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ABSTRACT

The interaction between a very thin macroscopic solenoid, and a single magnetic particle precessing in a external magnetic field $\mathbf{B}_0$, is described by taking into account the thermal and the zero-point fluctuations of Stochastic Electrodynamics. The inductor belongs to a RLC circuit without batteries and the random motion of the magnetic dipole generates in the solenoid a fluctuating current $I_{\text{dip}}(t)$, and a fluctuating voltage $\mathcal{E}_{\text{dip}}(t)$, with spectral distribution quite different from the Nyquist noise. We show that the mean square value $\langle I^2_{\text{dip}} \rangle$ presents an enormous variation when the frequency of precession approaches the frequency of the circuit, but it is still much smaller than the Nyquist current in the circuit. However, we also show that $\langle I^2_{\text{dip}} \rangle$ can reach measurable values if the inductor is interacting with a macroscopic sample of magnetic particles (atoms or nuclei) which are close enough to its coils.

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1. INTRODUCTION

Recently, we have shown [1] how to extend Boyer's classical model for free space paramagnetism [2] for dealing with the modifications that arise in the paramagnetic behaviour of a rigid magnetic dipole, as an effect of the interaction of the dipole with a macroscopic solenoid which is part of a RLC electric circuit without batteries.

The classical model for paramagnetism makes use of the random electromagnetic fluctuations of the zero-point and the thermal radiation fields of Stochastic Electrodynamics (SED) [3]. Besides the simple understanding of the paramagnetic phenomena provided by it, confidence in Boyer's model is further assured by the fact that it gives a very good quantitative account of the experimentally observed paramagnetic behaviour of a magnetic dipole at any temperature [4].

As part of a collective effort [1–8] to better understand the effects of the vacuum electromagnetic field — another name given to the zero-point and thermal radiation fields — in the interaction between a microscopic object as, for instance, an electric or magnetic dipole and some kind of environment, we will concentrate on the modifications in the electric current flowing along the RLC circuit. Such modifications are caused by the presence of the magnetic dipole in the neighbourhood of the inductor. In this regard we would like to call the reader attention to the recent paper by Blanco, Dechoum, França and Santos [7], which gives a detailed description of the Casimir interaction between a microscopic electric dipole and a macroscopic solenoid. Several new predictions, not yet observed, are presented clearly in this paper.

In the section 2 we show that the magnetic dipole precessing around a constant external magnetic field creates an additional electromotive force along the coils of the solenoid. The fluctuations in the movement of the dipole, due to the vacuum field and to the random magnetic field generated by the Nyquist electric current, create another fluctuating electromotive force in the solenoid, whose spectral distribution is different from the spectrum of the Nyquist fluctuating voltage. The conclusions are presented in the section 3.

2. EFFECT OF THE MAGNETIC DIPOLE ON THE RLC CIRCUIT

In the present section we deal with the effect of the presence of a magnetic dipole in the neighbourhood of the inductor of a RLC circuit.

The rigid magnetic dipole is at the origin of a coordinate system and it is subjected to an external constant magnetic field $\mathbf{B}_0 = B_0\,\hat{\mathbf{e}}_z$, to the fluctuating zero-point and thermal radiation fields of SED and to the magnetic field emitted from a solenoid which is part of a RLC circuit, disconnected from batteries or any other electric power supply.

The solenoid has radius a and N coils uniformly distributed along its length ℓ $(\ell \gg a)$. The distance from the axis of the solenoid to the dipole is y and the entire situation is depicted in Figure 1.

The current in the electric circuit obeys the following equation,

$$L\dot{I}(t) + RI(t) + \frac{1}{C}\int dt\, I(t) = \mathcal{E}_N(t) + \mathcal{E}_{\rm dip}(t) \equiv \mathcal{E}(t) \quad , \tag{1}$$

where $\mathcal{E}_N(t)$ is the Nyquist electromotive force [9] generated in the circuit as a consequence of the dynamical equilibrium between the dissipation of energy and the fluctuations [10]. $\mathcal{E}_{\rm dip}(t)$ denotes the electromotive force generated by the oscillating magnetic dipole which creates a time varying magnetic field $\mathbf{B}_{\rm dip}(t)$ through each coil of the solenoid. The result of this calculation is [1]

$$\mathcal{E}_{\rm dip}(t) \simeq -\frac{\pi a^2 N}{c}\int_{-\ell/2}^{\ell/2} dx \frac{\partial}{\partial t} B_x^{\rm dip}(t) = -\frac{N}{\ell}\,\frac{2\pi a^2}{cy^2}\,\dot{\mu}_z(t)\,. \tag{2}$$

