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Evidence from QCD Sum Rules for Large Violation of Heavy Quark Symmetry in Λ_b Semileptonic Decay

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Abstract

We set up sum rules for heavy lambda decays in a full QCD calculation which in the heavy quark mass limit incorporates the symmetries of heavy quark effective theory. For the semileptonic decay of the Λ_b we find at the zero recoil point a violation of the heavy quark symmetry of more than 20%. We test the method by calculating the decay width of the semileptonic Λ_c decay and obtain a reasonable agreement with experiment.

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The semileptonic decays of hadrons are the most valuable source of information for the determination of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, but unfortunately these fundamental quantities of the standard model have to be disentangled from the effects of strong interactions inside the hadrons. In the limit where the initial and final quarks are infinitely heavy the degrees of freedom can be integrated out and the remaining heavy quark effective theory (HQET) ([1–4], for a review see [5]) can make a number of rigorous statements independently of the details of the strong interaction. However, the corrections to the heavy quark effective theory due to finite quark masses are by no means negligible and therefore one has to look for more reliable treatments for the effects of strong interactions on the semileptonic decays. QCD sum rules [6] (for reviews see [7,8]) are one of the most widely used and best founded approaches for these purposes, treating non-perturbative effects analytically with a limited input of phenomenological parameters, with good success in the calculation of corrections to HQET (see [5] and references therein). Particularly, QCD sum rules have shown that the non-leading corrections to HQET can be quite appreciable, even for hadrons with b-quarks [9,10].

Semileptonic decays of charmed into strange hadrons are among the best investigated processes. The relevant CKM matrix element V_{cs} is well determined, so that calculations of these decays may provide very good tests of the applied method. On the other hand, there are quite serious discrepancies between experiments and HQET in the lifetime ratios of beautiful baryons to mesons (see e.g. [11]), and hence it is of prime interest to investigate all decay channels of the Λ_b.

In this letter we evaluate the semileptonic decays of heavy Λ-baryons in the QCD sum rule approach as developed for heavy meson decays in reference [12]. This approach treats full QCD but also reproduces the symmetries of HQET. It is therefore a very interesting approach in the investigation of deviations from HQET in the decay $\Lambda_b \to \Lambda_c \ell \nu_\ell$.

We label generically the initial channel by I and the final channel by F. For the decay $\Lambda_I \to \Lambda_F + \ell + \nu_\ell$ we start from the three-point function of the weak transition current from a initial to a final quark $J_\mu = \bar{Q}_F \gamma_\mu (1-\gamma_5) Q_I$ and the interpolating fields of the initial and final baryons η_{Λ_I} and η_{Λ_F}

$$\Pi_\mu(p_F,p_I) = (i)^2 \int d^4x d^4y \langle 0|T\{\eta_{\Lambda_F}(x)J_\mu(0)\overline{\eta}_{\Lambda_I}(y)\}|0\rangle e^{ip_Fx}e^{-ip_Iy} . \quad (1)$$

The experimental information is contained in the decay amplitude [13]

$$\langle \Lambda_F(p_F)|J_\mu|\Lambda_I(p_I)\rangle = \overline{u}(p_F)[\gamma_\mu(F_1^V + F_1^A\gamma_5) + i\sigma_{\mu\nu}q^\nu(F_2^V + F_2^A\gamma_5) + q_\mu(F_3^V + F_3^A\gamma_5)]u(p_I) , \quad (2)$$

where $q = p_F - p_I$ and the form factors are functions of $t = q^2$.

We relate the theoretical (1) with the physical (2) quantities by inserting intermediate states into Eq.(1) and evaluate it in the not so deep Euclidean region where $p_F^2 < M_{\Lambda_F}^2$ and $p_I^2 < M_{\Lambda_I}^2$.

