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EXAMPLES OF D=11 S-SUPERSYMMETRIC ACTIONS FOR POINT-LIKE DYNAMICAL SYSTEMS

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Examples of $D = 11$ S-supersymmetric actions for point-like dynamical systems.

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Abstract

A non standard super extensions of the Poincare algebra (S-algebra [1,2]), which seems to be relevant for construction of various $D = 11$ models, are studied. We present two examples of actions for point-like dynamical systems, which are invariant under off-shell closed realization of the S-algebra as well as under local fermionic κ-symmetry. On this ground, an explicit form of the S-algebra is advocated.

1 Introduction

The construction of higher-dimensional ($D > 10$) SYM [3,4] and superstring [5,6,2,7,8] models, which might be interesting in the M-theory context (see [9-14] and references therein), is under intensive investigation at present. It is known that consistency of the super Poincare and local symmetry transformations[1] imply rigid restrictions on possible dimensions of

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[1] We mean gauge transformations for the SYM-theory and local κ-symmetry transformations for the case of superstring.

space-time where these models can be formulated [15,9]. In particular, the standard methods can not be directly applied in $D > 10$ to construct the above mentioned super Poincare invariant models. One possibility to avoid these restrictions is to consider some different super extensions of the Poincare algebra[2]. In recent works [1-4,6,8,16,17] a relevant higher-dimensional superalgebra was discussed. It includes the Poincare generators as well as generators Q of new supertranslations with commutator may be written in the form[3]

$$\{Q_\alpha, Q_\beta\} \sim \Gamma^{\mu\nu} P_\mu n_\nu, \tag{1}$$

where $\Gamma^{\mu\nu}$ is antisymmetric product of $D = 11$ γ-matrices (we use γ-matrix conventions from [8]). It is known as S-algebra previously discussed in the M-theory context [1] (see [18] for discussion of a general case). For $D = 11$ case it can be realized in a superspace as follows:

$$\delta\theta = \epsilon, \qquad \delta x^\mu = i(\bar{\epsilon}\Gamma^{\mu\nu}\theta)n_\nu, \qquad \delta n^\mu = 0. \tag{2}$$

The appearance of a new variable n^μ seems to be an essential property of the construction (see discussion in [7,8]). In this relation it is interesting to clarify the role of the variable n^μ from the dynamical point of view, in particular, to present some examples of Lagrangian systems with n^μ incorporated on equal footing with other variables. Only in this case the corresponding theory can be actually SO(1,10) invariant.

It was also pointed out [2,8] that after substitution $n^\mu = (0, \cdots 0, 1)$ (which breaks SO(1,10) covariance up to SO(1,9) one) the transformations

[2] In recent works [5] $D = 11$ superstring action with second-class constraints simulating a gauge fixation for the κ-symmetry was suggested. The action was constructed by adding of an appropriately chosen terms to the GS action written in $D = 11$. Supersymmetry of quantum state spectrum for the model is under investigation now.

[3] As it will be demonstrated below (see also Ref.[2,8]) an explicit form of the algebra is $\{Q, Q\} \sim \Gamma^{\mu\nu} Z_{\mu\nu}$, with some additional bosonic generators $Z_{\mu\nu}$.

(2) reduce to

$$\delta\theta^\alpha = \epsilon^\alpha, \qquad \delta\bar{\theta}_\alpha = \bar{\epsilon}_\alpha,$$
$$\delta x^{\bar{\mu}} = -i\bar{\epsilon}_\alpha\tilde{\Gamma}^{\bar{\mu}\alpha\beta}\bar{\theta}_\beta - i\epsilon^\alpha\Gamma^{\bar{\mu}}_{\alpha\beta}\theta^\beta, \qquad \delta x^{10} = 0, \tag{3}$$

where $\theta = (\bar{\theta}_\alpha, \theta^\alpha)$, $\mu = (\bar{\mu}, 10)$, $\bar{\mu} = 0, 1, \cdots 9$, $\alpha = 1 \cdots 16$. One can see that (3) coincides exactly with the standard $D = 10$, type IIA supersymmetry transformations. In this sense the latter can be rewritten in a manifestly SO(1,10) covariant notations (2). Thus, it is naturally to ask about possibility of lifting the known $D = 10$ type IIA theories up to SO(1,10) invariant form. From the present discussion it is clear that the requirement of S-invariance instead of the super Poincare invariance might be a natural framework for construction of such a kind $D = 11$ formulations.

In this letter we present two examples of $D = 11$ finite-dimensional systems based on the S-algebra of global symmetries. For the first model the variable n^μ survives in the sector of physical degrees of freedom, while for the second one it turns out to be a nondynamical variable, which may be killed by a proper gauge fixing. It will be also demonstrated, that local κ-symmetry is consistent with global S-invariance in both cases.

