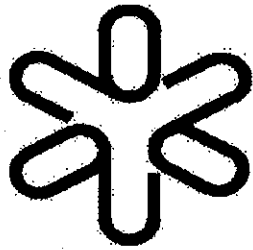


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**CLASSICAL DESCRIPTION OF SPINNING
DEGREES OF FREEDOM OF
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OF COMMUTING SPINORS**

DERIGLAZOV, A. A. AND GITMAN, D. M..
DEPTO. FÍSICA MATEMÁTICA

Publicação IF - 1324/98

Classical description of spinning degrees of freedom of relativistic particles by means of commuting spinors.

A.A. Deriglazov* and D.M. Gitman†

Instituto de Física, Universidade de São Paulo,
P.O. Box 66318, 05315-970, São Paulo, SP, Brasil.

Abstract

We consider a possibility to describe spin one-half and higher spins of massive relativistic particles by means of commuting spinors. We present two classical gauge models with the variables $x^\mu, \xi_\alpha, \chi_\alpha$, where ξ, χ are commuting Majorana spinors. In course of quantization both models reproduce Dirac equation. We analyze the possibility to introduce an interaction with an external electromagnetic background into the models and to generalize them to higher spin description. The first model admits a minimal interaction with the external electromagnetic field, but leads to reducible representations of the Poincare group being generalized for higher spins. The second model turns out to be appropriate for description of the massive higher spins. However, it seems to be difficult to introduce a minimal interaction with an external electromagnetic field into this model. We compare our approach with one, which uses Grassman variables, and establish a relation between them.

PAC codes: 0370

Keywords: Quantization, higher spins, classical models.

*deriglaz@fma.if.usp.br On leave of absence from Dept. Math. Phys., Tomsk Polytechnical University, Tomsk, Russia

†gitman@fma.if.usp.br

1 Introduction.

Classical relativistic spinning particle models and their quantization have been discussed intensively for a long time [1-13]. Such models have to reproduce one particle sector of the corresponding quantum field theory in course of quantization. There are two different approaches for description of the spinning degrees of freedom in such models. In the first one the anticommuting (Grassman) variables are used [2-13] following to the pioneer works [2-6]. In the second approach one uses commuting variables, which parametrize some compact manifolds [1, 14, 15]. Both approaches have some advantages and problems. In particular, it turns out to be problematic to generalize the first approach in $D = 3 + 1$ dimensions to massive higher spins, and to introduce an interaction with external backgrounds for the case of higher spins. The last problem appears also in the second approach.

In the present work we consider a possibility to describe spin one-half and higher spins of massive relativistic particles by means of commuting spinors. We present two classical gauge models in $D = 3 + 1$ dimensions with the variables $x^\mu, \xi_\alpha, \chi_\alpha$, where ξ, χ are commuting Majorana spinors. Both models are obtained by means of a localization of some global symmetries, which are characteristic for a simple action containing only kinetic terms for the above variables. In course of quantization both models reproduce Dirac equation. We analyze a possibility to introduce an interaction with an external background into the models and to generalize them to higher spin description. The first model admits a minimal interaction with the external electromagnetic field, but leads to reducible representations of the Poincare group being generalized for higher spins. The second model turns out to be appropriate for the description of the massive higher spins, it leads to Bargmann-Wigner wave equations [16] in course of quantization.

However, it seems to be difficult to introduce a minimal interaction with an external electromagnetic field into this model. We compare our approach with one, which uses Grassmann variables, and establish a relation between them. In particular, to this end we discuss a possibility to generalize basic models with Grassman variables to the case of massive higher spins

2 Description of the spinning degrees of freedom in terms of Grassman variables. Problem with higher spins.

In the most symmetric form, the action for spin one-half particle in $D = 3 + 1$ can be written as [2-6]

$$S = \int_0^1 \left[\frac{1}{2e} (\dot{x}^\mu - i\chi\psi^\mu)^2 - e\frac{m^2}{2} + im\psi^5\chi + i\psi_n\dot{\psi}^n \right] d\tau, \quad (1)$$

where x^μ , e are ordinary (bosonic or even) variables and ψ^n , χ are Grassman (fermionic or odd) variables which describe spinning degrees of freedom. Greek indices run over $\overline{0, 3}$ and Latin ones n, m run over $\overline{0, 3, 5}$. The metric tensors: $\eta_{\mu\nu} = \text{diag}(-1+1+1+1)$ and $\eta_{mn} = \text{diag}(-1+1+1+1+1)$. In addition to the reparametrizations, the action is invariant under local $N = 1$ (world-line) supersymmetry transformations.

