

UNIVERSIDADE DE SÃO PAULO

PUBLICAÇÕES

**INSTITUTO DE FÍSICA
CAIXA POSTAL 66318
05315-970 SÃO PAULO - SP
BRASIL**

IFUSP/P-1307

**EXAMPLES OF $D=11$ S-SUPERSYMMETRIC ACTIONS
FOR POINT-LIKE DYNAMICAL SYSTEMS**

A.A. Deriglazov and D.M. Gitman
Instituto de Física, Universidade de São Paulo

Abril/1998

Examples of $D = 11$ S-supersymmetric actions for point-like dynamical systems.

A.A. Deriglazov* and D.M. Gitman†

Instituto de Física, Universidade de São Paulo,
P.O. Box 66318, 05315-970, São Paulo, SP, Brasil.

Abstract

A non standard super extensions of the Poincare algebra (S-algebra [1,2]), which seems to be relevant for construction of various $D = 11$ models, are studied. We present two examples of actions for point-like dynamical systems, which are invariant under off-shell closed realization of the S-algebra as well as under local fermionic κ -symmetry. On this ground, an explicit form of the S-algebra is advocated.

1 Introduction

The construction of higher-dimensional ($D > 10$) SYM [3,4] and superstring [5,6,2,7,8] models, which might be interesting in the M-theory context (see [9-14] and references therein), is under intensive investigation at present. It is known that consistency of the super Poincare and local symmetry transformations¹ imply rigid restrictions on possible dimensions of

*deriglaz@fma.if.usp.br, On leave of absence from the Dept. of Mathematics, TPU, Tomsk, Russia.

†gitman@fma.if.usp.br

¹We mean gauge transformations for the SYM-theory and local κ -symmetry transformations for the case of superstring.

space-time where these models can be formulated [15,9]. In particular, the standard methods can not be directly applied in $D > 10$ to construct the above mentioned super Poincare invariant models. One possibility to avoid these restrictions is to consider some different super extensions of the Poincare algebra². In recent works [1-4,6,8,16,17] a relevant higher-dimensional superalgebra was discussed. It includes the Poincare generators as well as generators Q of new supertranslations with commutator may be written in the form³

$$\{Q_\alpha, Q_\beta\} \sim \Gamma^{\mu\nu} P_\mu n_\nu, \quad (1)$$

where $\Gamma^{\mu\nu}$ is antisymmetric product of $D = 11$ γ -matrices (we use γ -matrix conventions from [8]). It is known as S-algebra previously discussed in the M-theory context [1] (see [18] for discussion of a general case). For $D = 11$ case it can be realized in a superspace as follows:

$$\delta\theta = \epsilon, \quad \delta x^\mu = i(\bar{\epsilon}\Gamma^{\mu\nu}\theta)n_\nu, \quad \delta n^\mu = 0. \quad (2)$$

The appearance of a new variable n^μ seems to be an essential property of the construction (see discussion in [7,8]). In this relation it is interesting to clarify the role of the variable n^μ from the dynamical point of view, in particular, to present some examples of Lagrangian systems with n^μ incorporated on equal footing with other variables. Only in this case the corresponding theory can be actually $SO(1,10)$ invariant.

It was also pointed out [2,8] that after substitution $n^\mu = (0, \dots, 0, 1)$ (which breaks $SO(1,10)$ covariance up to $SO(1,9)$ one) the transformations

²In recent works [5] $D = 11$ superstring action with second-class constraints simulating a gauge fixation for the κ -symmetry was suggested. The action was constructed by adding of an appropriately chosen terms to the GS action written in $D = 11$. Supersymmetry of quantum state spectrum for the model is under investigation now.

³As it will be demonstrated below (see also Ref.[2,8]) an explicit form of the algebra is $\{Q, Q\} \sim \Gamma^{\mu\nu} Z_{\mu\nu}$, with some additional bosonic generators $Z_{\mu\nu}$.

(2) reduce to

$$\begin{aligned} \delta\theta^\alpha &= \epsilon^\alpha, & \delta\bar{\theta}_\alpha &= \bar{\epsilon}_\alpha, \\ \delta x^{\bar{\mu}} &= -i\bar{\epsilon}_\alpha \tilde{\Gamma}^{\bar{\mu}\alpha\beta} \bar{\theta}_\beta - i\epsilon^\alpha \Gamma_{\alpha\beta}^{\bar{\mu}} \theta^\beta, & \delta x^{10} &= 0, \end{aligned} \quad (3)$$

where $\theta = (\bar{\theta}_\alpha, \theta^\alpha)$, $\mu = (\bar{\mu}, 10)$, $\bar{\mu} = 0, 1, \dots, 9$, $\alpha = 1 \dots 16$. One can see that (3) coincides exactly with the standard $D = 10$, type IIA supersymmetry transformations. In this sense the latter can be rewritten in a manifestly $SO(1,10)$ covariant notations (2). Thus, it is naturally to ask about possibility of lifting the known $D = 10$ type IIA theories up to $SO(1,10)$ invariant form. From the present discussion it is clear that the requirement of S-invariance instead of the super Poincare invariance might be a natural framework for construction of such a kind $D = 11$ formulations.