The first example which we are going to study is in fact zero-tension limit of the $D = 11$ superstring action suggested in [8]. Physical degrees of freedom for the mechanical model may be considered as describing a composite system, the latter consists of a free moving particle and a superparticle (see also Refs.[6,16,17]). We present a Lagrangian action, which is invariant under local κ-symmetry as well as under off-shell closed realization of the S-algebra of global symmetries. The advantage of the present formulation (in comparison with [3,4,6,16,17]) is that an explicit Lagrangian action, with all the variables treated on equal footing is given. In particular, global symmetry transformations of the action form a super-

algebra in the usual sense, without appearance of nonlinear in generators terms in the right hand side of Eq.(1). In the result, a model-independent form of the S-algebra is presented.

From the discussion related to (2),(3) it is clear that a formulation where one may impose the gauge $n^\mu = (0, \cdots, 0, 1)$ would be at most preferable. As a second example, we present S-invariant model, which admits such a gauge, and which describes the propagation of a superparticle only. We hope that a similar construction may work for the case of $D = 11$ superstring as well.

The work is organized as follows. In the Sec.2 we present and discuss a $D = 11$ Poincare invariant action for the above mentioned composite system. In the Sec.3, a bosonic action which contains the nondynamical variable $n^\mu(\tau)$ related to S-symmetry is proposed. It is shown that the action describes a free propagating massless particle. On the base of this action S-supersymmetric version in $D = 11$ space-time is constructed in Sec.4. The latter action is invariant also under local fermionic κ-symmetry. Similarly to the Casalbuoni-Brink-Schwarz superparticle [19-21] it provides a free character of the dynamics for the physical sector variables.

2 D=11 composite system of a particle and a superparticle.

Let us consider the following $D = 11$ Lagrangian action

$$S = \int d\tau \left\{ v_\mu \Pi^\mu - \frac{1}{2}ev^2 + n_\mu \dot{z}^\mu - \frac{1}{2}\phi(n^2+1) \right\},$$
$$\Pi^\mu \equiv \dot{x}^\mu - i(\bar{\theta}\Gamma^{\mu\nu}\dot{\theta})n_\nu - \xi n^\mu, \tag{4}$$

where $x^\mu, v^\mu, z^\mu, n^\mu, e, \phi, \xi$ are Grassmann even and θ^α are Grassmann odd variables, dependent on the evolution parameter τ. The action is a direct mechanical analog of the $D = 11$ superstring suggested in [8]. Note [8] that eliminating the variable v^μ we can rewrite (4) in the second-order form relative to x^μ. Global bosonic symmetries of the action are both $D = 11$ Poincare transformations (with the variable n^μ being inert under the Poincare shifts) and the following transformations

$$\delta_b x^\mu = b^\mu{}_\nu n^\nu, \qquad \delta_b z^\mu = -b^\mu{}_\nu v^\nu, \tag{5}$$

with antisymmetric parameters $b^{\mu\nu} = -b^{\nu\mu}$. There is also a global symmetry with fermionic parameters ϵ^α,

$$\delta_\epsilon \theta = \epsilon, \qquad \delta_\epsilon x^\mu = -i(\bar\theta\Gamma^{\mu\nu}\epsilon)n_\nu, \qquad \delta_\epsilon z^\mu = i(\bar\theta\Gamma^{\mu\nu}\epsilon)v_\nu. \tag{6}$$

The algebra of the corresponding commutators turns out to be off-shell closed.[4] Thus, the S-algebra consist of Poincare subalgebra $(M^{\mu\nu}, P^\mu)$ and includes generators of new supertranslations Q_α as well as second-rank Lorentz tensor $Z_{\mu\nu}$. The nonzero commutators of the new generators are

$$\{Q_\alpha, Q_\beta\} = 2i(C\Gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu}. \tag{7}$$

Their commutators with the Poincare transformations have the standard form. Note, that it is not a modification of the super Poincare algebra but essentially different one, since the commutator of the supertranslations leads to Z-transformation instead of the Poincare shift.

The action (4) is also invariant under the local κ-symmetry transformations,

$$\begin{aligned} &\delta\theta = v_\mu\Gamma^\mu\kappa, \qquad \delta\xi = -2i(\dot{\bar\theta}\delta\theta), \qquad \delta e = 4i(\dot{\bar\theta}\Gamma^\mu\kappa)n_\mu. \\ &\delta x^\mu = i(\bar\theta\Gamma^{\mu\nu}\delta\theta)n_\nu, \qquad \delta z^\mu = -i(\bar\theta\Gamma^{\mu\nu}\delta\theta)v_\nu, \end{aligned} \tag{8}$$

This fact turns out to be crucial to verify that physical sector variables obey free equations of motion. Let us present the corresponding analysis