In the Hamiltonian formulation there are the following constraints

$$\begin{aligned} P_e = \frac{\partial L}{\partial \dot{e}} = 0, \quad P_\chi = \frac{\partial_r L}{\partial \dot{\chi}} = 0, \quad P_n - i\psi_n = 0, \quad \left(P_n = \frac{\partial_r L}{\partial \dot{\psi}^n} \right), \\ p_\mu\psi^\mu + m\psi^5 = 0, \quad p^2 + m^2 = 0, \quad \left(p_\mu = \frac{\partial L}{\partial \dot{x}^\mu} \right). \end{aligned} \quad (2)$$

The Dirac brackets are defined by means of the second-class constraints $P_n - i\psi_n = 0$. For the variables ψ^n these brackets are

$$\{\psi^n, \psi^m\}_D = -i\eta^{nm}, \quad (3)$$

and define commutation relations for the corresponding operators $\hat{\psi}^\mu$,

$$\left[\hat{\psi}^n, \hat{\psi}^m\right]_+ = \hbar \eta^{nm}, \quad (4)$$

The Dirac brackets for the remaining variables coincide with the Poisson ones. The commutation relations for operators $\hat{\psi}$ can be realized in a space of four-dimensional columns Ψ_α as follows:

$$\hat{\psi}^\mu = \left(\frac{\hbar}{2}\right)^{\frac{1}{2}} \Gamma^5 \Gamma^\mu, \quad \hat{\psi}^5 = \left(\frac{\hbar}{2}\right)^{\frac{1}{2}} \Gamma^5, \quad (5)$$

where Γ^μ are γ -matrices in $D = 3 + 1$ dimensions and $\Gamma^5 = i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$. Applying operators of the first-class constraints to state vectors, we specify according to Dirac [28] the physical sector. It follows from the structure of the first-class constraints that the physical sector contains only vectors of the form $\Psi_\alpha(x^\mu)$ subjected to the Dirac equation

$$(\hat{p}_\mu \Gamma^\mu + m)\Psi = 0. \quad (6)$$

Due to the fact that the Hamiltonian is zero in the case under consideration, no more equations on state vectors appear. Thus, the model (1) reproduces the Dirac equation in course of such simplified quantization. One can show that the consistent canonical quantization leads to the same result [7].

The basic model (1) admits a natural introduction of an interaction with electromagnetic and gravitational backgrounds [2-6]. The limit $m \rightarrow 0$ was studied in [17, 18] (actions, which may describe Weyl particles, were considered in [9, 19, 20, 32]). A generalization of the action (1) to arbitrary even $D = 2n$ dimensions turned out to be trivial [17], whereas the generalization to an odd dimensions $D = 2n + 1$ met complication due to the absence of Γ^5 in such a case. Different ways of the problem solution were proposed in [21, 22, 23] for the case $D = 2 + 1$. One of the corresponding actions was generalized then [24] to arbitrary $D = 2n + 1$ dimensions.

Construction of models for relativistic particles with higher spin turn out to be nontrivial and seams to be in general case an open problem until now. Using the basic action (1) one can try to construct a model in $D = 3 + 1$ which describes massive spin $s = N/2$ by means of an extension of the odd sector as $\psi_a^n, \chi_a, a = 1, \dots, N$ [11, 12, 8]

$$S = \int d\tau \left\{ \frac{1}{2e} (\dot{x}^\mu - i\chi_a \psi_a^\mu)^2 - e \frac{m^2}{2} + im\psi_a^5 \chi_a + i\psi_{an} \dot{\psi}_a^n + \frac{i}{2} f_{ab} (\psi_{an} \psi_b^n + k_{ab}) \right\}. \quad (7)$$