The magnetic dipole is subjected to various torques and its movement is described by the Bhabha equation [1, 2]

$$\dot{\mathbf{S}} = \vec{\mu} \times \left[\mathbf{B}_0 + \mathbf{B}_{VF}(t) + \mathbf{B}_{\rm sol}(t) - \frac{2}{3c^3}\,\dddot{\boldsymbol{\mu}}(t)\right] \quad , \tag{3}$$

where $\mathbf{S}$ is the spin angular momentum vector associated with the magnetic dipole by

$$\boldsymbol{\mu} = \frac{eg}{2mc}\,\mathbf{S}\,, \tag{4}$$

and e is the electric charge of a particle of mass m and gyromagnetic ratio g. We are denoting by $\mathbf{B}_{\rm sol}(t)$ the magnetic field generated by the solenoid.

The first term in the right hand side of equation (3) corresponds to a precession around the z axis with frequency

$$\eta = \frac{\mu\, B_0}{S}\,. \tag{5}$$

In that motion the magnetic dipole vector $\boldsymbol{\mu}$ makes a constant angle θ relative to the z axis and it creates an electromotive force given by (see (2))

$$\mathcal{E}_{\rm dip}(t) = -\frac{N}{\ell}\,\frac{2\pi a^2}{cy^2}\,\eta\mu\,{\rm sen}\theta\,{\rm sen}(\eta t + \alpha) \quad , \tag{6}$$

where α is an arbitrary phase. We shall take $\alpha = 0$ in what follows.

The electromotive force $\mathcal{E}_{\rm dip}(t)$ is responsible for establishing an electric current along the RLC circuit which, for an atomic dipole, is given by

$$I_{\rm dip}(t) = \frac{N}{\ell}\,\frac{2\pi a^2}{cy^2}\,\frac{S}{\hbar}\,g\,\mu_0\,\eta\,{\rm sen}\theta\left[\frac{R\,{\rm sen}(\eta t) - \left(\eta L - \frac{1}{\eta C}\right)\cos(\eta t)}{|Z(\eta)|^2}\right] \quad , \tag{7}$$

where μ_0 is the Bohr magneton, $\mu_0 = e\hbar/2mc$, and $Z(\omega)$ is the impedance of the RLC circuit,

$$Z(\omega) = R - i\left(\omega L - \frac{1}{\omega C}\right)\,. \tag{8}$$

If the particle is a nucleus one must replace μ_0 by the nuclear magneton.

Up to now the angle θ of precession remains undetermined. Its average value may be determined by taking into account the interplay of the three other terms in equation (3). As pointed out earlier [1, 2] the last term in that equation describes a monotonic tendency to align the magnetic moment $\boldsymbol{\mu}$ with the constant external magnetic field $\mathbf{B}_0$. The two middle terms represent fluctuations brought about by the zero-point and the thermal radiation fields of SED and by the fluctuating electric current in the RLC circuit.

The fluctuating magnetic field $\mathbf{B}_{VF}$ is usually written as [2]

$$\mathbf{B}_{VF} = \sum_{\lambda=1}^{2} \int d^3k\, \hat{\epsilon}(\mathbf{k},\lambda)\, H(\omega,T) \cos(\omega t + \xi(\mathbf{k},\lambda)) \equiv (B_x, B_y, B_z) \quad , \tag{9}$$

where the unit vectors $\hat{\epsilon}(\mathbf{k},\lambda)$ characterize the state of polarization λ of each plane wave of wave vector $\mathbf{k}$, and $\xi(\mathbf{k},\lambda)$ are random phases uniformly distributed in the interval $[0,2\pi]$. The function $H(\omega,T_d)$ relates $\mathbf{B}_{VF}$ to the energy spectral density of both the zero-point and the thermal fields at a temperature T_d and it is given by

$$H^2(\omega,T_d) = \frac{\hbar\omega}{2\pi^2} \coth\left(\frac{\hbar\omega}{2kT_d}\right) . \tag{10}$$

The solenoid magnetic field acting on the dipole is straightforwardly calculated and the result turns out to be [1]

$$\mathbf{B}_{\rm sol} = \hat{\mathbf{e}}_x \frac{N}{\ell} \frac{2\pi a^2}{cy^2} I(t) . \tag{11}$$