[4] S-algebra can be off-shell closed also for the action (4) written in the second order form [8]

in the Hamiltonian framework [22,23]. One finds the following trivial pairs of second-class constraints: $p_n^\mu = 0$, $p_z^\mu - n^\mu = 0$; $p_v^\mu = 0$, $p^\mu - v^\mu = 0$, among primary constraints of the theory (p_μ is conjugated momenta for x^μ, and momenta, conjugated to all the other configuration space variables q^i are denoted as p_{qi}). Then the canonical pairs $(n^\mu, p_{n\mu})$, $(v^\mu, p_{v\mu})$ can be omitted after introducing the associated Dirac bracket. Dirac brackets for the remaining variables coincide with Poisson ones [23] and the total Hamiltonian have the form

$$\begin{aligned} H^{(1)} = &\frac{1}{2}ep^2 + \xi(pp_z) + \frac{1}{2}(p_z^2 + 1) + \lambda_e p_e + \\ &+\lambda_\phi p_\phi + \lambda_\xi p_\xi + (\bar p_\theta - i\bar\theta\Gamma^{\mu\nu}p_\mu p_{z\nu})\lambda_\theta, \end{aligned} \tag{9}$$

where Lagrange multipliers corresponding to primary constraints are denoted as λ_*). The complete set of constraints can be written in the following form:

$$p_e = 0, \qquad p_\phi = 0, \qquad p_\xi = 0; \tag{10.a}$$

$$p_z^2 = -1, \qquad (pp_z) = 0, \qquad p^2 = 0; \tag{10.b}$$

$$L_\alpha \equiv \bar p_{\theta\alpha} - i(\bar\theta'\Gamma^\mu)_\alpha p_\mu = 0; \tag{10.c}$$

where $\theta' \equiv p_{z\mu}\Gamma^\mu\theta$. The matrix of the Poisson brackets of the fermionic constraints

$$\{L_\alpha, L_\beta\} = 2i(C\Gamma^{\mu\nu})_{\alpha\beta} p_\mu p_{z\nu}, \tag{11}$$

is degenerated on the constraint surface as a consequence of the identity $(\Gamma^{\mu\nu}p_\mu p_{z\nu})^2 = 4[(pp_z) - p^2 p_z^2]1 = 0$. It means that half of the constraints are first-class. From the condition $\{L_\alpha, H^{(1)}\} = 0$ one finds equation which determine λ_θ-multipliers,

$$p_\mu\Gamma^\mu\lambda'_\theta = 0, \qquad \lambda'_\theta \equiv p_{z\mu}\Gamma^\mu\lambda_\theta. \tag{12}$$

Imposing the gauge conditions $e = 1, \phi = 1, \xi = 0$ to the first-class constraints (10.a), one can omits the canonical pairs (e, p_e), (ϕ, p_ϕ), (ξ, p_ξ) from the consideration. The dynamics of the remaining variables is governed by the equations

$$\dot{z}^\mu = p_z^\mu + i(\bar\theta\Gamma^{\mu\nu}\lambda_\theta)p_\nu, \qquad \dot{p}_z^\mu = 0; \tag{13.a}$$

$$\dot{x}^\mu = p^\mu - i(\bar\theta\Gamma^{\mu\nu}\lambda_\theta)p_{z\nu}, \qquad \dot{p}^\mu = 0; \tag{13.b}$$

$$\dot\theta^\alpha = -\lambda_\theta^\alpha, \qquad \dot{\bar{p}}_{\theta\alpha} = 0. \tag{13.c}$$

As the next step we impose gauge conditions

$$\Gamma^+\theta' = 0 \tag{14}$$

to the first-class constraints which follow from the equations (10.c). By virtue of (12),(13.c) all λ_θ-multipliers can be determined, $\lambda_\theta = 0$, and (13.a-c) are reduced to free equations of motion.

The resulting picture corresponds to zero-tension limit of the $D = 11$ superstring action from [8]. Physical degrees of freedom for the model (4) may be considered as describing a composite system. It consists of the bosonic z^μ-particle (13.a) and the superparticle (13.b), (13.c), subject to the constraints (10.b). Both of them propagate freely except the kinematic constraint $(pp_z) = 0$, which means that the superparticle lives on $D = 10$ hyperplane orthogonal to the direction of motion of z^μ-particle.