The action (7) is invariant under reparametrizations, local N -extended supersymmetry transformations and local $O(N)$ -transformations. The Chern-Simons term $k_{ab} f_{ab}$ can only be added in the case $N = 2$ without breaking of $O(N)$ symmetry. Hamiltonian analysis for the action (7) leads to the following essential first-class constraints

$$\psi_{[a}^\mu \psi_{b]}^\mu + \psi_{[a}^5 \psi_{b]}^5 + k_{ab} = 0, \quad (8)$$

$$p_\mu \psi_a^\mu + m\psi_a^5 = 0, \quad (9)$$

$$p^2 + m^2 = 0. \quad (10)$$

and to Dirac brackets for the spinning degrees of freedom

$$\{\psi_a^n, \psi_b^m\}_D = -i\eta^{nm} \delta_{ab}. \quad (11)$$

Let us consider first the case $k_{ab} = 0$ in the action. To analyze this case it is convenient to impose the following gauge conditions

$$\psi_a^0 = 0, \quad (12)$$

for the first-class constraints (9). Then due to the constraint (9) and to the gauge condition (12), one stays with a set of three independent Grassman variables, which is convenient to select as [18],

$$\theta_a^i = \psi_a^i - \frac{p^i}{\mathbf{p}^2} p_i \psi_a^i + \frac{p^i}{\mathbf{p}^2} (\mathbf{p}^2 + m^2)^{\frac{1}{2}} \psi_a^5, \quad (13)$$

The Dirac brackets for the variables θ_a are

$$\{\theta_a^i, \theta_b^j\}_D = -i\eta^{ij}\delta_{ab}, \quad (14)$$

while the first-class constraints (8) take a form

$$\theta_{[a}^i \theta_{b]}^i = 0. \quad (15)$$

Operators, which correspond to the variables θ_a^i , can be realized as follows

$$\hat{\theta}_a^i = \left(\frac{\hbar}{2}\right)^{\frac{1}{2}} \Gamma^5 \otimes \dots \otimes \Gamma^5 \otimes \Gamma^5 \Gamma_{(a}^i \otimes 1 \otimes \dots \otimes 1, \quad (16)$$

in a space of spin-tensor functions $\Psi_{\alpha_1 \dots \alpha_N}(x^\mu)$. Applying the operators of the first-class constraints (15) to state vectors and using Fierz identities, we get the conditions

$$(\Gamma_i \Gamma^{(k)} \Gamma^i)_{\gamma\beta} \Psi_{\alpha_1 \dots \beta \dots \gamma \dots \alpha_N} = 0, \quad (17)$$

where $\Gamma^{(k)} \equiv \{1, \Gamma^\mu, \Gamma^{\mu\nu}, \Gamma^\mu \Gamma^5, \Gamma^5\}$ is a basis in the space of 4×4 matrices. The equations (17) are nontrivial for any $k = 0, 1, 2, 3, 4$. One can find by straightforward calculations that $\Psi_{\alpha_1 \dots \alpha_N} = 0$ as a consequence of (17), i.e. the physical subspace is empty. Thus, the direct generalization of the basic action (1) to higher spin massive case turns out to be problematic.

The modified action (7), which contains the Chern-Simons term ($k_{ab} \neq 0$), leads to the constraints

$$\theta_{[a}^i \theta_{b]}^i + k_{ab} = 0, \quad (18)$$

instead of (15). For $N > 2$ they are mixture of first and second-class constraints. The first-class constraints, being separated from the second-class ones, have the form (15), that immediately leads to the empty physical subspace. The case $N = 2$ is an exceptional since the rotational symmetry is not broken and the constraint (18) turns out to be first-class. It was

shown in [18] that the canonical quantization of such a model reproduces adequate quantum description of spin one massive particle.

The massless case can be obtained from (7-11) by the substitution $m = 0$, $\psi_a^5 = 0$, $k_{ab} = 0$. The model leads to Bargmann-Wigner equations for massless spin $s = N/2$ in course of the quantization, i.e. to the irreducible representation of the complete Poincare group. All the pseudoclassical constructions in $D = 3 + 1$ case may be extended to any even dimensions $D = 2n$. To construct higher spin models in $D = 2n + 1$ one may start with a model for $s = 1/2$ in the same dimension [25, 26]. For $D > 2 + 1$ a detailed elaboration of such a way seems to be still an open problem. Introduction of the interaction with external backgrounds remains also unsolved problem for higher spin pseudoclassical models.