The net result of the interplay in which all the various torques in equation (3) take a part is summarized in a stationary probability distribution $P(\theta)$ for the alignment angle θ [1],

$$P(\theta) = A \sin\theta \exp\left\{ \frac{S\eta \cos\theta}{\pi^2 H^2(\eta,T_d) + \frac{3\pi c^3}{16\eta^2}\left(\frac{N}{\ell}\frac{2\pi a^2}{cy^2}\right)^2 \frac{\left\langle \left|\tilde{\mathcal{E}}_N(\eta,T_c)\right|^2\right\rangle}{|Z(\eta)|^2}} \right\} , \tag{12}$$

where A is a normalization constant, and $\tilde{\mathcal{E}}_N(\omega,T_c)$ is the Fourier transform of the Nyquist electromotive force [9, 11],

$$\left\langle \left|\tilde{\mathcal{E}}_N(\omega,T_c)\right|^2\right\rangle = \frac{R\hbar\omega}{2\pi} \coth\left(\frac{\hbar\omega}{2kT_c}\right) , \tag{13}$$

because one can take $\mathcal{E}(t) \simeq \mathcal{E}_N(t)$ in equation (1).

We also indicate explicitly the possibility that the magnetic dipole be in a thermal bath at temperature T_d while the circuit is in another thermal bath at temperature T_c different from T_d. In particular, the average value of the z component of the magnetic moment $\boldsymbol{\mu}$ was found to be [1]

$$\frac{\langle\mu_z\rangle}{g\mu_0} \equiv \frac{\langle\mu\cos\theta\rangle}{g\mu_0} = \frac{S}{\hbar}\coth\left\{ \frac{2S/\hbar}{\coth\left(\frac{\hbar\eta}{2kT_d}\right) + \left(\frac{a}{y}\right)^4 \rho(\eta)\coth\left(\frac{\hbar\eta}{2kT_c}\right)} \right\}$$
$$- \frac{1}{2}\left\{ \coth\left(\frac{\hbar\eta}{2kT_d}\right) + \left(\frac{a}{y}\right)^4 \rho(\eta)\coth\left(\frac{\hbar\eta}{2kT_c}\right) \right\} , \tag{14}$$

where

$$\rho(\eta) = \frac{3\pi^2 N^2 c R}{\ell^2 \eta^2 |Z(\eta)|^2} . \tag{15}$$

Now we are in a position to complete the calculation referring to the average value of the current generated by the magnetic dipole. Performing the time average as well as the average over all the possible values of the angle θ we obtain, using (7) and (12), the result

$$\left\langle I^2_{\rm dip}\right\rangle \cong \frac{2}{3}\frac{\langle\mu_z\rangle g\mu_0\eta^4}{c^3 R}\left(\frac{a}{y}\right)^4 \rho(\eta)\left\{ \coth\left(\frac{\hbar\eta}{2kT_d}\right) + \left(\frac{a}{y}\right)^4 \rho(\eta)\coth\left(\frac{\hbar\eta}{2kT_c}\right) \right\} . \tag{16}$$

The first term inside the brackets corresponds to the contribution of the fields associated with the radiation bath with temperature T_d, that are scattered by the dipole into the solenoid coils. The second term has its origin in the radiation generated in the circuit (temperature T_c), which are scattered by the dipole, and return to the solenoid coils.

The Fourier transform $\tilde{\mathcal{E}}(\omega)$ of the total electromotive force (see eq. (1)) will be defined by

$$\mathcal{E}(t) \equiv \frac{1}{2}\int_0^\infty d\omega \tilde{\mathcal{E}}(\omega)\{\exp[-i\omega t - i\phi(\omega)] + c.c.\} , \tag{17}$$

where $\phi(\omega)$ are statistically independent random phases. The analitical expression for $\tilde{\mathcal{E}}(\omega)$ can be obtained, with sufficient accuracy, by taking the Fourier transform of eq. (1), which leads to

$$\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_N(\omega) - \frac{N}{\ell}\,\frac{2\pi a^2}{cy^2}\, i\omega\, \tilde{\mu}_x(\omega)\ . \tag{18}$$

To find $\tilde{\mu}_x(\omega)$ we linearize equation (3) by replacing $\langle\mu_z\rangle$ for μ_z and neglecting the z component of the vacuum magnetic field B_z in face of the constant applied magnetic field $\mathbf{B}_0$. With this procedure, due to Schiller and Tesser [12], we are left with only two coupled stochastic equations, one for $\mu_x(t)$, the other for $\mu_y(t)$. Defining a complex variable $\chi = \mu_x + i\mu_y$ it is possible to rewrite the corresponding part of equation (3) as

$$\frac{2mc}{eg}\,\dot{\chi} = i\,B_0\,\chi - \frac{i2\langle\mu_z\rangle}{3c^3}\,\dddot{\chi} - i\langle\mu_z\rangle\,(B_x + B_{\rm sol} + i\,B_y)\ , \tag{19}$$

where B_x and B_y are defined in (9) and $B_{\rm sol}$ is given by (11).

