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On zero modes of the eleven dimensional superstring

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Abstract

It is shown that recently pointed out by Berkovits on-shell degrees of freedom of the $D = 11$ superstring do not make contributions into the quantum states spectrum of the theory. As a concequence, the spectrum coincides with that of the $D = 10$ type IIA superstring.

In the recent work [1] the Green-Schwarz type formulation for the eleven dimensional superstring action has been proposed. The action is invariant under local fermionic κ-symmetry which eliminates half of θ-variables as well as under a number of global symmetries which can be considered as a realization of the "new supersymmetry" S-algebra [2–5]. It was also motivated that

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this $D = 11$ model is equivalent to the type IIA Green-Schwarz superstring.

In addition to the usual coordinates of $D = 11$ superspace $x^\mu(\sigma^a)$, $\theta^\alpha(\sigma^a)$, the action involves the auxilliary bosonic variables $n^\mu(\sigma^a)$, $A_a^\mu(\sigma^b)$, which are designed to provide the desired properties for the theory. A problem with these variables was formulated in Ref. [6], where it was pointed out that their zero modes can not be eliminated by means of gauge invariance and, hence, survive in the sector of physical degrees of freedom [1]. Thus, the question of the equivalence of this model to the GS superstring arises [6]. Since states spectrum of a string is determined by the action on a vacuum of oscillator modes only, one expects that the presence of zero modes for the case is inessential. In this short note we analyse this problem more detaily in the canonical quantization framework. It will be shown that taking into account of these zero modes do not spoil the final conclusion of Ref. [1]. Namely, quantum states spectrum of the model proposed coincides with that of the type IIA superstring.

Since we are dealing with zero modes of even variables, let us first consider a bosonic part of the action (we use the notations from Ref.1)

$$S = \int d^2\sigma \left\{ \frac{-g^{ab}}{2\sqrt{-g}}\partial_a x^\mu \partial_b x^\mu - \varepsilon^{ab}\xi_a(n^\mu \partial_b x^\mu) - n^\mu \varepsilon^{ab}\partial_a A_b^\mu - \frac{1}{\phi}(n^2+1) \right\} \tag{1}$$

[1]We are gratefull also to J. Gates for bringing this fact to our attention

It will be assumed that all the variables are periodical in the interval $\sigma \subset [0, \pi]$ functions. By direct application of the Dirac-Bergmann algorithm one finds the Hamiltonian

$$H = \int d\sigma \left\{ -\frac{N}{2}[\hat{p}^2 + (\partial_1 x)^2 - N_1(\hat{p}\partial_1 x) - \xi_0(n\partial_1 x) + \right.$$
$$(n\partial_1 A_0) + +\frac{1}{\phi}(n^2+1) + \omega^{ab}(\pi_g)_{ab} + \lambda_\phi \pi_\phi + \lambda_{\xi a}\pi_\xi{}^a +$$
$$\lambda_0 p_0 + \lambda_1(p_1 - n) + \lambda_n p_n, \tag{2}$$

where it was denoted

$$\hat{p}^\mu \equiv \ p^\mu + \xi_1 n^\mu, \qquad N \equiv \frac{\sqrt{-g}}{g^{00}}, \qquad N_1 \equiv \frac{g^{01}}{g^{00}}, \tag{3}$$

and p^μ, p^μ_a, p^μ_n, $(\pi_g)^{ab}$, π^a_ξ, π_ϕ are momenta conjugate to the variables x^μ, A^μ_a, n^μ, g^{ab}, ξ_a, ϕ respectively; λ_* are Lagrange multipliers corresponding to the primary constraints. The full system of constrains can be presented as follows

$$p^\mu_n = 0, \qquad n^\mu - p^\mu_1 = 0; \tag{4}$$

$$\pi^1_\xi = 0, \qquad \xi_1 - (p_1 p) = 0; \tag{5}$$

$$(\pi_g)_{ab} = 0, \qquad \pi_\phi = 0, \qquad \pi^0_\xi = 0, \qquad p^\mu_0 = 0; \tag{6}$$

$$(p_1)^2 = -1, \qquad \partial_1 p^\mu_1 = 0; \tag{7}$$

$$H_0 \equiv (p_1\partial_1 x) = 0, \qquad H_\pm \equiv (\hat{p}^\mu \pm \partial_1 x^\mu)^2 = 0; \tag{8}$$

where the constraint $n^\mu = p^\mu_1$ was used. Constraints (4),(5) are separated from others and form a system of second class, while

the remaining ones are first class. An appropriate gauge for the constraints from Eq.(6) is

$$g^{ab} = \eta^{ab}, \qquad \phi = 2, \qquad \xi_0 = 0, \qquad A^\mu_0 = \int_0^\sigma d\sigma' \xi_1 \hat{p}^\mu. \tag{9}$$