In this letter we present two examples of $D = 11$ finite-dimensional systems based on the S-algebra of global symmetries. For the first model the variable n^μ survives in the sector of physical degrees of freedom, while for the second one it turns out to be a nondynamical variable, which may be killed by a proper gauge fixing. It will be also demonstrated, that local κ -symmetry is consistent with global S-invariance in both cases.

The first example which we are going to study is in fact zero-tension limit of the $D = 11$ superstring action suggested in [8]. Physical degrees of freedom for the mechanical model may be considered as describing a composite system, the latter consists of a free moving particle and a superparticle (see also Refs.[6,16,17]). We present a Lagrangian action, which is invariant under local κ -symmetry as well as under off-shell closed realization of the S-algebra of global symmetries. The advantage of the present formulation (in comparison with [3,4,6,16,17]) is that an explicit Lagrangian action, with all the variables treated on equal footing is given. In particular, global symmetry transformations of the action form a super-

algebra in the usual sense, without appearance of nonlinear in generators terms in the right hand side of Eq.(1). In the result, a model-independent form of the S-algebra is presented.

From the discussion related to (2),(3) it is clear that a formulation where one may impose the gauge $n^\mu = (0, \dots, 0, 1)$ would be at most preferable. As a second example, we present S-invariant model, which admits such a gauge, and which describes the propagation of a superparticle only. We hope that a similar construction may work for the case of $D = 11$ superstring as well.

The work is organized as follows. In the Sec.2 we present and discuss a $D = 11$ Poincare invariant action for the above mentioned composite system. In the Sec.3, a bosonic action which contains the nondynamical variable $n^\mu(\tau)$ related to S-symmetry is proposed. It is shown that the action describes a free propagating massless particle. On the base of this action S-supersymmetric version in $D = 11$ space-time is constructed in Sec.4. The latter action is invariant also under local fermionic κ -symmetry. Similarly to the Casalbuoni-Brink-Schwarz superparticle [19-21] it provides a free character of the dynamics for the physical sector variables.

2 D=11 composite system of a particle and a superparticle.

Let us consider the following $D = 11$ Lagrangian action

$$\begin{aligned} S &= \int d\tau \left\{ v_\mu \Pi^\mu - \frac{1}{2} e v^2 + n_\mu \dot{z}^\mu - \frac{1}{2} \phi (n^2 + 1) \right\}, \\ \Pi^\mu &\equiv \dot{x}^\mu - i(\bar{\theta} \Gamma^{\mu\nu} \dot{\theta}) n_\nu - \xi n^\mu, \end{aligned} \quad (4)$$

where $x^\mu, v^\mu, z^\mu, n^\mu, e, \phi, \xi$ are Grassmann even and θ^α are Grassmann odd variables, dependent on the evolution parameter τ . The action is a direct

mechanical analog of the $D = 11$ superstring suggested in [8]. Note [8] that eliminating the variable v^μ we can rewrite (4) in the second-order form relative to x^μ . Global bosonic symmetries of the action are both $D = 11$ Poincare transformations (with the variable n^μ being inert under the Poincare shifts) and the following transformations

$$\delta_b x^\mu = b^\mu{}_\nu n^\nu, \quad \delta_b z^\mu = -b^\mu{}_\nu v^\nu, \quad (5)$$

with antisymmetric parameters $b^{\mu\nu} = -b^{\nu\mu}$. There is also a global symmetry with fermionic parameters ϵ^α ,

$$\delta_\epsilon \theta = \epsilon, \quad \delta_\epsilon x^\mu = -i(\bar{\theta}\Gamma^{\mu\nu}\epsilon)n_\nu, \quad \delta_\epsilon z^\mu = i(\bar{\theta}\Gamma^{\mu\nu}\epsilon)v_\nu. \quad (6)$$

The algebra of the corresponding commutators turns out to be off-shell closed.⁴ Thus, the S-algebra consist of Poincare subalgebra $(M^{\mu\nu}, P^\mu)$ and includes generators of new supertranslations Q_α as well as second-rank Lorentz tensor $Z_{\mu\nu}$. The nonzero commutators of the new generators are

$$\{Q_\alpha, Q_\beta\} = 2i(C\Gamma^{\mu\nu})_{\alpha\beta}Z_{\mu\nu}. \quad (7)$$

Their commutators with the Poincare transformations have the standard form. Note, that it is not a modification of the super Poincare algebra but essentially different one, since the commutator of the supertranslations leads to Z -transformation instead of the Poincare shift.

The action (4) is also invariant under the local κ -symmetry transformations,

$$\begin{aligned} \delta\theta &= v_\mu\Gamma^\mu\kappa, & \delta\xi &= -2i(\dot{\theta}\delta\theta), & \delta e &= 4i(\dot{\theta}\Gamma^\mu\kappa)n_\mu. \\ \delta x^\mu &= i(\bar{\theta}\Gamma^{\mu\nu}\delta\theta)n_\nu, & \delta z^\mu &= -i(\bar{\theta}\Gamma^{\mu\nu}\delta\theta)v_\nu, \end{aligned} \quad (8)$$

This fact turns out to be crucial to verify that physical sector variables obey free equations of motion. Let us present the corresponding analysis