The Fourier transform of the last equation gives

$$\tilde{\mu}_x(\omega) = \frac{\langle\mu_z\rangle}{\left(B_0 - \frac{i2\langle\mu_z\rangle\omega^3}{3c^3}\right)^2 - \left(\frac{2mc}{eg}\omega\right)^2}\left[\left(\tilde{B}_x(\omega) + \tilde{B}_{\rm sol}(\omega)\right)\left(B_0 - \frac{i2\langle\mu_z\rangle\omega^3}{3c^3}\right) - \frac{i2mc}{eg}\,\tilde{B}_y(\omega)\right]\ , \tag{20}$$

where ($j = x$ or y)

$$\tilde{B}_j(\omega) = \sum_{\lambda=1}^{2}\int d\,\Omega_k\ \hat{\epsilon}_j(\mathbf{k},\lambda)\,\frac{\omega^2}{2c^3}\,H(\omega,T_d)\exp[-i\xi(\mathbf{k}\cdot\lambda)]\ . \tag{21}$$

Considering the effect of the dipole on the circuit as a small perturbation equation (20) simplifies to

$$\tilde{\mu}_x(\omega) \simeq \frac{\langle\mu_z\rangle}{\left(B_0^2 - i2\frac{\langle\mu_z\rangle\omega^3}{3c^3}B_0\right) - \left(\frac{2mc}{eg}\omega\right)^2}\left[\left(\tilde{B}_x(\omega) + \frac{N}{\ell}\,\frac{2\pi a^2}{cy^2}\,\tilde{I}_N(\omega)\right)B_0 - \frac{i2mc}{eg}\,\omega\tilde{B}_y(\omega)\right]\ , \tag{22}$$

where we used the fact that $B_0 \gg 2\langle\mu_z\rangle\omega^3/3c^3$ for any frequency in a broad range centred at the frequency of precession of the dipole.

From eq. (18), a standard calculation for the autocorrelation function of the fluctuating voltage [9] allows us to write

$$\begin{aligned}\langle\,|\tilde{\mathcal{E}}(\omega)\,|^2\rangle \ \equiv\ & |\,Z(\omega)\,|^2\langle|\,\tilde{I}(\omega)\,|^2\rangle = \frac{R\hbar\omega}{2\pi}\,\coth\left(\frac{\hbar\omega}{2kT_c}\right) + \left(\frac{N}{\ell}\,\frac{2\pi a^2}{cy^2}\right)^4\left(\frac{g\mu_0\langle\mu_z\rangle}{\hbar}\right)^2 \times \\ & \times\left(\frac{R\hbar\omega^3}{2\pi}\right)\frac{\eta^2}{(\eta^2-\omega^2)^2 + \left(\frac{4}{3}\,\frac{g\mu_0\langle\mu_z\rangle\eta\omega^3}{\hbar c^3}\right)^2}\,\coth\left(\frac{\hbar\omega}{2kT_c}\right) + \\ & + \frac{2}{3c^3}\left(\frac{N}{\ell}\,\frac{2\pi a^2}{cy^2}\right)^2\left(\frac{g\mu_0\langle\mu_z\rangle}{\hbar}\right)^2 \times \\ & \times\left(\frac{\hbar\omega^5}{2\pi}\right)\frac{(\eta^2+\omega^2)}{(\eta^2-\omega^2)^2 + \left(\frac{4}{3}\,\frac{g\mu_0\langle\mu_z\rangle\eta\omega^3}{\hbar c^3}\right)^2}\,\coth\left(\frac{\hbar\omega}{2kT_d}\right)\ ,\end{aligned} \tag{23}$$

where $\langle\mu_z\rangle$ is given by our expression (14). In the above calculation we have considered that the fluctuations associated with the Nyquist noise are statistically independent from the vacuum electromagnetic fluctuations which characterize $\mathbf{B}_{VF}(t)$. It should be remarked that the above result is a new prediction. Up to now only the first term in (23) was experimentally observed [11] because the other terms, generated by the action of a single magnetic dipole, are very small.

