}^T \mathbf{G}^{-1} \mathbf{\tilde{X}}\}/\left[\pi\sqrt{\det(\mathbf{G})}\right], \tag{14}$$

where

$$\mathbf{X} = \begin{pmatrix} x - \langle x \rangle \\ y - \langle y \rangle \end{pmatrix}, \qquad \mathbf{G} = 2 \begin{pmatrix} \langle x^2 \rangle & \langle xy \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix},$$

with the notation

$$\langle x^m y^n \rangle = \int dx' \int dy'\, x'^m\, y'^n\, w(x', y'; \vec{b}), \tag{15}$$

where $w(x, y; \vec{b})$ is precisely the function given in (8).

In terms of E and P, the probability density is calculated as

$$\Gamma(E, P; \vec{b}) \simeq [2\sqrt{a_1 a_2}/\pi] \exp\{-a_1(E - \langle E \rangle)^2 - a_2 P^2\}, \tag{16}$$

where $a_1 = [s(\langle x^2 \rangle + \langle xy \rangle)]^{-1}$, $a_2 = [s(\langle x^2 \rangle - \langle xy \rangle)]^{-1}$ and (this is just a notation, don't confuse it with the average value; it is not because $w(x, y; \vec{b})$ is not normalized) $\langle E \rangle = (\langle x \rangle + \langle y \rangle)\sqrt{s}/2$.

Apparently, the expression (16) is normalized. However, both E and P are bounded because of the energy-momentum conservation constraint. There is also a minimum allowed fireball mass $M_{\min} = 2m_\pi$. So, we have to put some additional factor in (16) to recover the correct normalization,

$$\chi(E, P; \vec{b}) = \chi_0(\vec{b})\Gamma(E, P; \vec{b}), \tag{17}$$

where $\chi_0(\vec{b})$ should be determined by the condition

$$\int dP \int dE\, \chi(E, P; \vec{b})\, \theta(\sqrt{E^2 - P^2} - M_{\min}) = F_{pA}(\vec{b})/\sigma_{pA}^{inel}. \tag{18}$$

As implied by our parametrization (7), the gluon momentum distribution is independent of the particular type of nucleus, the only difference being their density. So, in computing the integral (15), x' and y' vary from some lower limit, defined by the condition $\sqrt{sx'y'} = M_{\min}$, up to 1, corresponding to the complete neglect of any collective effect of the nucleons in a nucleus. On the other hand, y in (11) may be larger than 1, because gluons from different nucleons may contribute to give the fireball a momentum transfer that is larger than $\sqrt{s}/2$,

which is just the incident momentum of a single nucleon in our e.v. frame. So, we take as the upper limit of $y(\vec{b})$ the overlap $h(\vec{b})$, given by (6), whenever it is larger than 1. When $h(\vec{b}) < 1$, we take it $= 1$, because in such a case the proton is interacting just with a single nucleon. It is clear that $x(\vec{b})$ corresponding to p is bounded by 1. These conditions, together with the lower bound $\sqrt{sxy} = M_{\min}$, determine the integration limits of (18).

Once $\chi(E, P; \vec{b})$ is determined, we are ready to compute the inelasticity distribution, which is the main object of the present note. In I, following the authors of [1], we have defined the inelasticity as the variable κ appearing in (13). However, the usual definition is $k = (E_0 - E')/E_0$, where E_0 is the incident energy and E' the leading (or surviving) particle energy. We shall adopt this terminology here and as for κ, call it just κ. There is also some difference between k defined in the lab. frame and the one given in the e.v. frame. However, since this is quite negligible, in this note we will compute everything in the latter, although data are not necessarily given in such a frame. The κ-distribution has been obtained in I and reads

$$\chi(\kappa) = \int d\vec{b} \int dE \int dP\, \chi(E, P; \vec{b})\, \delta(\sqrt{(E^2 - P^2)/s} - \kappa)\, \theta(\sqrt{E^2 - P^2} - M_{\min}). \tag{19}$$

Then, by using the only existing $\chi(\kappa)$ data [5] at $\sqrt{s} = 16.5$ GeV, we can fix the parameter α of the model. A comparison with the data is shown in Fig.1, where we have also put the result of [1]. It is seen that, due to the impact-parameter fluctuation, the small-κ events became enhanced and the overall shape flatter, in better agreement with the data. The enhancement of large-κ events is simply due to the larger value of α which is necessary in this case.