We introduce the couplings f_{F_1} and f_{I_1} of the currents with the respective hadronic states

$$\langle 0|\eta_{\Lambda_F}|\Lambda(p_F)\rangle = f_{F_1}u(p_F) \quad (3)$$

$$\langle \Lambda_I(p_I)|\overline{\eta}_{\Lambda_I}|0\rangle = f_{I_1}\overline{u}(p_I) , \quad (4)$$

and obtain

$$\Pi_\mu^{(\mathrm{phen})}(p_F,p_I) = \frac{(f_{F_1}\not{p}_F + f_{F_2})}{p_F^2 - M_{\Lambda_F}^2}\left[\gamma_\mu(F_1^V + F_1^A\gamma_5) + i\sigma_{\mu\nu}q^\nu(F_2^V + F_2^A\gamma_5)\right.$$
$$\left. + \; q_\mu(F_3^V + F_3^A\gamma_5)\right]\frac{(f_{I_1}\not{p}_I + f_{I_2})}{p_I^2 - M_{\Lambda_I}^2} + \text{higher resonances} , \quad (5)$$

where we have defined $f_{F_2} = f_{F_1}M_{\Lambda_F}$ and $f_{I_2} = f_{I_1}M_{\Lambda_I}$.

The theoretical contribution is obtained by performing the operator product expansion of the operator in Eq. (1) and then taking the expectation value with respect to the physical vacuum. The term from the unit operator gives the usual perturbative contribution, while the vaccum expectation values of the other operators in the expansion give the nonperturbative corrections proportional to the condensates of the respective operators. Thus

$$\Pi_\mu^{\mathrm{theor}} = \Pi_\mu^{\mathrm{pert}} + \sum_i \Pi_\mu^{\mathrm{nonpert}(i)} , \quad (6)$$

where the index i refers to the dimensions of the condensates.

As usual we evaluate the form factors $F_a^{V,A}$ of Eq.(2) ($a = 1,2,3$) by matching the phenomenological representation (5) of the three point function with the theoretical counterpart in Eq.(6). We project out sum rules for the products $F_a^{V,A}(q^2)f_{I_i}f_{F_k}$ $(i,k = 1,2)$ of the invariant amplitudes of Eq. (2) and the current couplings f of Eqs. (3,4), by performing appropriate traces of Eq. (5). We thus obtain 4 sum rules for each amplitude $F_a^{V,A}(t)$, but we use only those based on f_{I_1} and f_{F_1} since for them the imaginary part is positive definite. After this projection has been performed the treatment follows very closely the lines given in ref. [12]. We make the usual assumption that the contributions of the higher resonances (and the continuum) can be adequately approximated by the perturbative contributions above certain thresholds $s = p_I^2 \geq s_0$ and $u = p_F^2 \geq u_0$ and we use the Borel improvement by performing a double Laplace transform of the spectral functions of the theoretical expressions. A crucial ingredient for the incorporation of the HQET symmetries is to express the current couplings f_{I_i} and f_{F_k} also by QCD sum rules and relate the Borel parameters in the same way as explained in reference [12]. This also leads to a considerable reduction of the sensitivity to input parameters, like the continuum thresholds s_0 and u_0, and to radiative corrections [14].

As it is well known from two-point sum rules for baryons [15–17], there is a continuum of choices for the interpolating currents. Of course the results should in principle be independent of the choice of the current (except for pathological cases which couple very weakly to the ground state), but the justification of the approximations depends on the choice made. In this letter we concentrate on a very simple interpolating current which is motivated by a diquark picture where the two light quarks form a spin singlet state, namely

$$\eta_{\Lambda_Q} = \epsilon_{ABC}(\bar{u}^A\gamma^5 d^B)Q^C \quad (7)$$

where u^A and d^B stands for the Dirac field of light quarks of color A and B, and Q^C represents a heavy quark (b or c) of color C. With this choice of current the quark condensate and the mixed gluon-quark condensate do not contribute to Eq.(6). The gluon condensate does contribute, but experience with baryonic two-point functions and mesonic three point functions teaches us that it is of little influence. We are thus left with only the perturbative and the four quark condensate contributions.

In order to estimate the four quark condensate we use the factorization

$$\langle \bar{d}^A_\alpha \bar{u}^B_\beta u^{B'}_{\beta'} d^{A'}_{\alpha'} \rangle = \frac{\kappa}{12^2} \delta_{\beta\beta'} \delta_{\alpha\alpha'} \delta^{AA'} \delta^{BB''} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \ , \tag{8}$$

where the parameter κ, which ranges from 1 to 3, represents the deviation from the factorization hypothesis [7]. For the value of the quark condensate we take $< q\bar{q} >= -(230 \text{ MeV})^3$.