A few comments are in order. In the model considered variables (z^μ, p_z^μ) describe a tachyon[5] $p_z^2 = -1$. To avoid the problem, it was suggested in [6,16,17] to consider a target space of a non-standard signature (2,9) with the metric $\eta^{\mu\nu} = (+, - \cdots -, +)$. In such a space there is no of tachyon, but negative norm states appear in the model. Actually, four constraints are necessary to gauge out the undesirable components x^0, x^{10}, z^0, z^{10}. However, it is impossible to form four Poincare covariant constraints using only the variables p^μ, p_z^μ, which are in our disposal. This situation can be improved by considering of a modified action which describes a superparticle

[5]Note that it make no of special problem for the case of $D = 11$ superstring [8]

x^μ and a pair of particles $z_i^\mu, i = 1, 2$. Using the corresponding conjugate momenta $p_\mu, p_{i\mu}$ six constraints can be formed, which allow one to gauge out the six components $x^0, x^{10}, z_i^0, z_i^{10}$. We will not discuss such a construction in a more details, since our example considered in the next Sections do not have such problems.

3 SO(1,D-1)×SO(D-2)-invariant formulation for the bosonic particle.

In this Section we construct a free propagating bosonic particle action, which will be appropriate for our aims of supersymmetrization. Namely, it contains an auxiliary space-like variable π_{D-1}^μ, for which the gauge $\pi_{D-1}^\mu = (0, \cdots, 0, 1)$ is possible. We start from the Poincare invariant action which describes D particles in D-dimensional space-time

$$S_0 = \int d\tau \left\{ \pi_{\bar{a}\mu}\dot{x}_{\bar a}^\mu - \frac{1}{2}\sum_{\bar a=0}^{D-1}\phi_{\bar a}(\pi_{\bar a\mu}\pi_{\bar a}^\mu + c_{\bar a}^2)\right\}, \tag{15}$$

where $x_{\bar a}^\mu = (x_0^\mu \equiv x^\mu, x_a^\mu), a = 1, 2, \cdots, D-1$, and the number $c_{\bar a}$ determines the mass of a particle with the index $\bar a$. Let us consider the problem of reducing a number of physical degrees of freedom for the model by means of a localization of a part of global symmetries presented in the action. First, we note that the following transformation (without sum on $\bar a, \bar b$):

$$\delta_\lambda x_{\bar a}^\mu = \lambda_{\bar a\bar b}\pi_{\bar b}^\mu, \qquad \delta_\lambda x_{\bar b}^\mu = \lambda_{\bar b\bar a}\pi_{\bar a}^\mu, \qquad \lambda_{\bar a\bar b} \equiv \lambda_{\bar b\bar a}, \tag{16}$$

is a global symmetry of the action for any fixed pair of indices $\bar a \neq \bar b$ (note that for $\bar a = \bar b$ the symmetry is the local one, with the variable $\phi_{\bar a}$ being a corresponding gauge field). In order to localize this transformation it is sufficient to covariantize the time derivatives: $\dot{x}_{\bar a}^\mu \to \dot{x}_{\bar a}^\mu - \frac{1}{2}\phi_{\bar a\bar b}\pi_{\bar b}^\mu$, $\dot{x}_{\bar b}^\mu \to \dot{x}_{\bar b}^\mu - \frac{1}{2}\phi_{\bar b\bar a}\pi_{\bar a}^\mu$, where $\phi_{\bar a\bar b} \equiv \phi_{\bar b\bar a}$ is the corresponding "gauge field" with the transformation low $\delta_\lambda \phi_{\bar{a}\bar{b}} = \dot{\lambda}_{\bar{a}\bar{b}}$. It is useful to write the resulting locally invariant action in the form

$$S_1 = \int d\tau \left\{ \pi_{\bar{a}\mu} \dot{x}^\mu_{\bar{a}} - \frac{1}{2} \sum{}' \phi_{\bar{a}\bar{b}} (\pi_{\bar{a}\mu} \pi^\mu_{\bar{b}} + c^2_{\bar{a}} \delta_{\bar{a}\bar{b}}) \right\}, \tag{17}$$

where the touch means that the sum includes those pairs of indices for which the corresponding symmetry was localized. In particular, if all the symmetries are localized, one has $D(D+1)/2$ constraints and a number of physical degree of freedom for the model is equal to $D(D-1)/2$. Note, that it coincides exactly with the number of Lorentz symmetry generators. Further reduction of the physical degree of freedom can be achieved by a localization of the Lorentz symmetry transformations,

$$\delta x^\mu_{\bar{a}} = \omega^\mu{}_\nu x^\nu_{\bar{a}}, \qquad \delta \pi^\mu_{\bar{a}} = \omega^\mu{}_\nu \pi^\nu_{\bar{a}}. \tag{18}$$

