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## ON MINIMAL COUPLING OF THE ABC-SUPERPARTICLE TO SUPERGRAVITY BACKGROUND

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# On minimal coupling of the ABC–superparticle to supergravity background

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By rigorous application of the Hamiltonian methods we show that the $ABC$–formulation of the Siegel superparticle admits consistent minimal coupling to external supergravity. The consistency check proves to involve *all* the supergravity constraints.

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Recently, there has been considerable interest in the study of the superparticle (superstring) theories due to Siegel [1–7]. The first formulation of such a kind (the $AB$–model) can be viewed as the conventional superparticle [8] with only first–class constraints retained [9]. The second modification (the $ABC$–superparticle) appears if one tries to close the algebra of quantum $A$, $B$ constraint operators in the presence of external super Yang–Mills [10]. The third reformulation (the $ABCD$–, or "first–ilk"–superparticle) originated from the attempt to cure problems intrinsic in the $BRST$–quantized $ABC$–model [11,2]. Having advantage of being free of problematic second–class constraints, the latter two theories were proven to be physically equivalent to the conventional superparticle [12], thus suggesting an interesting alternative to attack the covariant quantization problem intrinsic in the original superparticle.

An important characteristic feature of the conventional superparticle, superstring theories is that they admit consistent (minimal) coupling to super Yang-Mills, supergravity backgrounds [13–15]. In particular, this allowed one to construct low energy effective action for the superstring theory within the framework of the sigma–model approach [16] and to get an elegant geometric interpretation of the super Yang–Mills, supergravity constraints themselves [14,15]. It is natural then to ask about the behaviour of the Siegel superparticles in external background superfields. For the $ABCD$–model in a curved superspace this question was previously addressed in Ref. [17], where it was shown that the system *can not* be minimally coupled to the supergravity background, thus showing serious drawback of the model.

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In this brief note we address the similar question for the $ABC$–superparticle. As shown below, this model does admit consistent minimal coupling to external supergravity. Interesting enough, the consistency check essentially involves *all* the supergravity constraints. This is in contrast with the conventional superparticle for which a smaller set is known to be sufficient to define the consistent coupling [14,15].

A conventional way to couple a superparticle model to a curved superspace is to rewrite its action in terms of the vielbein of a flat superspace and then set the latter to be that of a curved superspace. For the case at hand this yields[1]

$$S = \int d\tau \{ \tfrac{1}{2e}(\dot{z}^N e_N{}^a(z) + i\psi\sigma^a\bar{\rho} - i\rho\sigma^a\bar{\psi})^2 - \dot{z}^N e_N{}^\alpha(z)\rho_\alpha - \dot{z}^N e_{N\dot{\alpha}}(z)\bar{\rho}^{\dot{\alpha}} + \rho\sigma^a\bar{\rho}\Lambda_a + \rho^2 k + \bar{\rho}^2\bar{k} \}. \quad (1)$$

World indices appear on the coordinates $z^M$ and the vielbein $e_N{}^A(z)$ only, all other indices being tangent ones. Because consistent coupling has to preserve a number of degrees of freedom of the original model, we pass to the Hamiltonian formalism and analyze dynamics of the theory. Defining a phase space momentum to be the left derivative of a Lagrangian with respect to velocity, one finds the primary constraints

$$\begin{aligned} &p_e = 0, \\ &p_\psi = 0, \\ &p_{\bar{\psi}} = 0, \\ &p_\rho = 0, \\ &p_{\bar{\rho}} = 0, \\ &p_\Lambda = 0, \\ &p_k = 0, \\ &p_{\bar{k}} = 0, \\ &p_\alpha + \rho_\alpha = 0, \\ &\bar{p}_{\dot{\alpha}} + \bar{\rho}_{\dot{\alpha}} = 0, \end{aligned} \quad (2)$$

[1]For simplicity in this work we examine the problem in $d = 4$ superspace. The generalization to other dimensions is straightforward.