As general results we obtain

$$F_1^A(t) = -F_1^V(t);\ F_2^V(t) = F_3^V(t) = F_2^A(t) = F_3^A(t) = 0\ . \tag{9}$$

In Fig. 1 we show the behaviour of the contributions to the form factor F_1^V at t=0 for the process $\Lambda_b \to \Lambda_c \ell \nu_\ell$ for $\kappa = 2$ as function of the Borel mass M_F^2 . We have used $M_{\Lambda_b} = 5.65$ GeV, $M_{\Lambda_c} = 2.285$ GeV, $m_b = 4.6$ GeV, $m_c = 1.4$ GeV and $V_{cb} = 0.04$. We observe that the contributions from the continuum and from the four quark condensate are comparable and tend to stabilize each other for values of $M_F^2 \geq 10$ GeV^2. This seems to be a rather large value for the Borel mass. However, the influence of the 6-dimensional condensate is still large at that value and we expect the contributions of higher dimensional condensates to be very important at smaller Borel masses. A classical sum rule window where the perturbative and non-perturbative contributions are in equilibrium is thus the range above 10 GeV^2. In that range we obtain $F_1^V(0) = 0.49 \pm 0.03$ for $\kappa = 2$. We have also calculated the t-dependence of this form factor in the range $0 \leq t \leq 8$ GeV^2 which covers the major part of the kinematically allowed region $0 \leq t \leq 11.34$ GeV^2 and do not encounter difficulties with non-Landau singularities [12]. The t-dependence is represented with dots in Fig. 2, where it is shown that it can be very well approximated by a pole fit (solid line). The extrapolation of this fit to the maximal momentum transfer value, $t_{\max} = (m_{\Lambda_b} - m_{\Lambda_c})^2$ yields $F_1^V = -F_1^A = 0.74$. This value is remarkably stable against variations of the input parameters like κ, s_0, u_0 and the Borel mass. We estimate the errors to be

$$F_1^V(t_{\max}) = -F_1^A(t_{\max}) = 0.74 \pm 0.05\ . \tag{10}$$

In HQET this value is just the Isgur-Wise function at zero quark recoil and is predicted to be 1. We thus find a strong deviation of the heavy quark symmetry in the semileptonic decay $\Lambda_b \to \Lambda_c$. It is larger than in the $B \to D$ decays, where the corresponding value is about 0.9, in accordance with sum rules in full QCD [18], and corrections to HQET [19]. We have checked that this strong deviation is almost entirely due to the small mass of the charmed quark: by performing a sum rule analysis with a fictitious charm quark mass of 4.5 GeV (and correpondingly a mass of the Λ_c of 5.5 GeV) we obtained a deviation of only 5 % from HQET.

We have also calculated the semileptonic decay rate, which turns out to be much more sensitive to the input parameters than the value at the zero recoil amplitude. We obtained for the decay width

$$\Gamma(\Lambda_b \to \Lambda_c + e + \bar{\nu}_e) = (1.8 \pm 0.3) \times 10^{-14} \text{ , GeV} \tag{11}$$

where the errors reflect variations of κ from 1 to 2 and of the other parameters in reasonable limits. This value is consideraby smaller than other predictions [13,20,21] , which range from 3.5 to 6 $\times 10^{-14}$ GeV . It is interesting to note that the $1/m_Q$ corrections to HQET tend to increase the width [13,21], whereas our result clearly indicates that the width in full QCD is smaller than in HQET. The decay width obtained by us stays well below the experimental upper limit [22] given by $\Gamma(\Lambda_b \to \Lambda_c^+ \ell^- \bar{\nu}_\ell + \text{anything}) = (5.8 \pm 0.2) \times 10^{-14}$ GeV.