By the covariantization of the derivatives, $\dot{x}^\mu_{\bar{a}} \to D x^\mu_{\bar{a}} \equiv \dot{x}^\mu_{\bar{a}} - A^\mu{}_\nu x^\nu_{\bar{a}}$, where $\delta A^\mu{}_\nu = \dot{\omega}^\mu{}_\nu + \omega^\mu{}_\rho A^\rho{}_\nu - A^\mu{}_\rho \omega^\rho{}_\nu$, one obtains the action

$$S_2 = \int d\tau \left\{ \pi_{\bar{a}\mu} D x^\mu_{\bar{a}} - \frac{1}{2} \sum{}' \phi_{\bar{a}\bar{b}} (\pi_{\bar{a}\mu} \pi^\mu_{\bar{b}} + c^2_{\bar{a}} \delta_{\bar{a}\bar{b}}) \right\}, \tag{19}$$

which does not contain of physical degree of freedom if the sum runs over all indices. To get a model with nontrivial dynamics, let us retain nonlocalized a part of symmetries (16). The following action will be appropriate for our aims

$$\begin{aligned} S_3 = \int d\tau \Big\{ & \pi_\mu D x^\mu - \frac{1}{2} e \pi^2 - \xi (\pi_\mu \pi^\mu_{D-1}) + \\ & + \pi_{a\mu} D x^\mu_a - \frac{1}{2} \sum_{a,b=1}^{D-1} \phi_{ab} (\pi_{a\mu} \pi^\mu_b + \delta_{ab}) \Big\}. \end{aligned} \tag{20}$$

Here in addition to the local $SO(1, D-1)$ symmetry there is also a global symmetry $SO(D-2)$, acting on the indices $a, b = 1, 2, \cdots, D-2$. Let us demonstrate that the action (20) describes the propagation of a free massless particle. A straightforward Hamiltonian analysis reveal the following

first-class constraints:

$$\begin{aligned} & L_{ab} \equiv p_{a\mu} p^\mu_b + \delta_{ab} = 0, \\ & L^{\mu\nu} \equiv x^{[\mu} p^{\nu]} + \sum_{a=1}^{D-1} x^{[\mu}_a p^{\nu]}_a = 0; \end{aligned} \tag{21}$$

$$p_\mu p^\mu = 0, \qquad p_\mu p^\mu_{D-1} = 0, \tag{22}$$

Then the equations

$$x^\mu_a = \tau \delta^\mu_a, \qquad \mu \geq a; \qquad p^\mu_a = 0, \qquad \mu < a, \tag{23}$$

turn out to be a gauge fixation for the constraints (21). Then the unique solution of (21),(23) is

$$x^\mu_a = \tau \delta^\mu_a, \qquad p^\mu_a = \delta^\mu_a, \qquad a = 1, \cdots, D-1. \tag{24}$$

In particular, in this gauge, the variable $\pi^\mu_{D-1} \approx p^\mu_{D-1}$ acquires the desired form

$$\pi^\mu_{D-1} \approx p^\mu_{D-1} = (0, \cdots, 0, 1). \tag{25}$$

The dynamics of the remaining variables (x^μ, p_μ) is governed now by the free equations

$$\dot{x}^\mu = p^\mu, \qquad \dot{p}^\mu = 0, \tag{26}$$

which is accompanied by the constraints (22).

The $SO(1,9)$-covariance of the resulting system (26),(22) can be considered as a residual symmetry of the initial formulation (20), surviving in the gauge (23). Namely, one can see that the combination of $SO(1,10)$, $SO(9)$ and λ-transformations, which do not violates the gauge (23), are $SO(1,9)$ Lorentz transformations. As to the translation invariance, let us note that the action (20) is invariant also under transformations $\delta x^\mu = f^\mu$ with covariantly constant functions f^μ, $\quad Df^\mu = 0$. The general solution of this equation $f^\mu(a^\mu)$ consists of an arbitrary numbers a^μ, which are parameters of the global symmetry. In the gauge (23) this symmetry reduces to the standard Poincare shifts.

4 S-invariant action for the eleven-dimensional superparticle.

In this Section we present a supersymmetric version of the bosonic action (20) for the case $D=11$. It will be shown that global symmetry transformations for the model is a realization of $N=1, D=11$ S-algebra (7). These transformations are reduced to $N=2, D=10$ super Poincare one in the gauge (23)-(25). The action is also invariant under the local fermionic κ-symmetry which reduces a number of fermionic degree of freedom by one half. Similarly to the CBS superparticle it provides a free dynamics for the physical sector variables. Besides, the present action describes a superparticle only, in contrast to the example of Sec.2, where a composite system was considered.