The computation of the inelasticity distribution $\chi(k)$ is similar. Considering p as the projectile, we have $k = x$, so, by using (12),

$$\chi(k) = \int d\vec{b} \int dE \int dP\, \chi(E, P; \vec{b})\, \delta((E + P)/s - k)\, \theta(\sqrt{E^2 - P^2} - M_{\min}). \tag{20}$$

We show, in Fig.2, the results for several pA collisions at $\sqrt{s} = 550$ GeV. We do not have accelerator data at such a high energy, but it is seen that $\chi(k)$ is nearly k independent for pp, in agreement with ISR data. Recently [7], an estimate of hadron-Pb inelasticity distribution at an average energy of $\langle\sqrt{s}\rangle = 550$ GeV has been reported, in cosmic-ray experiment. The result is $\chi(k) = 3k^2$. We find that the qualitative features of our result agree well with the reported one. If one considers that our model does not include diffractive dissociation and that, seemingly, this component has not been separated from the non-diffractive one in those data, one may attribute the quantitative discrepancy to this effect. The energy-dependence of the inelasticity is quite small in our model, in opposition to the results of [1]. The main origin of this contrast is the factor σ_{pp}^{inel} which has been dropped out from (9). We show in Fig.3 the average value $\langle k\rangle$ as function of $\sqrt{s}$, for several target nuclei.

A related quantity is the leading-particle spectrum, as shown in Fig.4 at $\sqrt{s} = 14$ GeV. Assuming an approximate factorization of $x_l(= 2p_l/\sqrt{s})$ and p_T dependences, we have

$$E_l(d^3\sigma/dp^3) \approx f(x_l)h(p_T)\,, \tag{21}$$

where

$$f(p_l) = \int d\vec{b}\int dP\int dE\,\chi(E,P;\vec{b})\,\theta(\sqrt{E^2-P^2}-M_{\min})\,\delta(\left[\sqrt{s}-(E+P)\right]/2-p_l)\,. \tag{22}$$

The p_T dependence has been parametrized as

$$h(p_T) = (\beta/\pi)\,e^{-\beta p_T^2}, \tag{23}$$

where β has been determined by fitting the data [8]. One sees that the agreement is almost perfect. The result of [1] for pp is also shown for comparison. We did not put their curves for the other targets, but the behavior is similar, namely they are more bent showing a definite deviation from the data in the largest-x_l region. This is a consequence of the neglect of the peripheral events there.

We conclude the present note by summarizing that, except for the diffractive component, the IGM seems to describe well the p-A inelasticity, provided the peripheral events are correctly treated, by taking the impact-parameter fluctuation into account. The average inelasticity decreases very slowly with the energy, in this description.

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Figure Captions

Fig.1: κ-distribution for p-p at $\sqrt{s} = 16.5$ GeV. The data are from [5]. The solid line is our result, whereas the dashed one is from [1].

Fig.2: Inelasticity distribution for p-A collisions with several targets at $\sqrt{s} = 550$ GeV.

Fig.3: Energy dependence of the average inelasticity for p-A collisions.

Fig.4: Leading-particle spectra as fuction of x_l at $p_T = .3$ GeV. The data are from [8] at $\sqrt{s} = 14$ GeV. The solid curves are our results, whereas the dashed one is from [1]. We have chosen in (23) $\beta = 6$, 2.8, 2.6, 2.3, 2.2 and 2 GeV^{-2}, respectively for p, C, Al, Cu, Ag and Pb targets.