Considering the sharply peaked functions in the various terms of equation (23) it is possible to show that the total average current in the RLC circuit is

$$\begin{aligned}\left\langle I^2\right\rangle \ \equiv\ & \int_0^\infty d\omega\left\langle|\tilde{I}(\omega)|^2\right\rangle \cong \frac{\hbar\Omega}{2L}\,\coth\left(\frac{\hbar\Omega}{2kT_c}\right) + \frac{2}{3}\left(\frac{a}{y}\right)^8\frac{\eta^4}{c^3R}\,g\mu_0\langle\mu_z\rangle\,\rho^2(\eta)\coth\left(\frac{\hbar\eta}{2kT_c}\right) \\ & + \frac{2}{3}\left(\frac{a}{y}\right)^4\frac{\eta^4}{c^3R}\,g\mu_0\langle\mu_z\rangle\,\rho(\eta)\coth\left(\frac{\hbar\eta}{2kT_d}\right)\ ,\end{aligned} \tag{24}$$

plus other negligible contributions coming from the poles of $|Z(\omega)|^{-2}$.

Here $\Omega = (LC)^{-1/2}$ and the function $\rho(\eta)$ is given by equation (15). The first term in equation (24) is the Nyquist term while the other two are due to the motion of the dipole. Notice that this result is in agreement with the calculation presented in equation (16).

The same approximations can be used to calculate the mean squared charge stored in the capacitor, giving

$$\left\langle Q^2\right\rangle = \frac{\hbar}{2L\Omega}\coth\left(\frac{\hbar\Omega}{2kT_c}\right)+ + \frac{2}{3}\frac{\langle\mu_z\rangle g\mu_0\eta^2}{c^3R}\left(\frac{a}{y}\right)^4\rho(\eta)\left[\coth\left(\frac{\hbar\eta}{2kT_d}\right)+\left(\frac{a}{y}\right)^4\rho(\eta)\coth\left(\frac{\hbar\eta}{2kT_c}\right)\right] , \quad (25)$$

where the first term is the contribution of the Nyquist noise.

We shall show in what follows various numerical results which will help us to have a quantitative idea of the importance of the new terms in equations (23), (24) and (25).

3. CONCLUDING REMARKS

In section 2 we have shown that the presence of a precessing magnetic dipole in the neighbourhood of the inductor of a RLC circuit increases the mean squared current that flows through the circuit.

To grasp the magnitude of the effect we present in Figure 2 a plot of $\langle I_N^2\rangle$ and $\left\langle I_{\rm dip}^2\right\rangle$ given by equation (24) *vs.* the applied magnetic field $\mathbf{B}_0$ for a magnetic particle (like the ion Gd^{3+} considered in refs. [1] and [4]) that has $\mu=\mu_0$, $g=2$ and a ratio $S/\hbar=4$. We used the following set of parameters for the RLC circuit: $N=320$ turns, $\ell=2\,$cm, $a=2.8\times10^{-5}\,$cm, $y=2a$, $R=10^{-17}\,$s/cm, $L=10^{-24}\,$s^2/cm, and $C=10^3\,$cm. The temperature of the two thermal baths were chosen to be $T_d=T_c=2$K. These parameters are different from those we have used earlier [1] (see also refs. [7] and [13]). We have considered these particular values in order to facilitate the visualization of the effect in the Figure. 2.

We see from Figure 2 that $\left\langle I_{\rm dip}^2\right\rangle$ has an impressive variation of 10^{12} orders of magnitude when $\eta=\mu B_0/S$ approaches the frequency of the circuit Ω. However, even in the resonance situation, when $\eta\cong1/\sqrt{LC}$, the Nyquist mean squared current $\langle I_N^2\rangle$ is 10^{10} times larger than the dipole mean squared current $\left\langle I_{\rm dip}^2\right\rangle$. The smallness of the effect may be understood once we remember that we were using only one atomic (or nuclear), hence microscopic, dipole to perturbate a macroscopic electric circuit. If we have, as an experimentalist would, a macroscopic paramagnetic sample containing M atomic dipoles precessing independently around the external applied magnetic field $\mathbf{B}_0$ we would have, instead of equation (24), that the total current is given by

$$\left\langle I^2\right\rangle = \left\langle I_N^2\right\rangle + \beta M\left\langle I_{\rm dip}^2\right\rangle , \quad (26)$$

where β is a geometrical factor which depends on the particular way to distribute the particles arround the inductor. This suggests that under suitable circumstances it may be possible to observe a significant deviation from the Nyquist term in the current that flows in the circuit because for a macroscopic sample $M\sim10^{23}$.