⁴S-algebra can be off-shell closed also for the action (4) written in the second order form [8]

in the Hamiltonian framework [22,23]. One finds the following trivial pairs of second-class constraints: $p_n^\mu = 0$, $p_z^\mu - n^\mu = 0$; $p_v^\mu = 0$, $p^\mu - v^\mu = 0$, among primary constraints of the theory (p_μ is conjugated momenta for x^μ , and momenta, conjugated to all the other configuration space variables q^i are denoted as p_{qi}). Then the canonical pairs $(n^\mu, p_{n\mu})$, $(v^\mu, p_{v\mu})$ can be omitted after introducing the associated Dirac bracket. Dirac brackets for the remaining variables coincide with Poisson ones [23] and the total Hamiltonian have the form

$$\begin{aligned} H^{(1)} &= \frac{1}{2}ep^2 + \xi(pp_z) + \frac{1}{2}(p_z^2 + 1) + \lambda_e p_e + \\ &+ \lambda_\phi p_\phi + \lambda_\xi p_\xi + (\bar{p}_\theta - i\bar{\theta}\Gamma^{\mu\nu}p_\mu p_{z\nu})\lambda_\theta, \end{aligned} \quad (9)$$

where Lagrange multipliers corresponding to primary constraints are denoted as λ_* . The complete set of constraints can be written in the following form:

$$p_e = 0, \quad p_\phi = 0, \quad p_\xi = 0; \quad (10.a)$$

$$p_z^2 = -1, \quad (pp_z) = 0, \quad p^2 = 0; \quad (10.b)$$

$$L_\alpha \equiv \bar{p}_{\theta\alpha} - i(\bar{\theta}'\Gamma^\mu)_\alpha p_\mu = 0; \quad (10.c)$$

where $\theta' \equiv p_{z\mu}\Gamma^\mu\theta$. The matrix of the Poisson brackets of the fermionic constraints

$$\{L_\alpha, L_\beta\} = 2i(C\Gamma^{\mu\nu})_{\alpha\beta}p_\mu p_{z\nu}, \quad (11)$$

is degenerated on the constraint surface as a consequence of the identity $(\Gamma^{\mu\nu}p_\mu p_{z\nu})^2 = 4[(pp_z) - p^2 p_z^2]1 = 0$. It means that half of the constraints are first-class. From the condition $\{L_\alpha, H^{(1)}\} = 0$ one finds equation which determine λ_θ -multipliers,

$$p_\mu\Gamma^\mu\lambda'_\theta = 0, \quad \lambda'_\theta \equiv p_{z\mu}\Gamma^\mu\lambda_\theta. \quad (12)$$

Imposing the gauge conditions $e = 1, \phi = 1, \xi = 0$ to the first-class constraints (10.a), one can omit the canonical pairs (e, p_e) , (ϕ, p_ϕ) , (ξ, p_ξ) from

the consideration. The dynamics of the remaining variables is governed by the equations

$$\dot{z}^\mu = p_z^\mu + i(\bar{\theta}\Gamma^{\mu\nu}\lambda_\theta)p_\nu, \quad \dot{p}_z^\mu = 0; \quad (13.a)$$

$$\dot{x}^\mu = p^\mu - i(\bar{\theta}\Gamma^{\mu\nu}\lambda_\theta)p_{z\nu}, \quad \dot{p}^\mu = 0; \quad (13.b)$$

$$\dot{\theta}^\alpha = -\lambda_\theta^\alpha, \quad \dot{\bar{p}}_{\theta\alpha} = 0. \quad (13.c)$$

As the next step we impose gauge conditions

$$\Gamma^+\theta' = 0 \quad (14)$$

to the first-class constraints which follow from the equations (10.c). By virtue of (12),(13.c) all λ_θ -multipliers can be determined, $\lambda_\theta = 0$, and (13.a-c) are reduced to free equations of motion.

The resulting picture corresponds to zero-tension limit of the $D = 11$ superstring action from [8]. Physical degrees of freedom for the model (4) may be considered as describing a composite system. It consists of the bosonic z^μ -particle (13.a) and the superparticle (13.b), (13.c), subject to the constraints (10.b). Both of them propagate freely except the kinematic constraint $(pp_z) = 0$, which means that the superparticle lives on $D = 10$ hyperplane orthogonal to the direction of motion of z^μ -particle.

A few comments are in order. In the model considered variables (z^μ, p_z^μ) describe a tachyon⁵ $p_z^2 = -1$. To avoid the problem, it was suggested in [6,16,17] to consider a target space of a non-standard signature (2,9) with the metric $\eta^{\mu\nu} = (+, - \dots -, +)$. In such a space there is no of tachyon, but negative norm states appear in the model. Actually, four constraints are necessary to gauge out the undesirable components x^0, x^{10}, z^0, z^{10} . However, it is impossible to form four Poincare covariant constraints using only the variables p^μ, p_z^μ , which are in our disposal. This situation can be improved by considering of a modified action which describes a superparticle

⁵Note that it make no of special problem for the case of $D = 11$ superstring [8]

x^μ and a pair of particles $z_i^\mu, i = 1, 2$. Using the corresponding conjugate momenta $p_\mu, p_{i\mu}$ six constraints can be formed, which allow one to gauge out the six components $x^0, x^{10}, z_i^0, z_i^{10}$. We will not discuss such a construction in a more details, since our example considered in the next Sections do not have such problems.

3 $\text{SO}(1,D-1) \times \text{SO}(D-2)$ -invariant formulation for the bosonic particle.