The form factors and the decay width of the semileptonic Λ_c decay can be calculated in the same way. Here the relative importance of the four quark condensate is even larger than in the Λ_b decay and the relative error is correspondingly larger too. We obtain for the width

$$\Gamma(\Lambda_c \to \Lambda + e^+ + \nu_e) = (1.0 \pm 0.3) \times 10^{-13} \text{ GeV.} \tag{12}$$

Within the errors this value agrees with the reported experimental value [22]

$$\Gamma(\Lambda_c \to \Lambda + e^+ + \nu_e) = (0.74 \pm 0.15) \times 10^{-13} \text{ GeV.} \tag{13}$$

The relations (9) yield an asymmetry parameter $\alpha = -1$ whereas the observed value [22] is -0.82 ± 0.10.

In summary, by taking into account full QCD in the context of a sum rule approach that incorporates HQET in the heavy quark limit, we found a deviation larger than 20% from the HQET prediction for the $\Lambda_b \to \Lambda_c$ semileptonic decay form factor at zero recoil. We have calculated in our approach the semileptonic decay width for $\Lambda_c \to \Lambda + e^+ + \nu_e$, obtaining a reasonable agreement with the experimental result.

A detailed description of our procedure and the application to other decays and observables will be given in a forthcoming publication [23].

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REFERENCES

[1] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. **45**, 292 (1987); **47**,511 (1988).

[2] S. Nussinov and W. Wetzel, Phys. rev **D 36**, 130 (1987).

[3] N. Isgur and M.B. Wise, Phys. Lett. **B237**, 527 (1990); Nucl. Phys. **B348**, 276 (1991).

[4] H. Georgi, Phys. Lett. **B240**, 447 (1990); Nucl. Phys. **B348**, 293 (1991).

[5] M. Neubert, Phys. Rep. **245**, 259 (1994).

[6] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[7] S. Narison, *QCD Spectral Sum Rules* , World Scientific, Singapore 1989.

[8] M.A. Shifman, *Vacuum Structure and QCD Sum Rules*, North Holland, Amsterdam 1992.

[9] E. Bagan, P. Ball, V.M. Braun and H.G. Dosch, Phys. Lett. **B278**, 457 (1992).

[10] S. Narison, Z.Phys. **C55**, 671, (1992).

[11] M. Neubert, *Theory of Beauty Lifetimes*, Second International Conference on B Physics and CP Violation, Honolulu 1997, hep-ph/9707217.

[12] P. Ball, V.M. Braun and H.G. Dosch, Phys. Rev. **D44**, 3567 (1991).

[13] J.G. Körner, M. Krämer and D. Pirjol, Prog. Part. Nucl. Phys. **33** , 787 (1994).

[14] E. Bagan, P. Ball and P. Gosdzinsky, Phys. Lett **B301**, 249 (1993).

[15] B.L. Ioffe, Nucl. Phys. **B188**, 317 (1981) Erratum: **B191**, 591 (1981).

[16] Y. Chung, *et al.* Nucl. Phys. **B197**, 55 (1982).

[17] E. Bagan, M. Chabab, H.G. Dosch and S. Narison, Phys. Lett. **B278**, 367 (1992); **B287**, 176 (1992); **B301**, 243 (1993).

[18] P. Ball, Phys. Lett. **B281**, 133 (1992).

[19] M. Neubert, Phys. Lett. **B338**, 84 (1994).

[20] M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner and P. Kroll, Phys. Rev. **D56**, 348 (1997).

[21] Y. Dai, C. Huang, M. Huang and C. Liu, Phys. Lett. **B387**, 379 (1996).