where $p_A \equiv e_A{}^N p_N$ and $(p_e, p_\psi, p_{\bar\psi}, p_\rho, p_{\bar\rho}, p_\Lambda, p_k, p_{\bar k}, p_N)$ are momenta canonically conjugate to the configuration space variables $(e, \psi, \bar\psi, \rho, \bar\rho, \Lambda, k, \bar k, z^N)$ respectively. The canonical Hamiltonian reads

$$\begin{aligned}
H^{(1)} &= \lambda_e p_e + \lambda_\psi{}^\alpha p_{\psi\alpha} + \lambda_{\bar\psi\dot\alpha} p_{\bar\psi}{}^{\dot\alpha} \\
&+ \lambda_\rho{}^\alpha p_{\rho\alpha} + \lambda_{\bar\rho\dot\alpha} p_{\bar\rho}{}^{\dot\alpha} + \lambda_\Lambda{}^n p_{\Lambda n} \\
&+ \lambda_k p_k + \lambda_{\bar k} p_{\bar k} + \lambda^\alpha (p_\alpha + \rho_\alpha) \\
&+ \bar\lambda_{\dot\alpha}(\bar p^{\dot\alpha} + \bar\rho^{\dot\alpha}) + e\frac{(p^a)^2}{2} - i\psi\sigma^a\bar\rho p_a \\
&+ i\rho\sigma^a\bar\psi p_a - \rho\sigma^a\bar\rho\Lambda_a - \rho^2 k - \bar\rho^2\bar k,
\end{aligned} \tag{3}$$

where $\lambda_*$ denote the Lagrange multipliers corresponding to the primary constraints. To analyze consistency conditions for the primary constraints one introduces the Poisson bracket associated with the left derivatives [15] (note that under this bracket $\{\mu, p_\mu\} = -1$ with $\mu$ a fermion)

$$\begin{aligned}
\{A, B\} &= (-1)^{\epsilon_A\epsilon_N} \frac{\vec\partial A}{\partial z^N}\frac{\vec\partial B}{\partial p_N} \\
&- (-1)^{\epsilon_A\epsilon_B+\epsilon_B\epsilon_N} \frac{\vec\partial B}{\partial z^N}\frac{\vec\partial A}{\partial p_N},
\end{aligned} \tag{4}$$

where $\epsilon_A$ is the parity of a function A. In what follows, the important bracket

$$\begin{aligned}
\{p_A, p_B\} &= T_{AB}{}^C p_C - \omega_{AB}{}^C p_C \\
&+ (-1)^{\epsilon_A\epsilon_B}\omega_{BA}{}^C p_C,
\end{aligned} \tag{5}$$

proves to be useful. Here $T_{AB}{}^C$ and $\omega_{AB}{}^C$ are components of the super torsion and the super connection respectively. The preservation in time of the primary constraints gives now the secondary ones

$$\begin{aligned}
(p^a)^2 &= 0, \\
p_a(\sigma^a\bar\rho)_\alpha &= 0, \\
(\rho\sigma^a)_{\dot\alpha} p_a &= 0, \\
\rho_\alpha\bar\rho_{\dot\alpha} &= 0, \\
\rho^2 &= 0, \\
\bar\rho^2 &= 0,
\end{aligned} \tag{6}$$

and the equations to determine some of the Lagrange multipliers (together with their complex conjugates)

$$\lambda_\alpha = -ip_a(\sigma^a\bar\psi)_\alpha + \Lambda_a(\sigma^a\bar\rho)_\alpha + 2k\rho_\alpha,$$



$$\begin{aligned}
\lambda_{\rho\alpha} &= (T_{\alpha\beta}{}^D p_D - \omega_{\beta\alpha}{}^\gamma p_\gamma)\lambda^\beta \\
&- (T_{\alpha\dot\beta}{}^D p_D - \omega_{\dot\beta\alpha}{}^\gamma p_\gamma)\bar\lambda^{\dot\beta} \\
&- ep^a(T_{\alpha a}{}^D p_D + \omega_{a\alpha}{}^\gamma p_\gamma) \\
&+ i(\psi\sigma^a\bar\rho)T_{\alpha a}{}^c p_c - i(\rho\sigma^a\bar\psi)T_{\alpha a}{}^c p_c.
\end{aligned} \tag{7}$$