In this Section we construct a free propagating bosonic particle action, which will be appropriate for our aims of supersymmetrization. Namely, it contains an auxiliary space-like variable π_{D-1}^μ , for which the gauge $\pi_{D-1}^\mu = (0, \dots, 0, 1)$ is possible. We start from the Poincare invariant action which describes D particles in D -dimensional space-time

$$S_0 = \int d\tau \left\{ \pi_{\bar{a}\mu} \dot{x}_a^\mu - \frac{1}{2} \sum_{\bar{a}=0}^{D-1} \phi_{\bar{a}} (\pi_{\bar{a}\mu} \pi_{\bar{a}}^\mu + c_{\bar{a}}^2) \right\}, \quad (15)$$

where $x_{\bar{a}}^\mu = (x_0^\mu \equiv x^\mu, x_a^\mu), a = 1, 2, \dots, D-1$, and the number $c_{\bar{a}}$ determines the mass of a particle with the index \bar{a} . Let us consider the problem of reducing a number of physical degrees of freedom for the model by means of a localization of a part of global symmetries presented in the action. First, we note that the following transformation (without sum on \bar{a}, \bar{b}):

$$\delta_\lambda x_{\bar{a}}^\mu = \lambda_{\bar{a}\bar{b}} \pi_{\bar{b}}^\mu, \quad \delta_\lambda x_{\bar{b}}^\mu = \lambda_{\bar{b}\bar{a}} \pi_{\bar{a}}^\mu, \quad \lambda_{\bar{a}\bar{b}} \equiv \lambda_{\bar{b}\bar{a}}, \quad (16)$$

is a global symmetry of the action for any fixed pair of indices $\bar{a} \neq \bar{b}$ (note that for $\bar{a} = \bar{b}$ the symmetry is the local one, with the variable $\phi_{\bar{a}}$ being a corresponding gauge field). In order to localize this transformation it is sufficient to covariantize the time derivatives: $\dot{x}_{\bar{a}}^\mu \rightarrow \dot{x}_{\bar{a}}^\mu - \frac{1}{2} \phi_{\bar{a}\bar{b}} \pi_{\bar{b}}^\mu, \dot{x}_{\bar{b}}^\mu \rightarrow \dot{x}_{\bar{b}}^\mu - \frac{1}{2} \phi_{\bar{b}\bar{a}} \pi_{\bar{a}}^\mu$, where $\phi_{\bar{a}\bar{b}} \equiv \phi_{\bar{b}\bar{a}}$ is the corresponding ‘‘gauge field’’ with the

transformation law $\delta_\lambda \phi_{\bar{a}\bar{b}} = \lambda_{\bar{a}\bar{b}}$. It is useful to write the resulting locally invariant action in the form

$$S_1 = \int d\tau \left\{ \pi_{\bar{a}\mu} \dot{x}_{\bar{a}}^\mu - \frac{1}{2} \sum' \phi_{\bar{a}\bar{b}} (\pi_{\bar{a}\mu} \pi_{\bar{b}}^\mu + c_{\bar{a}}^2 \delta_{\bar{a}\bar{b}}) \right\}, \quad (17)$$

where the touch means that the sum includes those pairs of indices for which the corresponding symmetry was localized. In particular, if all the symmetries are localized, one has $D(D+1)/2$ constraints and a number of physical degree of freedom for the model is equal to $D(D-1)/2$. Note, that it coincides exactly with the number of Lorentz symmetry generators. Further reduction of the physical degree of freedom can be achieved by a localization of the Lorentz symmetry transformations,

$$\delta x_{\bar{a}}^\mu = \omega^\mu{}_\nu x_{\bar{a}}^\nu, \quad \delta \pi_{\bar{a}}^\mu = \omega^\mu{}_\nu \pi_{\bar{a}}^\nu. \quad (18)$$

By the covariantization of the derivatives, $\dot{x}_{\bar{a}}^\mu \rightarrow Dx_{\bar{a}}^\mu \equiv \dot{x}_{\bar{a}}^\mu - A^\mu{}_\nu x_{\bar{a}}^\nu$, where $\delta A^\mu{}_\nu = \dot{\omega}^\mu{}_\nu + \omega^\mu{}_\rho A^\rho{}_\nu - A^\mu{}_\rho \omega^\rho{}_\nu$, one obtains the action

$$S_2 = \int d\tau \left\{ \pi_{\bar{a}\mu} Dx_{\bar{a}}^\mu - \frac{1}{2} \sum' \phi_{\bar{a}\bar{b}} (\pi_{\bar{a}\mu} \pi_{\bar{b}}^\mu + c_{\bar{a}}^2 \delta_{\bar{a}\bar{b}}) \right\}, \quad (19)$$

which does not contain of physical degree of freedom if the sum runs over all indices. To get a model with nontrivial dynamics, let us retain nonlocalized a part of symmetries (16). The following action will be appropriate for our aims

$$S_3 = \int d\tau \left\{ \pi_\mu Dx^\mu - \frac{1}{2} e \pi^2 - \xi (\pi_\mu \pi_{D-1}^\mu) + \pi_{a\mu} Dx_a^\mu - \frac{1}{2} \sum_{a,b=1}^{D-1} \phi_{ab} (\pi_{a\mu} \pi_b^\mu + \delta_{ab}) \right\}. \quad (20)$$