This choice simplifies the subsequent analysis of the (A^μ_1, p^μ_1)-sector since Hamiltonian equations of motion for these variables look now as

$$\partial_0 A^\mu_1 = p^\mu_1, \qquad \partial_0 p^\mu_1 = 0. \tag{10}$$

In order to find a correct gauge for the constraints (7) let us consider Fourier decomposition of periodical in the interval $\sigma \subset [0, \pi]$ functions

$$A^\mu_1(\tau, \sigma) = Y^\mu(\tau) + \sum_{n\neq 0} y^\mu_n(\tau) e^{i2n\sigma},$$
$$p^\mu_1(\tau, \sigma) = P^\mu_y(\tau) + \sum_{n\neq 0} p^\mu_n(\tau) e^{i2n\sigma}. \tag{11}$$

Then the constraint $\partial_1 p^\mu_1 = 0$ is equivalent to $p^\mu_n = 0$, $n \neq 0$, and an appropriate gauge is $y^\mu_n = 0$, or, in the covariant form $\partial_1 A^\mu_1 = 0$. Thus, physical degrees of freedom in the sector (A^μ_1, p^μ_1) are zero modes of these variables, and the corresponding dynamics is

$$A^\mu_1(\tau, \sigma) = Y^\mu + P^\mu_y \tau,$$
$$p^\mu_1(\tau, \sigma) = P^\mu_y = const, \qquad (P_y)^2 = -1. \tag{12}$$

This sector of the theory (1) can be considered as describing a string-like object $A^\mu_1(\tau, \sigma)$ which propagates without oscillations with the center of mass $Y^\mu + P_y^\mu \tau$ and the corresponding momenta P_y^μ. The only quantum state of the string is its ground state $| P_y >$ with mass $m_y^2 = P_y^2 = -1$.

Dynamics of the remaining variables is governed now by

$$\partial_0 x^\mu = -p^\mu - (P_y p)P_y^\mu, \qquad \partial_0 p^\mu = -\partial_1\partial_1 x^\mu. \tag{13}$$

In addition, the constraints

$$\begin{aligned} H_0 &\equiv (P_y\partial_1 x) = 0, \\ H_\pm &\equiv (p^\mu + (P_y p)P_y^\mu \pm \partial_1 x^\mu)^2 = 0, \end{aligned} \tag{14}$$

hold, which obey the following algebra

$$\begin{aligned} &\{H_\pm, H_\pm\} = \pm 4[H_\pm(\sigma) \pm (P_y p)H_0(\sigma) + (\sigma \to \sigma')]\partial_\sigma \delta(\sigma - \sigma'), \\ &\{H_+, H_-\} = 4[(P_y p)H_0(\sigma) + (\sigma \to \sigma')]\partial_\sigma \delta(\sigma - \sigma'), \\ &\{H_0, H_\pm\} = \pm 2 H_0(\sigma')\partial_\sigma \delta(\sigma - \sigma'). \end{aligned} \tag{15}$$

On the $D = 10$ hyperplane extracted by the constraint $H_0(\sigma) = 0$ it reduces to the standard Virasoro algebra. Note also that variable $x^\mu(\tau, \sigma)$ obeys the free equation $(\partial_\tau^2 - \partial_\sigma^2)x^\mu = 0$, as a consequence of Eqs.(13),(14).

To proceed further, it is usefull to impose the gauge

$$(P_y \partial_1 p) = 0, \tag{16}$$

to the constraint $H_0 = 0$. By virtue of Eqs.(13),(16) one finds, in particular, that $(P_y p) = (P_y P)$, where P^μ is the zero mode of

$p^\mu(\tau, \sigma)$. Then the final solution to Eq.(13) for the case of closed world sheet reads

$$\begin{aligned} x^\mu(\tau,\sigma) &= X^\mu - \tfrac{1}{\pi}(P^\mu + (P_y P)P_y^\mu)\tau + \\ &\quad \tfrac{i}{2\sqrt{\pi}}\sum \tfrac{1}{n}[\bar\alpha_n^\mu e^{i2n(\tau+\sigma)} + \alpha_{-n}^\mu e^{-i2n(\tau-\sigma)}], \\ p^\mu(\tau,\sigma) &= \tfrac{1}{\pi}P^\mu + \tfrac{1}{\sqrt{\pi}}\sum[\bar\alpha_n^\mu e^{i2n(\tau+\sigma)} - \alpha_{-n}^\mu e^{-i2n(\tau-\sigma)}], \end{aligned} \tag{17}$$

which is accompanied by the constraints

$$P_y^\mu \bar\alpha_n^\mu = 0, \qquad P_y^\mu \alpha_{-n}^\mu = 0, \tag{18}$$

$$\begin{aligned} H_+ &= \tfrac{8}{\pi}\sum_{-\infty}^{\infty} L_n e^{i2n(\tau-\sigma)} = 0, \qquad L_n \equiv \tfrac{1}{2}\sum_{-\infty}^{\infty}\alpha_{n-k}^\mu\alpha_k^\mu, \\ H_- &= \tfrac{8}{\pi}\sum_{-\infty}^{\infty} \bar L_n e^{i2n(\tau+\sigma)} = 0, \qquad \bar L_n \equiv \tfrac{1}{2}\sum_{-\infty}^{\infty}\bar\alpha_{n-k}^\mu\bar\alpha_k^\mu, \end{aligned} \tag{19}$$

where $\alpha_0^\mu = -\bar\alpha_0^{-\mu} \equiv \frac{1}{2\sqrt{\pi}}(P^\mu + (P_y P)P_y^\mu)$.