The action under consideration is

$$
\begin{aligned}
S=\int d\tau\, \{ \pi_\mu \left[Dx^\mu - i(\bar\theta\Gamma^{\mu\nu}D\theta)\pi_{10\nu} - \xi\pi_{10\mu}\right] - \\
-\frac{1}{2}e\pi^2 + \pi_{a\mu}Dx_a^\mu - \frac{1}{2}\phi_{ab}(\pi_{a\mu}\pi_b^\mu + \delta_{ab}) \Big\},
\end{aligned}
\tag{27}
$$

where $a=1,2,\cdots,10$, and

$$
Dx_a^\mu \equiv \dot{x}_a^\mu - A^\mu{}_\nu x_a^\nu, \qquad D\theta \equiv \dot\theta + \frac{1}{4}A_{\mu\nu}\Gamma^{\mu\nu}\theta. \tag{28}
$$

The local bosonic symmetries for the action are both $SO(1,10)$ transformations,

$$
\begin{aligned}
&\delta x_{\bar a}^\mu = \omega^\mu{}_\nu x_{\bar a}^\nu, \qquad \delta\pi_{\bar a}^\mu = \omega^\mu{}_\nu \pi_{\bar a}^\nu, \\
&\delta\theta = -\frac{1}{4}\omega_{\mu\nu}\Gamma^{\mu\nu}\theta, \qquad \delta A^\mu{}_\nu = \dot\omega^\mu{}_\nu + \omega^\mu{}_\rho A^\rho{}_\nu - A^\mu{}_\rho\omega^\rho{}_\nu,
\end{aligned}
\tag{29}
$$

and the transformations (without sum on a, b),

$$
\begin{aligned}
&\delta x^\mu = \alpha\pi^\mu, \qquad \delta e = \dot\alpha; \\
&\delta x^\mu = \lambda\pi_{10}^\mu, \qquad \delta x_{10}^\mu = \lambda\pi^\mu, \qquad \delta\xi = \dot\lambda; \\
&\delta x_a^\mu = \lambda_{ab}\pi_b^\mu, \qquad \delta x_b^\mu = \lambda_{ba}\pi_a^\mu, \qquad \delta\phi_{ab} = \dot\lambda_{ab}, \qquad \lambda_{ab}\equiv\lambda_{ba}.
\end{aligned}
\tag{30}
$$

There are also local fermionic κ-symmetry transformations with the parameter κ^α being $SO(1,10)$ Majorana spinor,

$$
\begin{aligned}
&\delta\theta = \pi^\mu\Gamma^\mu\kappa, \\
&\delta x^\mu = i(\bar\theta\Gamma^{\mu\nu}\delta\theta)\pi_{10\nu}, \qquad \delta x_{10}^\mu = -i(\bar\theta\Gamma^{\mu\nu}\delta\theta)\pi_\nu, \\
&\delta e = 4i(\overline{D\theta}\Gamma^\mu\kappa)\pi_{10\mu}, \qquad \delta\xi = -2i(\overline{D\theta}\Gamma^\mu\kappa)\pi_\mu.
\end{aligned}
\tag{31}
$$

The global new supersymmetry transformations are realized as follows:

$$
\begin{aligned}
&\delta_\epsilon\theta^\alpha = f^\alpha(\epsilon), \\
&\delta_\epsilon x^\mu = i(\bar f\Gamma^{\mu\nu}\theta)\pi_{10\nu}, \qquad \delta_\epsilon x_{10}^\mu = -i(\bar f\Gamma^{\mu\nu}\theta)\pi_\nu,
\end{aligned}
\tag{32}
$$

with covariantly constant odd functions $f^\alpha(\epsilon)$, $Df^\alpha=0$. The general solution of this equation consists of arbitrary constants ϵ^α, which are parameters of global symmetry (32). Besides, there is global bosonic symmetry with the parameters $b^{\mu\nu}=-b^{\nu\mu}$,

$$
\delta_b x^\mu = f^\mu{}_\nu(b)\pi_{10}^\nu, \qquad \delta x_{10}^\mu = -f^\mu{}_\nu(b)\pi^\nu. \tag{33}
$$

Note that $\delta A^{\mu\nu}=0$ under these transformations, and there are no of derivatives in (32), (33). As a consequence, the algebra of the generators $Q_\alpha, Z_{\mu\nu}$, corresponding to the transformations (32), (33), coincides with (7). Thus, (32), (33) is a realization of the S-algebra for the model under consideration.

Let us study the dynamics of the model in the Hamiltonian framework. The total Hamiltonian is

$$
\begin{aligned}
H^{(1)} = \frac{1}{2}ep^2 + \xi p_\mu p_{10}^\mu + \frac{1}{2}\phi_{ab}L_{ab} + A_{\mu\nu}L^{\mu\nu} + \lambda_{x\bar a\mu}(p_{\bar a}^\mu - \pi_{\bar a}^\mu) + \\
+\lambda_e p_e + \lambda_\xi p_\xi + \lambda_{\phi ab}p_{\phi ab} + \lambda_{\pi\bar a\mu}p_{\pi\bar a}^\mu + \lambda_A^{\mu\nu}p_{A\mu\nu} + L_\alpha\lambda_\theta^\alpha,
\end{aligned}
\tag{34}
$$