Here in addition to the local $SO(1, D-1)$ symmetry there is also a global symmetry $SO(D-2)$, acting on the indices $a, b = 1, 2, \dots, D-2$. Let us demonstrate that the action (20) describes the propagation of a free massless particle. A straightforward Hamiltonian analysis reveal the following

first-class constraints:

$$L_{ab} \equiv p_{a\mu} p_b^\mu + \delta_{ab} = 0, \quad (21)$$

$$L^{\mu\nu} \equiv x^{[\mu} p^{\nu]} + \sum_{a=1}^{D-1} x_a^{[\mu} p_a^{\nu]} = 0;$$

$$p_\mu p^\mu = 0, \quad p_\mu p_{D-1}^\mu = 0, \quad (22)$$

Then the equations

$$x_a^\mu = \tau \delta_a^\mu, \quad \mu \geq a; \quad p_a^\mu = 0, \quad \mu < a, \quad (23)$$

turn out to be a gauge fixation for the constraints (21). Then the unique solution of (21),(23) is

$$x_a^\mu = \tau \delta_a^\mu, \quad p_a^\mu = \delta_a^\mu, \quad a = 1, \dots, D-1. \quad (24)$$

In particular, in this gauge, the variable $\pi_{D-1}^\mu \approx p_{D-1}^\mu$ acquires the desired form

$$\pi_{D-1}^\mu \approx p_{D-1}^\mu = (0, \dots, 0, 1). \quad (25)$$

The dynamics of the remaining variables (x^μ, p_μ) is governed now by the free equations

$$\dot{x}^\mu = p^\mu, \quad \dot{p}^\mu = 0, \quad (26)$$

which is accompanied by the constraints (22).

The $SO(1, 9)$ -covariance of the resulting system (26),(22) can be considered as a residual symmetry of the initial formulation (20), surviving in the gauge (23). Namely, one can see that the combination of $SO(1, 10)$, $SO(9)$ and λ -transformations, which do not violates the gauge (23), are $SO(1, 9)$ Lorentz transformations. As to the translation invariance, let us note that the action (20) is invariant also under transformations $\delta x^\mu = f^\mu$ with covariantly constant functions f^μ , $Df^\mu = 0$. The general solution of this equation $f^\mu(a^\mu)$ consists of an arbitrary numbers a^μ , which are parameters of the global symmetry. In the gauge (23) this symmetry reduces to the standard Poincare shifts.

4 S-invariant action for the eleven-dimensional superparticle.

In this Section we present a supersymmetric version of the bosonic action (20) for the case $D = 11$. It will be shown that global symmetry transformations for the model is a realization of $N = 1, D = 11$ S-algebra (7). These transformations are reduced to $N = 2, D = 10$ super Poincare one in the gauge (23)-(25). The action is also invariant under the local fermionic κ -symmetry which reduces a number of fermionic degree of freedom by one half. Similarly to the CBS superparticle it provides a free dynamics for the physical sector variables. Besides, the present action describes a superparticle only, in contrast to the example of Sec.2, where a composite system was considered.

The action under consideration is

$$S = \int d\tau \left\{ \pi_\mu [Dx^\mu - i(\bar{\theta}\Gamma^{\mu\nu}D\theta)\pi_{10\nu} - \xi\pi_{10\mu}] - \frac{1}{2}e\pi^2 + \pi_{a\mu}Dx_a^\mu - \frac{1}{2}\phi_{ab}(\pi_{a\mu}\pi_b^\mu + \delta_{ab}) \right\}, \quad (27)$$

where $a = 1, 2, \dots, 10$, and

$$Dx_a^\mu \equiv \dot{x}_a^\mu - A^\mu{}_\nu x_a^\nu, \quad D\theta \equiv \dot{\theta} + \frac{1}{4}A_{\mu\nu}\Gamma^{\mu\nu}\theta. \quad (28)$$

The local bosonic symmetries for the action are both $SO(1, 10)$ transformations,

$$\begin{aligned} \delta x_a^\mu &= \omega^\mu{}_\nu x_a^\nu, & \delta \pi_a^\mu &= \omega^\mu{}_\nu \pi_a^\nu, \\ \delta \theta &= -\frac{1}{4}\omega_{\mu\nu}\Gamma^{\mu\nu}\theta, & \delta A^\mu{}_\nu &= \dot{\omega}^\mu{}_\nu + \omega^\mu{}_\rho A^\rho{}_\nu - A^\mu{}_\rho \omega^\rho{}_\nu, \end{aligned} \quad (29)$$

and the transformations (without sum on a, b),

$$\begin{aligned} \delta x^\mu &= \alpha\pi^\mu, & \delta e &= \dot{\alpha}; \\ \delta x^\mu &= \lambda\pi_{10}^\mu, & \delta x_{10}^\mu &= \lambda\pi^\mu, & \delta \xi &= \dot{\lambda}; \end{aligned}$$

$$\delta x_a^\mu = \lambda_{ab}\pi_b^\mu, \quad \delta x_b^\mu = \lambda_{ba}\pi_a^\mu, \quad \delta \phi_{ab} = \dot{\lambda}_{ab}, \quad \lambda_{ab} \equiv \lambda_{ba}. \quad (30)$$