Existence of the above mentioned problems in pseudoclassical approach motivates a development of alternative descriptions of spinning degrees of freedom in terms of the bosonic variables. Below we present two new models of such a kind.

3 First model. Spin one-half from the commuting spinors.

We start from a reparametrization invariant action of spinless relativistic particle in $D = 3 + 1$ dimensions and add to it a simplest reparametrization invariant term, which may be constructed from two additional (to x^μ and e) variables χ_α and ξ_β , $\alpha, \beta = 1, 2, 3, 4$, the latter are commuting Majorana spinors of SO(1,3) group,

$$S = \int d\tau \left\{ \frac{1}{2e} \dot{x}^2 - e \frac{m^2}{2} + i \bar{\chi} \dot{\xi} \right\}. \quad (19)$$

Besides of the manifest Poincare invariance, the action (19) is also invariant under global symmetry transformations with parameters β, γ, ϵ , where β, γ are scalars, while ϵ is a commuting spinor,

$$\delta\xi = -\beta\xi, \quad \delta\bar{\chi} = \beta\bar{\chi}, \quad (20)$$

$$\delta\xi = -\frac{\gamma}{e}\dot{x}_\mu\Gamma^\mu\xi, \quad \delta\bar{\chi} = \frac{\gamma}{e}\dot{x}_\mu\bar{\chi}\Gamma^\mu, \quad \delta x^\mu = i\gamma(\bar{\chi}\Gamma^\mu\xi), \quad (21)$$

$$\delta\bar{\xi} = \frac{1}{e}\dot{x}_\mu\bar{\epsilon}\Gamma^\mu, \quad \delta x^\mu = -i\bar{\epsilon}\Gamma^\mu\chi. \quad (22)$$

Let us consider some local versions of the theory (19) which can be obtained by means of gauging the symmetries (20), (21), (22). First, we consider a model which arises after localization of the transformations (20), (21). Following the usual manner, one has to consider the parameters β, γ as arbitrary functions of the evolution parameter τ and to introduce a ‘‘covariant’’ derivative,

$$D\xi \equiv \dot{\xi} + \phi\xi, \quad \delta\phi = -\dot{\beta}, \quad (23)$$

for the symmetry (20), and another one

$$Dx^\mu \equiv \dot{x}^\mu - i\omega(\bar{\xi}\Gamma^\mu\chi), \quad \delta\omega = \dot{\gamma}, \quad (24)$$

for the symmetry (21). The new variables ϕ, ω play a role of ‘‘gauge fields’’ for the corresponding symmetries, as it can be seen from their transformation law. Besides, one can add a terms $ik\phi, ik_1m\omega$ (where k and k_1 are some constants to be specified below), which does not break both the reparametrization symmetry and $\beta(\tau), \gamma(\tau)$ symmetries. Thus, a local version for the model (19) can be written as follows

$$S = \int d\tau \left\{ \frac{1}{2e} Dx^\mu Dx_\mu + i\bar{\chi}D\xi - e\frac{m^2}{2} + ik_1m\omega + ik\phi \right\}, \quad (25)$$

$$\delta\xi = -\beta\xi, \quad \delta\bar{\chi} = \beta\bar{\chi}, \quad \delta\phi = -\dot{\beta} \quad (26)$$

$$\delta\xi = -\frac{\gamma}{e}\dot{x}_\mu\Gamma^\mu\xi, \quad \delta\bar{\chi} = \frac{\gamma}{e}\dot{x}_\mu\bar{\chi}\Gamma^\mu, \quad \delta x^\mu = i\gamma(\bar{\chi}\Gamma^\mu\xi), \quad \delta\omega = \dot{\gamma}. \quad (27)$$

The action can be written also in the first order form

$$S = \int d\tau \left\{ \pi_\mu \dot{x}^\mu - \frac{1}{2} e (\pi^2 + m^2) + i \bar{\chi} \dot{\xi} - i \omega (\pi_\mu \bar{\xi} \Gamma^\mu \chi - k_1 m) - i \phi (\bar{\xi} \chi - k) \right\}. \quad (28)$$

It shows explicitly the structure of secondary first-class constraints, which correspond to the symmetries (26), (27), with ω, ϕ being considered as the Lagrange multipliers.