where $p_{\bar a\mu}\equiv(p_\mu, p_{a\mu})$, $p_{\pi\bar a\mu}=(p_{\pi\mu}, p_{\pi a\mu})$ are momenta conjugated to the variables $x_{\bar a}^\mu \equiv (x^\mu, x_a^\mu)$, $\pi_{\bar a}^\mu \equiv (\pi^\mu, \pi_a^\mu)$. The complete set of constraints can be written in the form

$$p^{\mu}_{\pi\bar{a}} = 0, \qquad p^{\mu}_{\bar{a}} - \pi^{\mu}_{\bar{a}} = 0; \tag{35.a}$$

$$p_e = 0, \qquad p_\xi = 0, \qquad p_{\phi ab} = 0, \qquad p_{A\mu\nu} = 0; \tag{35.b}$$

$$p^2 = 0, \qquad pp_{10} = 0; \tag{35.c}$$

$$L_{ab} \equiv p_{a\mu}p^{\mu}_{b} + \delta_{ab} = 0, \qquad L^{\mu\nu} \equiv x^{[\mu}_{\bar{a}}p^{\nu]}_{\bar{a}} - \frac{1}{4}\bar{p}_\theta\Gamma^{\mu\nu}\theta = 0; \tag{35.d}$$

$$L_\alpha \equiv \bar{p}_{\theta\alpha} - i(\bar{\theta}\Gamma^{\mu\nu})_\alpha p_\mu p_{10\nu} = 0. \tag{35.e}$$

Besides, some equations for the Lagrange multipliers can be determined in the course of Dirac procedure,

$$\begin{aligned} &\lambda^{\mu}_{x\bar{a}} = \delta_{\bar{a},0}ep^{\mu} + \delta_{\bar{a},10}\xi p^{\mu}_{10} - \phi_{\bar{a}\bar{b}}p^{\mu}_{\bar{b}} - A^{\mu}{}_{\nu}x^{\nu}_{\bar{a}}, \qquad \lambda^{\mu}_{\pi\bar{a}} = A^{\mu}{}_{\nu}p^{\nu}_{\bar{a}}; \\ &\Gamma^{\mu\nu}\lambda_\theta p_\mu p_{10\nu} = 0. \end{aligned} \tag{36}$$

Imposing the gauge conditions

$$e = 1, \qquad \xi = 0, \qquad \phi_{ab} = \delta_{ab}, \qquad A^{\mu\nu} = 0, \tag{37}$$

to the first-class constraints (35.b) and taking into account the second-class constraints (35.a), one can eliminate the canonical pairs (e, p_e), (ξ, p_ξ), $(\phi_{ab}, p_{\phi ab})$, $(A^{\mu\nu}, p_{A\mu\nu})$, $(\pi^{\mu}_{\bar{a}}, p^{\mu}_{\pi\bar{a}})$ from the consideration. The constraints (35.d,e) obey the following algebra:

$$\begin{aligned} &\{L^{\mu\nu}, L^{\rho\delta}\} = \eta^{\mu\rho}L^{\nu\delta} + (permutations\ \mu\nu\rho\delta) \approx 0, \\ &\{L^{\mu\nu}, L_\alpha\} = -\frac{1}{4}(\Gamma^{\mu\nu}L)_\alpha \approx 0, \\ &\{L_\alpha, L_\beta\} = 2i(C\Gamma^{\mu\nu})_{\alpha\beta}p_\mu p_{10\nu} \end{aligned} \tag{38}$$

whereas all other Poisson brackets vanish identically. It follows from the last equation (38) and from the identity $(\Gamma^{\mu\nu}p_\mu p_{10\nu})^2 = 4\left[(pp_{10}) - p^2p^2_{10}\right] = 0$ that half of the constraints $L_\alpha = 0$ are first-class. They correspond to the local κ-symmetry (31). The next step is to impose the gauge conditions (23) for the first-class constraints $L_{ab} = 0$, $L^{\mu\nu} = 0$ and the gauge condition

$x^{10} = 0$ for the equation $(pp_{10}) = 0$. Then, in particular, $p^{\mu}_{10} = (0, \cdots, 0, 1)$, which breaks the manifest $D = 11$ S-invariance (32), (33) up to $D = 10$, type IIA super Poincare one. It is useful on this stage to introduce SO(1,9) notations for the SO(1,10) objects [8],

$$\begin{aligned} &\Gamma^\mu = (\Gamma^{\bar{\mu}}, \Gamma^{10}) = \left(\begin{pmatrix} 0 & \Gamma^{\bar{\mu}} \\ \tilde{\Gamma}^{\bar{\mu}} & 0 \end{pmatrix}, \begin{pmatrix} 1_{16} & 0 \\ 0 & -1_{16} \end{pmatrix} \right), \\ &\theta = (\bar{\theta}_\alpha, \theta^\alpha), \qquad \bar{p}_\theta = (\bar{p}^{\alpha}_{\theta}, p_{\theta\alpha}), \\ &\bar{\mu} = 0, 1, \cdots, 9, \qquad \alpha = 1, \cdots, 16, \end{aligned} \tag{39}$$