There are also local fermionic κ -symmetry transformations with the parameter κ^α being $SO(1, 10)$ Majorana spinor,

$$\begin{aligned} \delta \theta &= \pi^\mu\Gamma^\mu\kappa, \\ \delta x^\mu &= i(\bar{\theta}\Gamma^{\mu\nu}\delta\theta)\pi_{10\nu}, & \delta x_{10}^\mu &= -i(\bar{\theta}\Gamma^{\mu\nu}\delta\theta)\pi_\nu, \\ \delta e &= 4i(\bar{D}\theta\Gamma^\mu\kappa)\pi_{10\mu}, & \delta \xi &= -2i(\bar{D}\theta\Gamma^\mu\kappa)\pi_\mu. \end{aligned} \quad (31)$$

The global new supersymmetry transformations are realized as follows:

$$\begin{aligned} \delta_\epsilon \theta^\alpha &= f^\alpha(\epsilon), \\ \delta_\epsilon x^\mu &= i(\bar{f}\Gamma^{\mu\nu}\theta)\pi_{10\nu}, & \delta_\epsilon x_{10}^\mu &= -i(\bar{f}\Gamma^{\mu\nu}\theta)\pi_\nu, \end{aligned} \quad (32)$$

with covariantly constant odd functions $f^\alpha(\epsilon)$, $Df^\alpha = 0$. The general solution of this equation consists of arbitrary constants ϵ^α , which are parameters of global symmetry (32). Besides, there is global bosonic symmetry with the parameters $b^{\mu\nu} = -b^{\nu\mu}$,

$$\delta_b x^\mu = f^\mu{}_\nu(b)\pi_{10}^\nu, \quad \delta_b x_{10}^\mu = -f^\mu{}_\nu(b)\pi^\nu. \quad (33)$$

Note that $\delta A^{\mu\nu} = 0$ under these transformations, and there are no of derivatives in (32), (33). As a consequence, the algebra of the generators $Q_\alpha, Z_{\mu\nu}$, corresponding to the transformations (32), (33), coincides with (7). Thus, (32), (33) is a realization of the S-algebra for the model under consideration.

Let us study the dynamics of the model in the Hamiltonian framework. The total Hamiltonian is

$$\begin{aligned} H^{(1)} &= \frac{1}{2}ep^2 + \xi p_\mu p_{10}^\mu + \frac{1}{2}\phi_{ab}L_{ab} + A_{\mu\nu}L^{\mu\nu} + \lambda_{x\bar{a}\mu}(p_{\bar{a}}^\mu - \pi_{\bar{a}}^\mu) + \\ &+ \lambda_e p_e + \lambda_\xi p_\xi + \lambda_{\phi ab} p_{\phi ab} + \lambda_{\pi\bar{a}\mu} p_{\pi\bar{a}}^\mu + \lambda_A^{\mu\nu} p_{A\mu\nu} + L_\alpha \lambda_\theta^\alpha, \end{aligned} \quad (34)$$

where $p_{\bar{a}\mu} \equiv (p_\mu, p_{a\mu})$, $p_{\pi\bar{a}\mu} = (p_{\pi\mu}, p_{\pi a\mu})$ are momenta conjugated to the variables $x_{\bar{a}}^\mu \equiv (x^\mu, x_a^\mu)$, $\pi_{\bar{a}}^\mu \equiv (\pi^\mu, \pi_a^\mu)$. The complete set of constraints can

be written in the form

$$p_{\pi\bar{a}}^\mu = 0, \quad p_{\bar{a}}^\mu - \pi_{\bar{a}}^\mu = 0; \quad (35.a)$$

$$p_e = 0, \quad p_\xi = 0, \quad p_{\phi ab} = 0, \quad p_{A\mu\nu} = 0; \quad (35.b)$$

$$p^2 = 0, \quad pp_{10} = 0; \quad (35.c)$$

$$L_{ab} \equiv p_{a\mu} p_b^\mu + \delta_{ab} = 0, \quad L^{\mu\nu} \equiv x_{\bar{a}}^{[\mu} p_{\bar{a}}^{\nu]} - \frac{1}{4} \bar{p}_\theta \Gamma^{\mu\nu} \theta = 0; \quad (35.d)$$

$$L_\alpha \equiv \bar{p}_{\theta\alpha} - i(\bar{\theta} \Gamma^{\mu\nu})_\alpha p_\mu p_{10\nu} = 0. \quad (35.e)$$

Besides, some equations for the Lagrange multipliers can be determined in the course of Dirac procedure,

$$\lambda_{x\bar{a}}^\mu = \delta_{\bar{a},0} e p^\mu + \delta_{\bar{a},10} \xi p_{10}^\mu - \phi_{\bar{a}\bar{b}} p_b^\mu - A^\mu{}_\nu x_{\bar{a}}^\nu, \quad \lambda_{\pi\bar{a}}^\mu = A^\mu{}_\nu p_{\bar{a}}^\nu, \quad \Gamma^{\mu\nu} \lambda_\theta p_\mu p_{10\nu} = 0. \quad (36)$$