As mentioned elsewhere [1] we may interpret these results as a consequence of an interplay between the RLC circuit, the magnetic dipole and the entire environment represented by the random fields of SED. The RLC circuit picks up thermal and zero-point energy from the environment and a fraction of the fluctuating energy is radiated to the dipole [7, 8]. There, the extra electromagnetic noise brought in by the circuit alters the dipole tendency to align itself with the external field $\mathbf{B}_0$ as is shown in ref. [1] and in the Figure 3. On the other hand, the dipole also radiates the energy it picks up from the environment and the solenoid is able to absorb a part of it. According to the Figures 2 and 3, both the energy of the circuit and the energy of the magnetic particle increases in the resonance condition, even when the temperature is very close to the absolute zero. Therefore, one can conclude, on the basis of the energy conservation principle, that the zero-point radiation bath is the relevant energy reservoir for this phenomenon.

It is possible to have a better estimate of the order of magnitude of the effect if we consider the solenoid surrounded by 1 g of water (a cilinder of radius 0.6 cm and height 1 cm involving the inductor). This sample has 10^{22} protons which, at resonance, become demagnetized by the random radiation generated by the solenoid (see Figure 3). Considering the parameters of the circuit described above, we see, from equation (26) and Figure 2, that $\left\langle I^2_{\rm dip}\right\rangle \sim 10^6 \left\langle I^2_N\right\rangle$ (notice that the nuclear magneton is only $10^{-3}\,\mu_0$). Therefore, we conclude that the effect can be measured. The experiment would be similar to and no more difficult than the observation of the Nyquist spectrum by Koch et al. [11].

Another useful estimate is the order of magnitude of 10^{-3} watts obtained for $R\left\langle I^2_{\rm dip}\right\rangle$ if we use an atomic paramagnetic sample of 10^{22} particles and the data of Figure 2. This small but significant power dissipated in the circuit, is being continuously exchanged between the radiation bath, the RLC circuit and the magnetic sample since the entire system is supposed to be in dynamical equilibrium.

We hope that our simplified theoretical analysis will encourage the experimental investigation of these phenomena. It is well known that the study of atomic and nuclear magnetism is one of those areas of science which has remarkable versatility of applications in Physics, Chemistry and Biology, and which seems to go beyond all expectations.

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FIGURE CAPTIONS

FIGURE 1

Schematic picture showing the magnetic dipole orientation with respect to the applied magnetic field $\mathbf{B}_0$, and at a distance y from the inductor of the RLC circuit. The magnetic field $\mathbf{B}_{\rm dip}(t)$ and the fluctuating current $I(t)$ are also indicated. The solenoid axis is parallel to the x direction and extends from $-\ell/2$ to $\ell/2$.

FIGURE 2

The mean square value of the Nyquist current $\langle I_N^2 \rangle$ is compared with the current induced by a single dipole $I_{\rm dip}$. We see that $\left\langle I_{\rm dip}^2 \right\rangle$, given by equation (16), presents an impressive variation of 10^{12} orders of magnitude as a function of B_0. The resonance peak of $\left\langle I_{\rm dip}^2 \right\rangle$ occurs when $\eta = \mu B_0/S = 1/\sqrt{LC}$ and it is due mainly to the contribution of the second term in (16). The temperature of the system was taken as $T_c = T_d = 2°$K but the effect also appears at high temperatures.

FIGURE 3

Magnetization per nucleon as predicted by our eq. (14). The parameters of the circuit are similar to those used in refs. [1], [7], [8] and [13], namely $R = 10^{-12}$ sec/cm, $L = 5 \times 10^{-20}$ sec^2/cm, $N \simeq 3200$, $\ell = 5$ cm, $a = 7 \times 10^{-4}$cm and $C \cong 3 \times 10^2$ cm. The temperature was chosen $T_c = T_d = 0.001°$K. We have considered that the magnetic particle is a proton ($S/\hbar \simeq 1$, $g \simeq 2.8$ and $2\mu \simeq \mu_0 \times 10^{-3}$). In the upper curve $y = 1.4$ cm whereas in the lower curve $y = 0.5$ cm. The broken line corresponds to taking $y \to \infty$. The suppression of the magnetization occurs when we vary B_0 up to the resonant value $B_0 = 20$ kG (see also Figure 2).