In the Hamiltonian formulation [28, 29], a complete set of constraints for the action (28) under consideration has the form

$$p_e = p_\omega = p_\phi = 0, \quad (29)$$

$$p^\mu - \pi^\mu = 0, \quad p_\pi^\mu = 0, \quad \bar{p}_\xi - i \bar{\chi} = 0, \quad \bar{p}_\chi = 0, \quad (30)$$

$$p_\mu \bar{\xi} \Gamma^\mu \chi - k_1 m = 0, \quad p^2 + m^2 = 0, \quad \bar{\xi} \chi - k = 0, \quad (31)$$

where $p, p_\pi, \bar{p}_\xi, \dots$ are canonical momenta for the variables x, π, ξ, \dots , and the equations (31) represent secondary constraints. There are no more constraints in the problem.

Imposing the gauge conditions

$$e = 1, \quad \phi = \omega = 0, \quad (32)$$

we define the Dirac bracket associated with the second-class set (29), (30), (32). For the independent variables $x^\mu, p_\nu, \bar{\xi}, \chi$ they are

$$\{x^\mu, p_\nu\}_D = \delta^\mu_\nu, \quad \{\bar{\xi}^\alpha, \chi_\beta\}_D = -i \delta^\alpha_\beta. \quad (33)$$

One can see that in the gauge chosen, the variables obey the free equations of motion: $\dot{x}^\mu = p^\mu, \dot{p}^\mu = 0, \dot{\bar{\xi}} = \dot{\chi} = 0$, and the first-class constraints (31).

The Dirac brackets (33) define the commutation relations for the corresponding operators

$$[\hat{x}^\mu, \hat{p}_\nu]_- = i \hbar \delta^\mu_\nu, \quad [\hat{\bar{\xi}}^\alpha, \hat{\chi}_\beta]_- = \hbar \delta^\alpha_\beta. \quad (34)$$

The algebra (34) can be realized on the space of $\bar{\xi}$ -regular functions

$$\Psi(x, \bar{\xi}) = \sum_{N=0}^{\infty} \Psi^{(N)} \equiv \sum_{N=0}^{\infty} \bar{\xi}^{\alpha_1} \dots \bar{\xi}^{\alpha_N} \Psi_{\alpha_1 \dots \alpha_N}(x^\mu), \quad (35)$$

where $\Psi_{\alpha_1 \dots \alpha_N}$ is spin-tensor of N-order. By the construction, it is symmetric in all its indexes: $\Psi_{\alpha_1 \dots \alpha_N} \equiv \Psi_{(\alpha_1 \dots \alpha_N)}$. We select the standard coordinate realization

$$\hat{x}^\mu = x^\mu, \quad \hat{p}_\mu = -i\hbar\partial_\mu, \quad \hat{\xi}^\alpha = \bar{\xi}^\alpha, \quad \hat{\chi}_\alpha = -\hbar\frac{\partial}{\partial\bar{\xi}^\alpha}, \quad (36)$$

and specify the physical sector applying operators of first-class constraints (31) to state vectors. Let us choose $k_1 = k = \hbar$ in the initial action (25), then the last constraint from (31) leads to the conditions

$$(N-1)\Psi^{(N)} = 0, \quad N = 0, 1, \dots, \quad (37)$$

which means that only the subspace $\Psi^{(1)} = \bar{\xi}^\alpha \Psi_\alpha(x)$ from Ψ -space (35) contains physical vectors. Then the remaining constraints from (31) demand that vectors $\Psi_\alpha(x)$ must obey the Dirac equation

$$(\hat{p}_\mu \Gamma^\mu + m)\Psi = 0. \quad (38)$$

Thus, under an appropriate choice of the parameters, the action (28) describes spin one-half free particle.