From Eq.(18)and the equality $(P^\mu + (P_y P)P_y^\mu)P_y^\mu = 0$ for momenta of center of mass, it follows that the sector (x^μ, p^μ) of the theory (1) describes in fact a closed string which lives on the (D-1)-dimensional hyperplane ortogonal to the P_y^μ - direction.

From the zero modes X^μ, P^μ, Y^μ, P_y^μ, one can construct the following quantities

$$\mathcal{X}^\mu \equiv X^\mu - \frac{1}{2}\frac{P_y Y}{P_y P}P_y^\mu, \qquad \mathcal{P}^\mu \equiv P^\mu + (P_y P)P_y^\mu, \tag{20}$$

with the properties

$$\{\mathcal{X}^\mu, \mathcal{P}^\nu\} = \eta^{\mu\nu}, \qquad \{\mathcal{X}^\mu, \mathcal{X}^\nu\} = \{\mathcal{P}^\mu, \mathcal{P}^\nu\} = 0. \tag{21}$$

So the quantities $\mathcal{P}^\mu$, $\mathcal{L}^{\mu\nu} = \frac{1}{2}(\mathcal{X}^\mu\mathcal{P}^\nu - \mathcal{X}^\nu\mathcal{P}^\mu)$ are generators of the Poincare group. This allows one to obtain the mass formulae for physical states. We adopt the Gupta-Bleuler prescription by requiring that physical states be annihilated by half of $: L_n :$, $: \bar{L}_n :$ operators

$$(L_n - a\delta_{n,0}) \mid phys >= (\bar{L}_n - a\delta_{n,0}) \mid phys >= 0, \;\; n > 0. \quad (22)$$

By virtue of Eq.(19) for n=0 one finds the mass of the states

$$m^2 = \mathcal{P}^2 = -4\pi \left\{ \sum_{n>0} (\alpha^{\mu}_{-n}\alpha^{\mu}_{n} + \bar{\alpha}^{\mu}_{-n}\bar{\alpha}^{\mu}_{n}) + 2a \right\} \quad (23)$$

As it should be, mass of the state is determined by oscillator exitations of $x^{\mu}(\tau, \sigma)$ -string only, zero modes of the (A^{μ}_1, p^{μ}_1) -sector do not make of contributions into this expression.

In order to describe a spectrum of the superstring suggested in Ref.1, it is more convenient to consider noncovariant quantization in an appropriately chosen coordinate system. By making use of Lorentz transformation one can consider coordinate system where $P^{\mu}_y = (0, ...0, 1)$. (Note that it is admissible procedure in the canonical quantization framework, since the Lorentz transformation is a particular example of the canonical one). This breaks manifest $SO(1, D-1)$ covariance up to $SO(1, D-2)$ one. In this basis Eq.(13)-(16) reduces to

$$\partial_0 x^D = 0, \qquad \partial_0 p^D = 0; \quad (24)$$

$$\partial_0 x^{\bar{\mu}} = -p^{\bar{\mu}}, \qquad \partial_0 p^{\bar{\mu}} = -\partial_1\partial_1 x^{\bar{\mu}}, \qquad (p^{\bar{\mu}} \pm \partial_1 x^{\bar{\mu}})^2 = 0; \quad (25)$$

where $\mu = (\bar{\mu}, D)$. Thus, zero modes of the theory (1) along the direction P^{μ}_y decouples from (D-1) - dimensional sector (25),

while oscillator modes along the direction P^{μ}_y are absent as a consequence of the equations $(P_y\partial_1 x) = (P_y\partial_1 p) = 0$. As was shown in Ref.1, the same holds for the supersymmetrical case, where equalities like $\bar{\theta}\Gamma^{\mu\nu}P^{\nu}_y\psi = -\theta\Gamma^{\bar{\mu}}\psi - \bar{\theta}\tilde{\Gamma}^{\bar{\mu}}\bar{\psi}$ must be taken into account.

Let us discuss the obtained results. The D - dimensional theory (1) can be considered as describing a pair of strings. The sector of auxilliary variables (A^{μ}_1, p^{μ}_1) corresponds to a string-like object which can not have oscillator exitations. The only physical degrees of freedom of this non-oscillating string (NO-string) are zero modes Y^{μ}, P^{μ}_y which correspond to propagation of the center of mass. After quantization, the only state of NO–string is its ground state with the mass $m^2 = -1$.

The sector of variables (x^{μ}, p^{μ}) describes the closed string (25),(23) which lives on (D-1) - dimensional hyperplane ortogonal to P^{μ}_y - direction. (Constraints (8) relating NO - string and the closed string mean that the last has no component of center of mass momenta as well as of oscillator exitations in the P^{μ}_y - direction, see Eqs.(17),(18)). From the mass formulae (23), and Eq.(25) it follows that quantum states spectrum of the theory (1) coincides with that of the (D-1) - dimensional closed bosonic string. In a similar fashion, spectrum of the D=11 superstring suggested in Ref.1 coincides with that of the D=10 type IIA superstring.

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