Figure 1

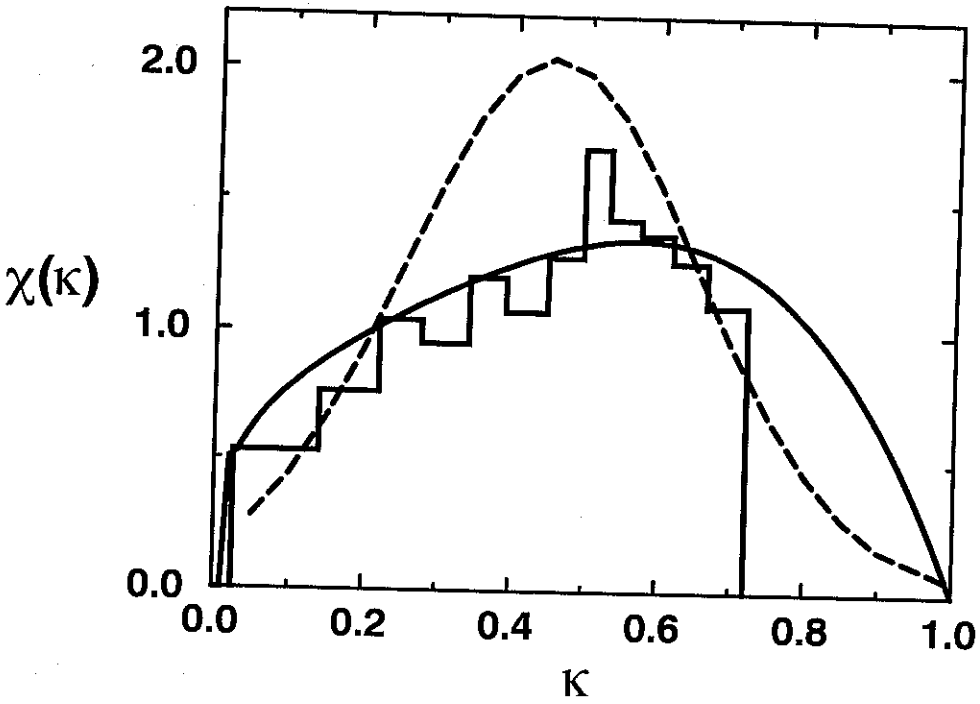

Figure 2

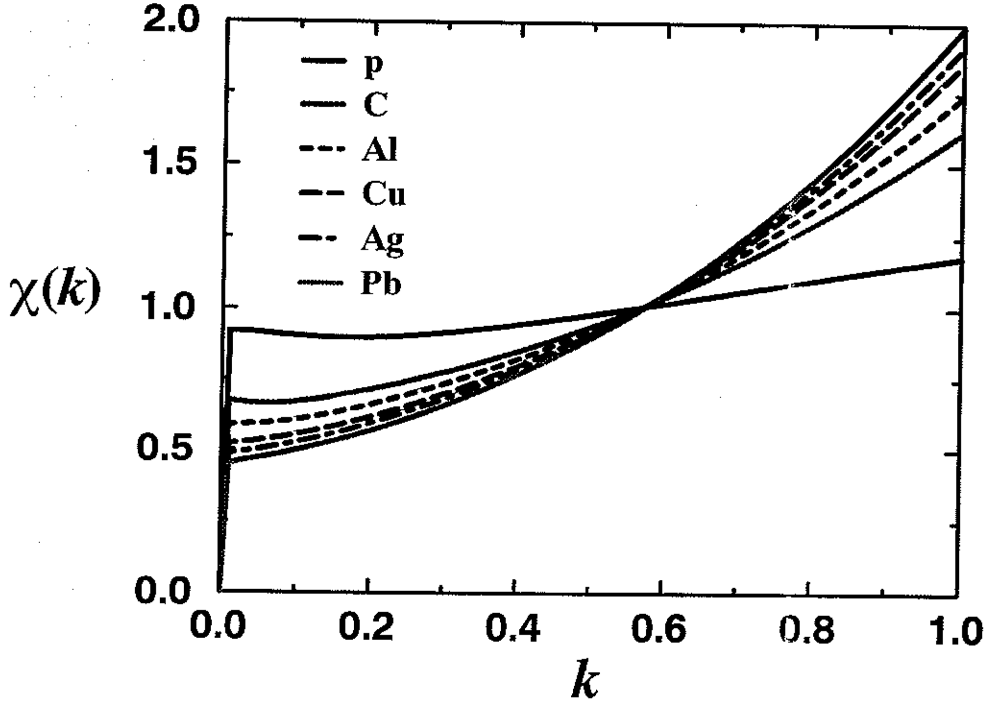

Figure 3

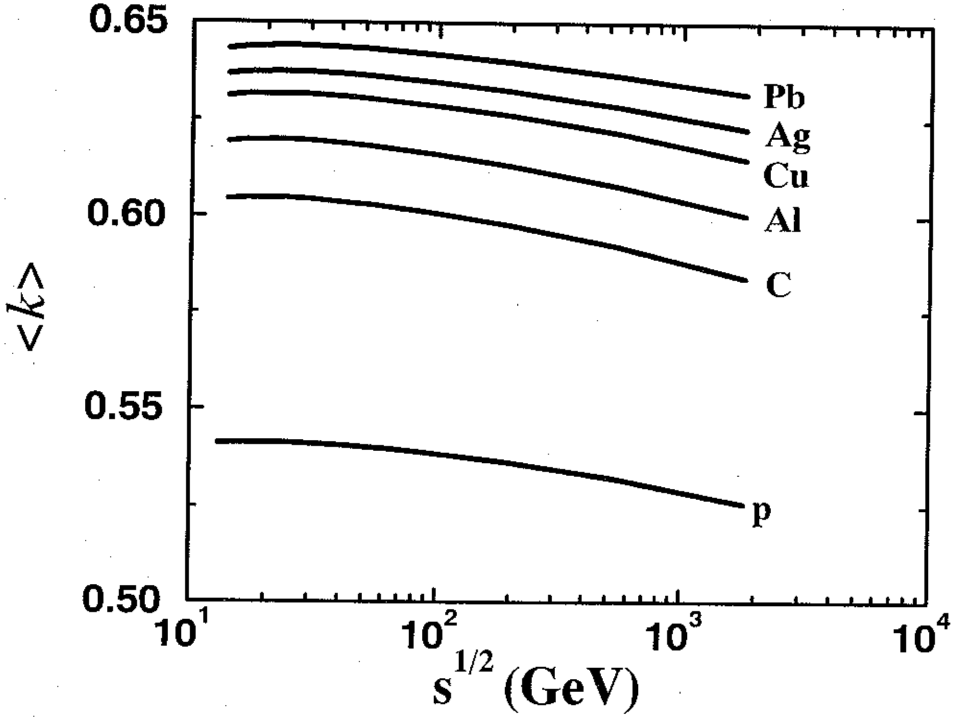

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Figure 4