Imposing the gauge conditions

$$e = 1, \quad \xi = 0, \quad \phi_{ab} = \delta_{ab}, \quad A^{\mu\nu} = 0, \quad (37)$$

to the first-class constraints (35.b) and taking into account the second-class constraints (35.a), one can eliminate the canonical pairs (e, p_e) , (ξ, p_ξ) , $(\phi_{ab}, p_{\phi ab})$, $(A^{\mu\nu}, p_{A\mu\nu})$, $(\pi_{\bar{a}}^\mu, p_{\pi\bar{a}}^\mu)$ from the consideration. The constraints (35.d,e) obey the following algebra:

$$\begin{aligned} \{L^{\mu\nu}, L^{\rho\delta}\} &= \eta^{\mu\rho} L^{\nu\delta} + (\text{permutations } \mu\nu\rho\delta) \approx 0, \\ \{L^{\mu\nu}, L_\alpha\} &= -\frac{1}{4} (\Gamma^{\mu\nu} L)_\alpha \approx 0, \\ \{L_\alpha, L_\beta\} &= 2i(\bar{C}\Gamma^{\mu\nu})_{\alpha\beta} p_\mu p_{10\nu} \end{aligned} \quad (38)$$

whereas all other Poisson brackets vanish identically. It follows from the last equation (38) and from the identity $(\Gamma^{\mu\nu} p_\mu p_{10\nu})^2 = 4[(pp_{10}) - p^2 p_{10}^2] = 0$ that half of the constraints $L_\alpha = 0$ are first-class. They correspond to the local κ -symmetry (31). The next step is to impose the gauge conditions (23) for the first-class constraints $L_{ab} = 0$, $L^{\mu\nu} = 0$ and the gauge condition

$x^{10} = 0$ for the equation $(pp_{10}) = 0$. Then, in particular, $p_{10}^\mu = (0, \dots, 0, 1)$, which breaks the manifest $D = 11$ S-invariance (32), (33) up to $D = 10$, type IIA super Poincare one. It is useful on this stage to introduce $SO(1,9)$ notations for the $SO(1,10)$ objects [8],

$$\begin{aligned} \Gamma^\mu &= (\Gamma^{\bar{\mu}}, \Gamma^{10}) = \left(\begin{pmatrix} 0 & \Gamma^{\bar{\mu}} \\ \tilde{\Gamma}^{\bar{\mu}} & 0 \end{pmatrix}, \begin{pmatrix} \mathbf{1}_{16} & 0 \\ 0 & -\mathbf{1}_{16} \end{pmatrix} \right), \\ \theta &= (\bar{\theta}_\alpha, \theta^\alpha), \quad \bar{p}_\theta = (\bar{p}_\theta^\alpha, p_{\theta\alpha}), \\ \bar{\mu} &= 0, 1, \dots, 9, \quad \alpha = 1, \dots, 16, \end{aligned} \quad (39)$$

where $\bar{\theta}_\alpha, \theta^\alpha$ are $SO(1,9)$ Majorana-Weyl spinors of opposite chirality. In such notations equations of motion for the remaining variables can be written as

$$\begin{aligned} \dot{x}^{\bar{\mu}} &= p^{\bar{\mu}} + i\theta \Gamma^{\bar{\mu}} \lambda_\theta + i\bar{\theta} \tilde{\Gamma}^{\bar{\mu}} \lambda_{\bar{\theta}}, \quad \dot{p}^{\bar{\mu}} = 0, \\ \dot{\theta}^\alpha &= -\lambda_\theta^\alpha, \quad \dot{\bar{\theta}}_\alpha = -\bar{\lambda}_{\theta\alpha}, \end{aligned} \quad (40)$$

while for the remaining constraints one finds the expressions

$$p^2 = 0, \quad (41.a)$$

$$p_{\theta\alpha} + i\theta^\beta \Gamma_{\beta\alpha}^{\bar{\mu}} p_{\bar{\mu}} = 0, \quad \bar{p}_\theta^\alpha + i\theta_\beta \tilde{\Gamma}^{\bar{\mu}\beta\alpha} p_{\bar{\mu}} = 0. \quad (41.b)$$

To get the final form of the dynamics, we pass to the light-cone coordinates $x^{\bar{\mu}} \rightarrow (x^+, x^-, x^i)$, $i = 1, 2, \dots, 8$, and to $SO(8)$ notations for spinors, $\bar{\theta}_\alpha = (\theta_a, \bar{\theta}'_a)$, $\theta^\alpha = (\theta'_a, \bar{\theta}_a)$, $\bar{p}_\theta^\alpha = (p_{\theta a}, \bar{p}'_{\theta a})$, $p_{\theta\alpha} = (p'_{\theta a}, \bar{p}_{\theta a})$, $a, \dot{a} = 1, 2, \dots, 8$. It allows one to write an equivalent to (41.b) set of constraints, which is explicitly classified as first- and second-class respectively,

$$\sqrt{2} p^+ p'_\theta + \bar{p}_\theta \tilde{\gamma}^i p^i = 0, \quad \sqrt{2} p^+ \bar{p}'_\theta - p_\theta \gamma^i p^i = 0; \quad (42.a)$$

$$\bar{p}_\theta + i\sqrt{2} p^+ \bar{\theta} - i\theta' \gamma^i p^i = 0, \quad p_\theta - i\sqrt{2} p^+ \theta - i\bar{\theta}' \tilde{\gamma}^i p^i = 0. \quad (42.b)$$

Then the equations $\theta'_a = 0$, $\bar{\theta}'_a = 0$ (or, equivalently, $\Gamma_{32}^+ \theta_{32} = 0$) are the gauge conditions for the first-class constraints (42.a). It follows from (36),

(40) that $\lambda_\theta = 0$ in this gauge. Thus the dynamics of the physical variables is described by the equations

$$\begin{aligned} \dot{x}^{\bar{\mu}} &= p^{\bar{\mu}}, & \dot{p}^{\bar{\mu}} &= 0, & \dot{p}^2 &= 0; \\ \dot{\theta}_\alpha &= 0, & \dot{\bar{\theta}}_{\bar{\alpha}} &= 0. \end{aligned} \quad (43)$$

Using the same arguments as in Sec.3, one can prove that $D = 10$ super Poincare symmetry transformations for (43) are some combinations of the symmetries (29)-(33), which do not spoil the gauge chosen. Besides the S-algebra (7) reduces to the type IIA supersymmetry algebra in this gauge.