An interaction with an external electromagnetic field A^μ , may be introduced by means of a minimal coupling,

$$S_{int} = \int d\tau \left\{ qA_\mu \dot{x}^\mu - \frac{1}{2}eqF_{\mu\nu}(\bar{\xi}\Gamma^{\mu\nu}\chi) \right\}, \quad (39)$$

where q is an electric charge and the coefficient $-\frac{1}{2}$ in the second term was fixed from the requirement of invariance of (39) under the transformations (27). Repeating the above Hamiltonian analysis, one gets the equations of motion

$$\begin{aligned} \dot{x}^\mu &= \tilde{p}^\mu, & \dot{p}^\mu &= -\frac{1}{2}q\partial^\mu F_{\nu\rho}(\bar{\xi}\Gamma^{\nu\rho}\chi), \\ \dot{\bar{\xi}} &= 0, & \dot{\chi} &= -\frac{i}{2}qF_{\mu\nu}\Gamma^{\mu\nu}\chi, \end{aligned} \quad (40)$$

and the constraints

$$\tilde{p}^2 + m^2 + qF_{\mu\nu}(\bar{\xi}\Gamma^{\mu\nu}\chi) = 0, \quad \tilde{p}_\mu(\bar{\xi}\Gamma^\mu\chi) - \hbar m = 0, \quad \bar{\xi}\chi - \hbar = 0, \quad (41)$$

where $\tilde{p}^\mu = p^\mu - qA^\mu$. Being considered as constraints on the state vectors (35), the equations (41) reproduce formally a description of spin one-half particle on an external electromagnetic field.

It follows from (37) that in the realization under consideration the last constraint from (31) plays a role of the second Casimir operator for the Poincare group. Thus, it is interesting to consider the case when $k_1 = k = \hbar N$ in the initial action. Then the equation (37) specifies the subspace $\Psi^{(N)} = \bar{\xi}^{\alpha_1} \dots \bar{\xi}^{\alpha_N} \Psi_{\alpha_1 \dots \alpha_N}(x)$ from Ψ -space (35), while the first two constraints from (31) lead to the conditions

$$\begin{aligned} (p_\mu \Gamma_{\alpha_1 \beta}^\mu + m \delta_{\alpha_1 \beta}) \Psi_{\beta \alpha_2 \dots \alpha_N} + (p_\mu \Gamma_{\alpha_2 \beta}^\mu + m \delta_{\alpha_2 \beta}) \Psi_{\alpha_1 \beta \alpha_3 \dots \alpha_N} + \dots &= 0, \\ (p^2 + m^2) \Psi_{\alpha_1 \dots \alpha_N} &= 0. \end{aligned} \quad (42)$$

The subspace which is defined by (42) contains, in particular, an irreducible representation of spin $s = N/2$,

$$(p_\mu \Gamma_{\alpha_k \beta}^\mu + m \delta_{\alpha_k \beta}) \Psi_{\alpha_1 \dots \beta \dots \alpha_N} = 0, \quad k = 1, 2, \dots, N. \quad (43)$$

It would be interesting to study in more detail the spin content of the space (42).

Thus, the action (28) with commuting spinor variables reproduces after a quantization an adequate quantum theory of spin one-half particle. However, it seems to be not appropriate for minimal description of higher spins. In the next section we are going to consider another action, which solves the latter problem. This action corresponds to localization of symmetries (20), (22) instead of (20), (21).

4 Second model. Higher spins from the commuting spinors.

Action functional to be examined is

$$S = \int d\tau \left\{ \pi_\mu \dot{x}^\mu - \frac{1}{2} e (\pi^2 + m^2) + i \bar{\chi} \dot{\xi} - i \bar{\lambda} (\pi_\mu \Gamma^\mu \chi - k_1 m \chi) - i \phi (W^2 - k) \right\}, \quad (44)$$

where $\bar{\lambda}^\alpha$ is an additional commuting Majorana spinor, $W_\mu \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\delta} S^{\nu\rho} \pi^\delta$ is the Pauli-Lubanski vector, and $S^{\nu\rho} = -\frac{1}{2} \bar{\xi} \Gamma^{\nu\rho} \chi$. The latter quantity is a generator of Lorentz transformations, which are induced by the Dirac brackets (33): $\delta_\omega \bar{\xi} = \omega_{\mu\nu} \{S^{\mu\nu}, \bar{\xi}\}_D = -\frac{i}{2} \omega_{\mu\nu} \bar{\xi} \Gamma^{\mu\nu}$.