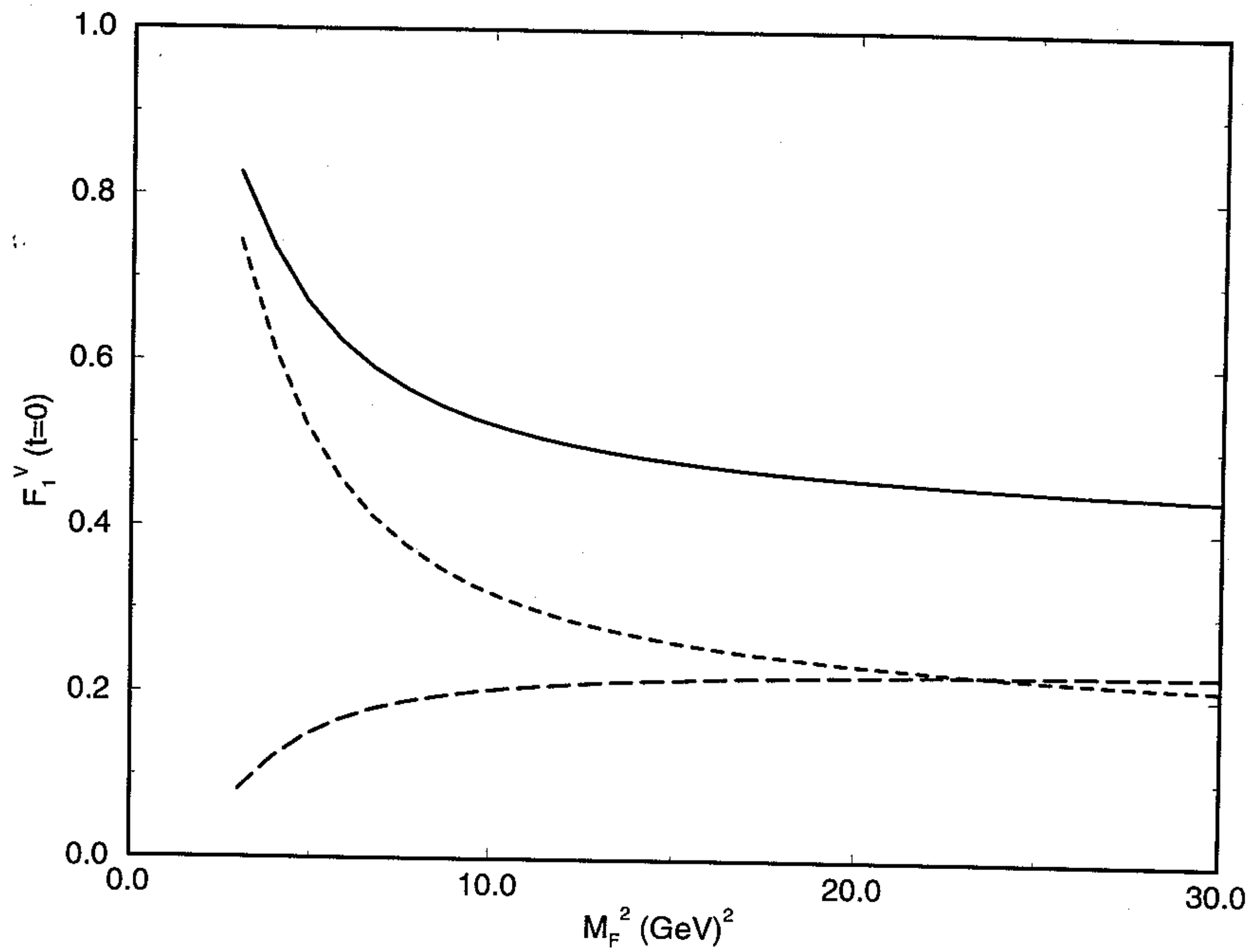
[22] Particle Data Group, Phys. Rev. **54**, 1 (1996).

[23] H.G. Dosch, E. Ferreira, M. Nielsen and R. Rosenfeld, in preparation.