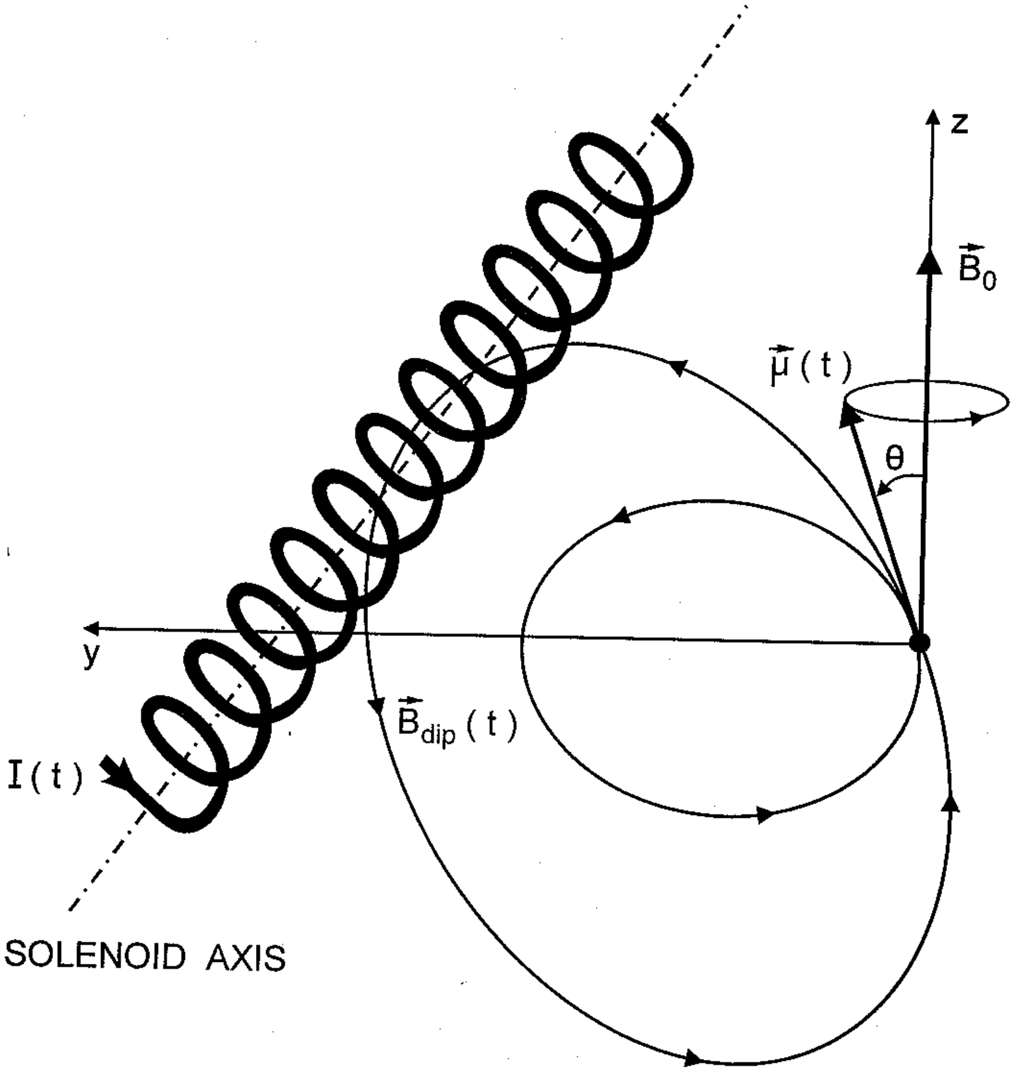

FIGURE 1

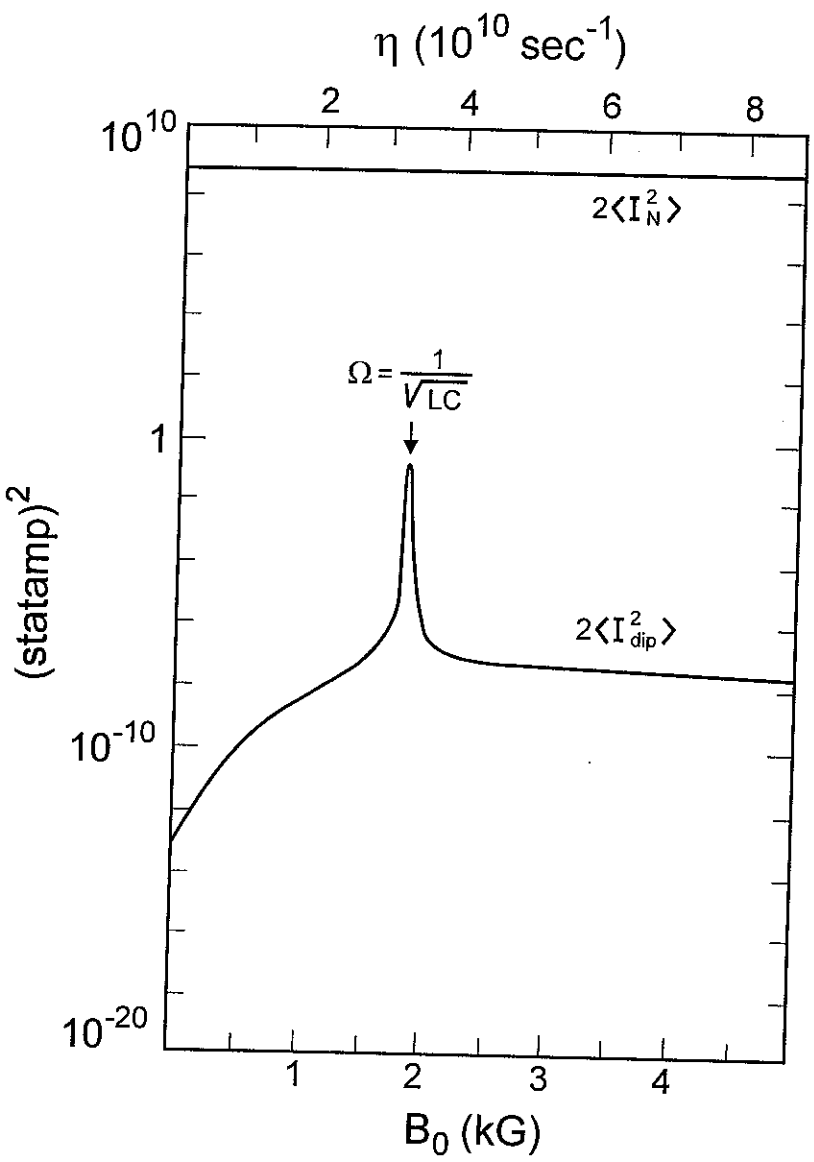