Let us demonstrate that the action (44) can be considered as a local version of the model (19). For this aim we rewrite (44) in the second order form by means of an integration over the variable π^μ . First, by using of γ -matrix identities as well as the identity

$$\begin{aligned} \epsilon^{dabc} \epsilon_{d\mu\nu\rho} &= -\delta_\mu^a (\delta_\nu^b \delta_\rho^c - \delta_\nu^c \delta_\rho^b) + \\ &\delta_\mu^b (\delta_\nu^a \delta_\rho^c - \delta_\nu^c \delta_\rho^a) - \delta_\mu^c (\delta_\nu^a \delta_\rho^b - \delta_\nu^b \delta_\rho^a), \end{aligned} \quad (45)$$

the second Casimir operator W^2 can be rewritten as

$$\begin{aligned} W^2 &= -\frac{1}{8} \pi^2 (\bar{\chi} \Gamma^{\mu\nu} \xi) (\bar{\chi} \Gamma_{\mu\nu} \xi) + \frac{1}{4} (\bar{\chi} \Gamma^{\mu\rho} \xi) (\bar{\chi} \Gamma_{\mu\nu} \xi) p_\rho p^\nu \\ &= -\frac{1}{16} \pi^2 (\bar{\xi} \chi)^2 + \frac{1}{4} \pi_\mu (\bar{\xi} \bar{\sigma}^\mu \chi) \xi (\pi_\nu \sigma^\nu \bar{\chi} - m \chi) + \\ &\frac{m}{4} (\bar{\xi} \chi) \bar{\xi} (\pi_\mu \bar{\sigma}^\mu \chi - m \chi) + \frac{1}{4} (\bar{\xi} \bar{\chi}) (\xi \chi) (\pi^2 + m^2), \end{aligned} \quad (46)$$

where the last three terms are written in two dimensional spinor notations, and $\sigma^\mu_{ba}, \bar{\sigma}^{\mu\dot{a}b}$ are $D = 4$ matrices Pauli [27]. After substitution of (46) into (44), the last three terms can be included into redefinition of the variables $e, \lambda^a, \bar{\lambda}_{\dot{a}}$. As a result, one obtains the action in the form

$$S = \int d\tau \left\{ \pi_\mu \dot{x}^\mu + i \bar{\chi} \dot{\xi} - \frac{1}{2} e (\pi^2 + m^2) + \right.$$

$$\left. \frac{i}{16}\phi\pi^2(\bar{\xi}\chi)^2 - i\bar{\lambda}(\pi_\mu\Gamma^\mu\chi - k_1m\chi) + ik\phi \right\}. \quad (47)$$

Equations of motion $\delta S/\delta\pi_\mu = 0$ can be solved as

$$\pi^\mu = \frac{\dot{x}^\mu - i\bar{\lambda}\Gamma^\mu\chi}{e - \frac{i}{8}\phi(\bar{\xi}\chi)^2}, n \quad (48)$$

and substituted into (47). After a additional redefinition of the variable $e : e \rightarrow e + \frac{i}{8}\phi(\bar{\xi}\chi)^2$, the action takes the final form

$$S = \int d\tau \left\{ \frac{1}{2e} Dx^\mu Dx_\mu + i\bar{\chi}D\xi - \frac{1}{2}em^2 + ik_1m\bar{\lambda}\chi + ik\phi \right\}, \quad (49)$$

where $Dx^\mu \equiv \dot{x}^\mu - i\bar{\lambda}\Gamma^\mu\chi$, $D\xi \equiv \dot{\xi} - \frac{1}{16}\phi m^2(\bar{\chi}\xi)\xi$. Local symmetries of the action are both reparametrizations and the transformations

$$\delta\xi = -\beta\xi, \quad \delta\bar{\chi} = \beta\bar{\chi}, \quad \delta\phi = -\frac{16}{m^2(\bar{\chi}\xi)}\dot{\beta}, \quad (50)$$