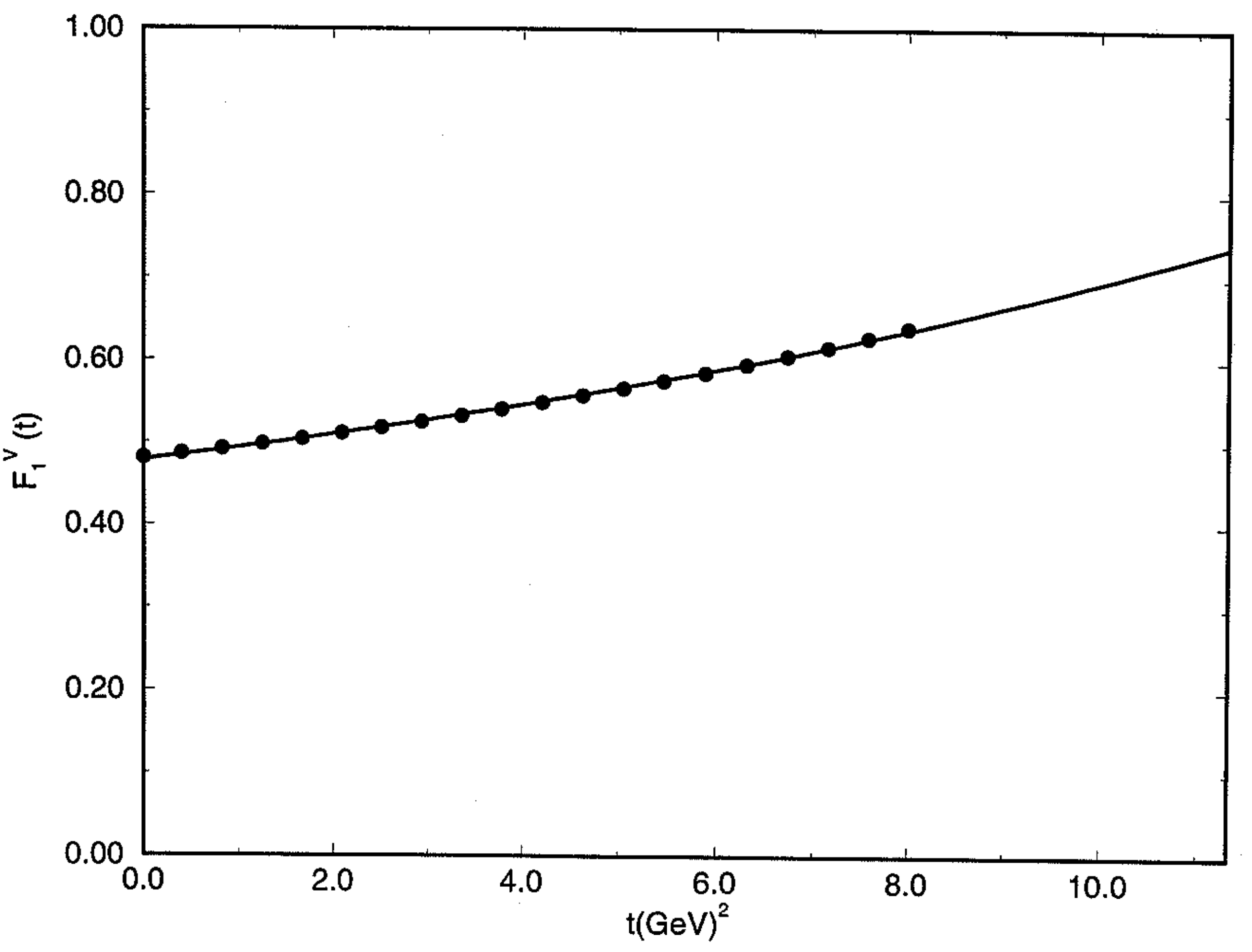
FIGURES

FIG. 1. The sum rule for the decay amplitude F_1^V at t=0 for the process $\Lambda_b \to \Lambda_c \ell \nu_\ell$ as function of the Borel mass M_P^2 . The long-dashed line is the perturbative contribution, the short-dashed line that of the four quark condensate for $\kappa = 2$. The solid line is the total contribution.

FIG. 2. The decay amplitude F_1^V at t=0 for the process $\Lambda_b \to \Lambda_c \ell \nu_\ell$ as function of the squared momentum transfer t to the leptons. The dots show the sum-rule results and the solid line is a pole fit $F_1^V(t) = 15.32/(32.03 - t)$.



$F_1^V(t=0)$
$M_F^2\ (\mathrm{GeV})^2$
1.0
0.8
0.6
0.4
0.2
0.0
0.0
10.0
20.0
30.0



$F_1^V(t)$
$t(\mathrm{GeV})^2$
1.00
0.80
0.60
0.40
0.20
0.00
0.0
2.0
4.0
6.0
8.0
10.0