where $\bar{\theta}_\alpha, \theta^\alpha$ are SO(1,9) Majorana-Weyl spinors of opposite chirality. In such notations equations of motion for the remaining variables can be written as

$$\begin{aligned} &\dot{x}^{\bar{\mu}} = p^{\bar{\mu}} + i\theta\Gamma^{\bar{\mu}}\lambda_\theta + i\bar{\theta}\tilde{\Gamma}^{\bar{\mu}}\lambda_{\bar{\theta}}, \qquad \dot{p}^{\bar{\mu}} = 0; \\ &\dot{\theta}^\alpha = -\lambda^{\alpha}_{\theta}, \qquad \dot{\bar{\theta}}_\alpha = -\bar{\lambda}_{\theta\alpha}, \end{aligned} \tag{40}$$

while for the remaining constraints one finds the expressions

$$p^2 = 0, \tag{41.a}$$

$$p_{\theta\alpha} + i\theta^\beta\Gamma^{\bar{\mu}}_{\beta\alpha}p_{\bar{\mu}} = 0, \qquad \bar{p}^{\alpha}_{\theta} + i\theta_\beta\tilde{\Gamma}^{\bar{\mu}\beta\alpha}p_{\bar{\mu}} = 0. \tag{41.b}$$

To get the final form of the dynamics, we pass to the light-cone coordinates $x^{\bar{\mu}} \to (x^+, x^-, x^i)$, $i = 1, 2, \cdots, 8$, and to SO(8) notations for spinors, $\bar{\theta}_\alpha = (\theta_a, \bar{\theta}'_{\dot{a}})$, $\theta^\alpha = (\theta'_a, \bar{\theta}_{\dot{a}})$, $\bar{p}^{\alpha}_{\theta} = (p_{\theta a}, \bar{p}'_{\theta\dot{a}})$, $p_{\theta\alpha} = (p'_{\theta a}, \bar{p}_{\theta\dot{a}})$, $a, \dot{a} = 1, 2, \cdots, 8$. It allows one to write an equivalent to (41.b) set of constraints, which is explicitly classified as first- and second-class respectively,

$$\sqrt{2}p^+p'_\theta + \bar{p}_\theta\tilde{\gamma}^ip^i = 0, \qquad \sqrt{2}p^+\bar{p}'_\theta - p_\theta\gamma^ip^i = 0; \tag{42.a}$$

$$\bar{p}_\theta + i\sqrt{2}p^+\bar{\theta} - i\theta'\gamma^ip^i = 0, \qquad p_\theta - i\sqrt{2}p^+\theta - i\bar{\theta}'\tilde{\gamma}^ip^i = 0. \tag{42.b}$$

Then the equations $\theta'_a = 0, \bar{\theta}'_{\dot{a}} = 0$ (or, equivalently, $\Gamma^{+}_{32}\theta_{32} = 0$) are the gauge conditions for the first-class constraints (42.a). It follows from (36),

(40) that $\lambda_\theta = 0$ in this gauge. Thus the dynamics of the physical variables is described by the equations

$$\begin{aligned} &\dot{x}^{\bar{\mu}} = p^{\bar{\mu}}, \qquad \dot{p}^{\bar{\mu}} = 0, \qquad p^2 = 0; \\ &\dot{\theta}_a = 0, \qquad \dot{\bar{\theta}}_{\dot{a}} = 0. \end{aligned} \tag{43}$$

Using the same arguments as in Sec.3, one can prove that $D = 10$ super Poincare symmetry transformations for (43) are some combinations of the symmetries (29)-(33), which do not spoil the gauge chosen. Besides the S-algebra (7) reduces to the type IIA supersymmetry algebra in this gauge.

5 Summary.

In the present paper we have constructed explicitly several Lagrangian actions for $D = 11$ S-invariant mechanical models. In particular, it was shown that $D = 10$ type IIA superparticle (40), (41) can be presented in the S-invariant formulation (27). In course of the consideration an explicit form of the S-algebra (7) was obtained. Being model-independent, it may be used as a basis for a systematic construction of various $D = 11$ models. In particular, it follows from the consideration of Sec.4 that there may exist a more transparent algebraic formulation for the $D = 11$ superparticle in terms of the Lorentz-harmonic variables [24-28]. We consider these models as a preliminary step towards a construction of $D = 11$ S-invariant formulations for SYM and superstring actions, which might contribute to a better understanding of the uncompactified M-theory [10-13].

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