5 Summary.

In the present paper we have constructed explicitly several Lagrangian actions for $D = 11$ S-invariant mechanical models. In particular, it was shown that $D = 10$ type IIA superparticle (40), (41) can be presented in the S-invariant formulation (27). In course of the consideration an explicit form of the S-algebra (7) was obtained. Being model-independent, it may be used as a basis for a systematic construction of various $D = 11$ models. In particular, it follows from the consideration of Sec.4 that there may exist a more transparent algebraic formulation for the $D = 11$ superparticle in terms of the Lorentz-harmonic variables [24-28]. We consider these models as a preliminary step towards a construction of $D = 11$ S-invariant formulations for SYM and superstring actions, which might contribute to a better understanding of the uncompactified M-theory [10-13].

Acknowledgments.

D.M.G. thanks Brazilian foundation CNPq for permanent support. The work of A.A.D. has been supported by the Joint DFG-RFBR project No

96-02-00180G, by Project INTAS-96-0308, and by FAPESP.

References

- [1] I. Bars, Phys. Rev. **D55** (1997) 2373; hep-th/9607112.
- [2] A.A. Deriglazov, *D=11 Superstring with New Supersymmetry and D=10 type IIA Green-Schwarz Superstring*, hep-th/9709025.
- [3] H. Nishino and E. Sezgin, Phys. Lett. **B388** (1996) 569; hep-th/9607185.
- [4] H. Nishino, *Supergravity in 10+2 Dimensions as Consistent Background for Superstring*, hep-th/9703214; *SYM Theories in $D \geq 12$* , hep-th/9708064; *Lagrangian and Covariant Field Equations for Supersymmetric YM Theory in 12D*, hep-th/9710141.
- [5] A. A. Deriglazov and A. V. Galajinsky, *On Possible Generalization of the Superstring Action to Eleven Dimensions*, hep-th/9706152, Phys. Rev. **D** (1998) (in print); Mod. Phys. Lett. **A12** (1997) 2993.
- [6] I. Bars and C. Deliduman, Phys. Rev. **D56** (1997) 6579; hep-th/9707215.
- [7] N. Berkovits, *A Problem with the Superstring Action of Deriglazov and Galajinsky*, hep-th/9712056.
- [8] A.A. Deriglazov and D.M. Gitman, *On Zero Modes of D=11 Superstring*, hep-th/9801176; *Green-Schwarz Type Formulation of D=11 Superstring Action with "New Supersymmetry" S-Algebra*, hep-th/9804055.
- [9] M.J. Duff, *Supermembranes*, hep-th/9611203.

- [10] E. Witten, Nucl. Phys. **B443** (1995) 85; hep-th/9503124.
- [11] M.J. Duff, Int. J. Mod. Phys. **A11** (1996) 5623; hep-th/9608117.
- [12] J.H. Schwarz, *Lectures on Superstring and M-Theory Dualities*, Nucl. Phys. Proc. Suppl. **B55** (1997) 1; hep-th/9607201.
- [13] P. K. Townsend, *Four Lectures on M-Theory*, hep-th/9612121.
- [14] T. Banks, W. Fishler, S. Shenker and L. Susskind, Phys. Rev. **D55** (1997) 5112, hep-th/9610043.
- [15] A. Achucarro, J. Evans, P. Townsend and D. Wiltshire, Phys. Lett. **D198** (1987) 441.
- [16] I. Bars and C. Kounnas, *A New Supersymmetry*, hep-th/9612119; Phys. Rev. **D56** (1997) 3664; hep-th/9705205.
- [17] I. Rudychev and E. Sezgin, Phys. Lett. **B415** (1997) 363; hep-th/9704057.
- [18] J. W. van Holten and A. van Proeyen, J. Phys. **A15** (1982) 3763.
- [19] R. Casalbuoni, Phys. Lett **B62** (1976) 49.
- [20] L. Brink and J. H. Schwarz, Phys. Lett. **B100** (1981) 310.
- [21] W. Siegel, Phys. Lett. **B128** (1983) 397.
- [22] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva University, New York 1964).
- [23] D. M. Gitman and I. V. Tyutin, *Quantization of Fields with Constraints* (Springer-Verlag, Berlin 1990).
- [24] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, Class. Quant. Gravity, **1** (1984) 469; **2** (1985) 155.
- [25] E. Sokatchev, Phys. Lett. **B169** (1986) 209.
- [26] D.P. Sorokin, V.I. Tkach and D.V. Volkov, Mod. Phys. Lett. **A4** (1989) 901.
- [27] N. Berkovits, Phys. Lett. **B247** (1990) 45.
- [28] A.S. Galperin, P.S. Howe and K.S. Stelle, Nucl. Phys. **B368** (1992) 248.