In obtaining Eq. (7) we used the constraints (2),(6) and the explicit form of the connection $\omega_{Nab} = -\omega_{Nba}, \omega_{N\alpha}{}^\beta = \frac{1}{2}\omega_{Nab}(\sigma^{ab})_\alpha{}^\beta, \omega_{N\dot\alpha}{}^{\dot\beta} = \frac{1}{2}\omega_{Nab}(\tilde\sigma^{ab})^{\dot\beta}{}_{\dot\alpha}$. It is worth mentioning that the last two lines in Eq. (6) follow from the second and the third ones and, hence, can be omitted. We find it convenient to keep the corresponding trivial contributions to the Lagrangian (1) in oder to write the local $\kappa$-symmetry in the simplest form (see Eqs. (12),(13) below).

Consistency conditions for the secondary constraints produce the equations (together with their complex conjugates)

$$\begin{aligned}
& p_a\sigma^a{}_{\alpha\dot\gamma}\lambda_{\bar\rho}{}^{\dot\gamma} + (\sigma^a\bar\rho)_\alpha\{-(T_{a\beta}{}^c p_c \\
&+ \omega_{\beta a}{}^b p_b)\lambda^\beta + (T_{a\dot\alpha}{}^c p_c + \omega_{\dot\alpha a}{}^b p_b)\bar\lambda^{\dot\alpha} \\
&+ ep^b(T_{ab}{}^c p_c + \omega_{ba}{}^c p_c)\} = 0, \\
& p^a(-T_{a\alpha}{}^D p_D\lambda^\alpha + T_{a\dot\alpha}{}^D p_D\bar\lambda^{\dot\alpha} \\
&- i(\psi\sigma^b\bar\rho)T_{ab}{}^c p_c + i(\rho\sigma^b\bar\psi)T_{ab}{}^c p_c) = 0, \\
& \rho_\alpha\lambda_{\bar\rho\dot\alpha} - \bar\rho_{\dot\alpha}\lambda_{\rho\alpha} = 0,
\end{aligned} \tag{8}$$

which, after the substitution of Eq. (7), imply further (highly nonlinear) constraints and, hence, change a number of degrees of freedom in the problem as compared to that in a flat superspace. Thus some restrictions on the background geometry are necessary to define consistent coupling. Taking these to be the full set of $d = 4, N = 1$ supergravity constraints [18],

$$\begin{aligned}
T_{ab}{}^c &= 0, \\
T_{\breve\alpha b}{}^c &= 0, \\
T_{\breve\alpha\breve\beta}{}^{\breve\gamma}, &= 0 \\
T_{\alpha\beta}{}^c &= 0, \\
T_{\dot\alpha\dot\beta}{}^c &= 0, \\
T_{\alpha\dot\beta}{}^c &= 2i\sigma^c{}_{\alpha\dot\beta},
\end{aligned} \tag{9}$$

where $\breve\alpha$ means either $\alpha$ or $\dot\alpha$, one can check that equations (8) vanish and, moreover, the constraints[2]

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[2]The variables $(e, \psi, \bar\psi, \rho, \bar\rho, \Lambda, k, \bar k)$ together with the corresponding momenta can be omitted

$$
\begin{aligned}
&(p^a)^2 = 0, \\
&p_a(\sigma^a \bar{p})_\alpha = 0, \\
&(p\sigma^a)_{\dot\alpha} p_a = 0, \\
&p_a \bar{p}_{\dot\alpha} = 0,
\end{aligned} \tag{10}
$$

form a closed algebra and completely determine dynamics of the model just as in the free case. It is interesting to note that checking this one essentially needs to use *all* the supergravity constraints (9) as well as the solutions of the Bianchi identities involving $T_{a\check\alpha}{}^{\check\beta}$ (see Ref. [19] for the explicit relations). This is in contrast with the conventional superparticle [8] for which the similar analysis shows that the equations

$$
\begin{aligned}
T_{\check\alpha(ac)} &= \eta_{ac} T_{\check\alpha}, \\
T_{\alpha\beta}{}^c &= 0, \\
T_{\dot\alpha\dot\beta}{}^c &= 0, \\
T_{\alpha\dot\beta}{}^c &= 2i\sigma^c{}_{\alpha\dot\beta},
\end{aligned} \tag{11}
$$

with $T_{\check\alpha}$ an arbitrary superfield, are sufficient to determine consistent coupling (see also Refs. [14,15,20]).