FIGURE 2

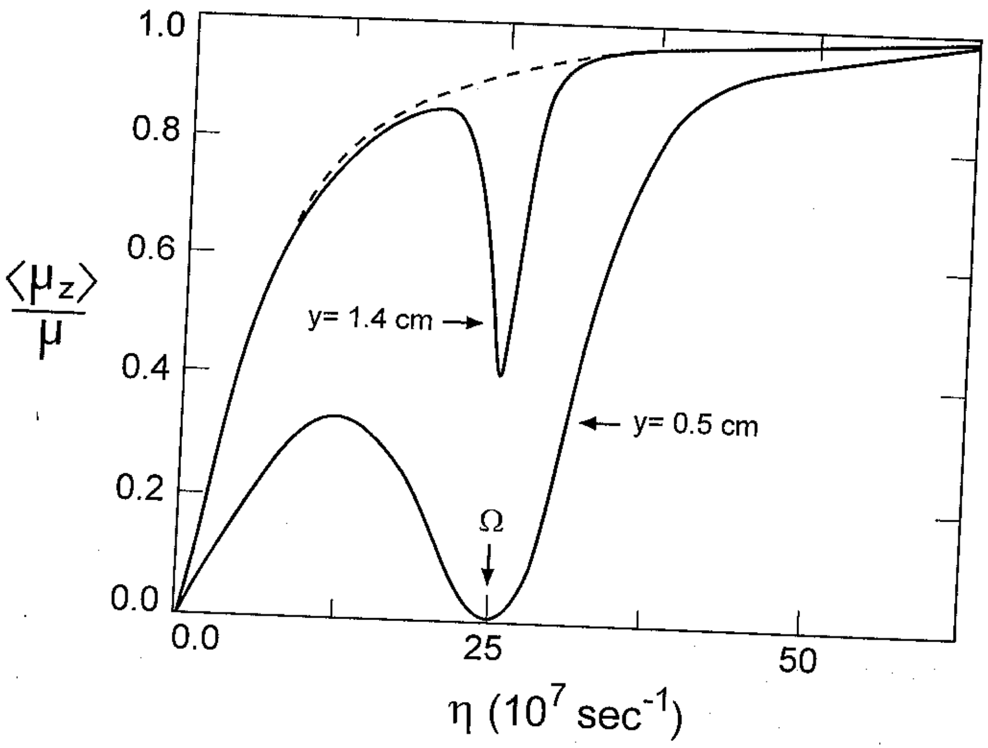

FIGURE 3