$$\delta\bar{\xi} = \bar{\epsilon}\left(\frac{1}{e}Dx_\mu\Gamma^\mu - m\right), \quad \delta x^\mu = -i\bar{\epsilon}\Gamma^\mu\chi, \quad \delta\bar{\lambda} = -\dot{\bar{\epsilon}} + \frac{m^2}{8}(\bar{\chi}\xi)\bar{\epsilon}, \quad (51)$$

with the parameters $\beta(\tau)$, $\epsilon_\alpha(\tau)$. Taking into account that the combination $(\bar{\chi}\xi)$ is invariant under the transformations (50), and comparing (50), (51) with the equations (20), (22), one may see that the action (49) can be obtained from (19) by localization of the symmetries (20), (22). The variables ϕ , λ_α play a role of the corresponding gauge fields. It is interesting to remark that if one starts from the action (19) with the anticommuting spinor variables ξ , χ , the analogous procedure leads to the model of Siegel superparticle [30, 31].

Hamiltonian analysis for the action (44) leads to the following first-class constraints

$$\frac{1}{16}(\epsilon_{\mu\nu\rho\delta}(\bar{\xi}\Gamma^{\nu\rho}\chi)p^\delta)^2 - k = 0, \quad (52)$$

$$p^2 + m^2 = 0, \quad (p_\mu\Gamma^\mu - k_1m)\chi = 0, \quad (53)$$

while the remaining variables $x^\mu, p_\nu, \bar{\xi}, \chi$ obey the Dirac brackets (33). Commutation relations for the corresponding operators (34) can be realized similar to (35), (36). After tedious calculations, one may see that the first-class constraint (52) leads to the following condition on the physical states

$$\left[\frac{1}{16} \hbar^2 m^2 N(N+2) - k \right] \Psi^{(N)} = 0, \quad N = 0, 1, 2, \dots \quad (54)$$

Let us take $k = \frac{1}{16} \hbar^2 m^2 N(N+2)$, $k_1 = -1$ in the action (49). Then the condition (54) selects the subspace $\Psi^{(N)}$ from (35) as physical one, while the constraints (53) leads to the Bargmann-Wigner equations (43) for the functions $\Psi_{(\alpha_1 \dots \alpha_N)}(x)$, which describe spin $s = N/2$ relativistic particles.

Thus, the action (44) provides classical description of massive higher spin relativistic particles and leads to an adequate minimal quantum theory of these particles. However, the problem how to introduce an interaction with an external electromagnetic background is still open for the case under consideration since the minimal interaction does not retain the first-class character of the constraints (53).

5 Discussion.

Thus, it was demonstrated that two simple classical models (25), (44), in which commuting spinors are used to describe spinning degrees of freedom, lead to adequate quantum description of spinning particles, in particular, irreducible representations of the complete Poincare group in the Bargmann-Wigner realization (43) are selected in course of the quantization.

One may mention some advantages of using the commuting spinors. In course of the quantization we get directly a realization in which state

vectors are symmetric in all their spinor indexes by construction. As it was demonstrated above, in the models, based on odd variables, an additional $O(N)$ local symmetry has to be introduced to provide such a realization. Beside the mass-shell condition, the second Casimir operator (46) can be naturally incorporated into the action [15], which allows one to define a subspace of a given spin. For models with odd variables the classical analog of this operator is identically zero for $D = 4$.

However, the approach proposed has not be treated as an alternative to the pseudoclassical description of spinning particles. It has to be rather considered as a combination of classical description of space motion with semi-classical description of spin. Namely, the models proposed may be understood as those which where obtained from some pseudoclassical models by a partial quantization of Grassman variables. Indeed, let us take the action (1) and quantize only the odd variables ψ , considering x^μ and e as external given fields. Then we arrive to the γ -matrix realization for the operators $\hat{\psi}$. In this case, the first-class constraint being applied to state vectors coincides with the classical equation of motion (53) of the theory (44). That confirms the above interpretation of the status of the models proposed.

Acknowledgments.

D.M.G. thanks Brazilian foundation CNPq for permanent support. The work of A.A.D. has been supported by the Joint DFG-RFBR project No 96-02-00180G, by Project INTAS-96-0308, and by FAPESP.

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