In complete agreement with the Hamiltonian analysis, the Lagrangian (1) becomes invariant under the local $\kappa$–symmetry when the restrictions (9) hold. Actually, varying the action (1) with respect to the direct generalization of the flat $\kappa$–symmetry to a curved superspace (for technical details see Ref. [20])

$$
\begin{aligned}
\delta z^N e_{N\alpha} &= -ie^{-1}\Pi_a(\sigma^a\bar\kappa)_\alpha, \\
\delta z^N e_{N\dot\alpha} &= ie^{-1}\Pi_a(\kappa\sigma^a)_{\dot\alpha}, \\
\delta z^N e_N{}^a &= i\rho\sigma^a\bar\kappa - i\kappa\sigma^a\bar\rho, \\
\delta e &= 4\dot z^N e_N{}^\alpha \kappa_\alpha + 4\bar\kappa_{\dot\alpha}\dot z^N e_N{}^{\dot\alpha}, \\
\delta\psi^\alpha &= D(\kappa^\alpha), \\
\delta\bar\psi^{\dot\alpha} &= D(\bar\kappa^{\dot\alpha}),
\end{aligned} \tag{12}
$$

where $\Pi^a \equiv \dot z^N e_N{}^a + i\psi\sigma^a\bar\rho - i\rho\sigma^a\bar\psi$ and $D(k^A)$ is the covariant derivative, and making use of Eq. (9) and the solutions of the Bianchi identities involving $T_{a\check\alpha}{}^{\check\beta}$ [19], one finds that

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after imposing the gauge conditions $e = 1, \psi = 0, \bar\psi = 0, \Lambda = 0, k = 0, \bar k = 0$, and constructing the Dirac bracket associated with the second class constraints $p_{\rho\alpha} = 0, p_\alpha + \rho_\alpha = 0, p_{\bar\rho\dot\alpha} = 0, \bar p_{\dot\alpha} + \bar\rho_{\dot\alpha} = 0$.



all the terms entering the variation are proportional to $\rho\bar\rho, \rho^2, \bar\rho^2$, provided the additional variations of the fields $e, \psi$

$$
\begin{aligned}
\delta e &= 2e(R\kappa^\alpha\rho_\alpha + \bar R\bar\rho_{\dot\alpha}\bar\kappa^{\dot\alpha} \\
&\quad - \tfrac{3}{8}\rho^\alpha G_{\alpha\dot\alpha}\bar\kappa^{\dot\alpha} - \tfrac{3}{8}\kappa^\alpha G_{\alpha\dot\alpha}\bar\rho^{\dot\alpha}), \\
\delta\psi^\alpha &= -\tfrac{i}{8}\Pi_a(\kappa\sigma^a)_{\dot\alpha}G^{\alpha\dot\alpha} + \tfrac{3i}{8}\kappa^\alpha\Pi_a\tilde\sigma^{a\dot\beta\beta}G_{\beta\dot\beta},
\end{aligned} \tag{13}
$$

have been done. The superfields $R, G_{\alpha\dot\alpha}$ are those entering the solutions of the Bianchi identities [19]. Obviously, the remnant can be canceled by an appropriate variation of the fields $\Lambda, k, \bar k$.

Finally, let us briefly comment on the possibility to couple the $AB$–model to a curved superspace. The Lagrangian to start with is given by Eq. (1) with the three last terms omitted (the Hamiltonian analogue is the omitting of the three last lines in Eq. (6)). Exploiting the same machinery as above, it is easy to check that the consistency conditions like Eq. (8) do not vanish even if the full set of the supergravity constraints holds. They involve terms proportional to $\rho\bar\rho$ (times background superfields), thus giving further higher order fermionic constraints in the problem and changing the original number of degrees of freedom. This suggests that another way to formulate the $ABC$–superparticle is to require the closure of the algebra of the $A, B$–constraints in a curved superspace.

To summarize, we conclude that the $ABC$–model is the only one in the family of the Siegel superparticles which admits consistent minimal coupling to external supergravity.

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