Instituto de Física
Universidade de São Paulo

NONLINEAR ENHANCEMENT OF THE MULTIPHONON COULOMB EXCITATION IN RELATIVISTIC HEAVY ION COLLISIONS

HUSSEIN, M. S.; TOLEDO PIZA, A. F. R.; VOROV, O. K.
DEPTO. FÍSICA NUCLEAR

Publicação IF - 1335/98

UNIVERSIDADE DE SÃO PAULO
Instituto de Física
Cidade Universitária
Caixa Postal 66.318
05315-970 - São Paulo - Brasil

DEPARTAMENTO DE FÍSICA NUCLEAR
INSTITUTO DE FÍSICA
UNIVERSIDADE DE SÃO PAULO

IFUSP - DFN/98-011

Nonlinear Enhancement of the Multiphonon Coulomb Excitation in Relativistic Heavy Ion Collisions

M.S. Hussein
A.F.R. de Toledo Piza
O.K. Vorov

Nonlinear Enhancement of the Multiphonon Coulomb Excitation in Relativistic Heavy Ion Collisions

M.S. Hussein, A.F.R. de Toledo Piza, O.K. Vorov

Instituto de Fisica, Universidade de Sao Paulo Caixa Postal 66318, 05315-970, Sao Paulo, SP, Brasil

(24 November 1998)

Abstract

We propose a soluble model of the nonlinear effects in the Coulomb excitation of the multiphonon Giant Dipole Resonances. Analytical expressions for the multi-phonon transition probabilities are derived, based on the SU(1,1) algebra. For reasonably small magnitude of nonlinearity $x \simeq 0.1 - 0.2$ enhancement factor for the Double Giant Resonance excitation probabilities and the cross sections reaches values $1.3 - 2$ compatible with experimental data. The enhancement factor is found to decrease with increasing bombarding energy.

Typeset using REVTEX

Coulomb Excitation in collisions of relativistic ions is one of the most promising methods in modern nuclear physics [1–6]. One of the most interesting applications of this method to studies of nuclear structure is the possibility to observe and study the multi phonon Giant Resonances [1]. The double Dipole Giant Resonances (DGDR) have been observed in a number of nuclei [7–9]. The "bulk properties" of the one–and two-phonon GDR are now partly understood [1] and they are in a reasonable agreement with the theoretical picture based on the concept of GDR-phonons as almost harmonic quantized vibrations.

Despite that, there is a persisting discrepancy between the theory and the data, observed in various experiments [7–11] that still remains to be understood: the double GDR excitation cross sections are found enhanced by factor $1.3 - 2$ with respect to the predictions of the harmonic phonon picture [1], [3], [12], [13]. This discrepancy, which almost disappears at high bombarding energy, has attracted much attention in current literature [4], [14–18], [19–21]; among the approaches to resolve the problem are the higher order perturbation theory treatment [18], and studies of anharmonic/nonlinear aspects of GDR dynamics [4], [19,20]; [21]. Recently, the concept of hot phonons [16], [17] within Brink-Axel mechanism was proposed that provides microscopic explanation of the effect. These seemingly orthogonal explanations deserve clarification which we try to supply here.

The purpose of this work is to examine, within a a soluble model the role of the nonlinear effects on the transition amplitudes that connect the multiphonon states in a heavy-ion Coulomb excitation process. Most studies of anharmonic corrections [19–21] concentrated on their effect in the spectrum [22,23]. Within our model, the nonlinear effects are described by a single parameter, and the model contains the harmonic model as its limiting case when the nonlinearity goes to zero. We obtain analytical expressions for the probabilities of excitation of multiphonon states which substitute the Poisson formula of the harmonic phonon theory. For reasonably small values of the nonlinearity, the present model is able to reproduce the observed enhancement of the double GDR cross sections and its energy dependence.

We work in a semiclassical approach [12] to the coupled-channels problem, i.e., the projectile motion is approximated by a classical trajectory (straight line) and the excitation of the Giant Resonances is treated quantum mechanically [12], [13], [14]. The use of this method is justified due to the small wavelenghts associated with the relative motion in relativistic heavy ion collisions. The intrinsic state $|\psi(t)>$ of excited nucleus is the solution of the time dependent Schrödinger equation

$$i\frac{\partial|\psi(t)>}{\partial t}=[H_0+V(t))]\ |\psi(t)>,\quad |\psi(t)>=\sum_{N=0} a_N(t)\ |N>\ \exp\Big(-iE_Nt\Big), \tag{1}$$

where H_0 is the intrinsic Hamiltonian and V is the channel-coupling interaction, (we set $\hbar=c=1$). The problem is to find the expansion amplitudes $a_N(t)$ in the wave packet $|\psi\rangle$ as functions of impact parameter b where E_N is the energy of the state $|N>$ with the numbers of excited GDR phonons N. The excitation probability W_N of an intrinsic state $|N>$ in a collision with impact parameter b and the total cross section σ_N for excitation of the state $|N>$ are

$$W_N(b)=|a_N(\infty)|^2,\qquad \sigma_N=2\pi\int_{b_{gr}}^{\infty} bW_N(b)db \tag{2}$$

where $b_{gr}=1.2(A_{exc}^{1/3}+A_{sp}^{1/3})$ is the grazing impact parameter, the labels exc (sp) refer to excited (spectator) nucleus in a colliding pair. It is convenient to treat the coupled channel equations (1) in terms of the unitary evolution operator U_I such that $|\psi(t)\rangle=U_I(t)|0\rangle$:

$$i\frac{d}{dt}U_I(t)=V_I(t)U_I(t),\qquad V_I(t)=e^{iH_0t}V(t)e^{-iH_0t},\qquad U_I(t=-\infty)=I, \tag{3}$$

where the time-dependent Hamiltonian $H(t)=H_0+V(t)$ that acts in the intrinsic multi-GDR states with the GDR frequency ω is given by $H_0=\omega N_d,\quad N_d\equiv\sum_m d_m^+d_m$ and

$$V(t)=v_1(t)[(E1_{-1})^\dagger-(E1_{+1})^\dagger]+v_0(t)(E1_0)^\dagger+Herm.Conj. \tag{4}$$

where $E1_m^\dagger$ and $E1_m$ are the dimensionless operators acting in the space of the multi-GDR states created by the boson operators d_m^+, m is the angular momentum projection. The functions v_m are given in [2], e.g.,

$$v_1(t)=\frac{F}{[1+(\frac{\gamma v}{b}t)^2]^{3/2}},\quad F=\frac{Z_{sp}e^2\gamma}{2b^2}\sqrt{\frac{N_{exc}Z_{exc}}{A_{exc}^{2/3}m_N\cdot 80MeV}}. \tag{5}$$

Here, m_N and e are the proton mass and charge, Z, N and A denote the nuclear charge, the neutron number and the mass number of the colliding partners, $\gamma=(1-v^2)^{-1/2}$ is relativistic factor and v is the velocity. [1].

In the harmonic approximation, the operators $E1^\dagger_m, E1_m$ are linear in the GDR phonons, $E1^\dagger_m = d^\dagger_m$. This model of "ideal bosons" coupled linearly to the Coulomb field admits well known exact nonperturbative solution (see, e.g. [13]) for the excitation probabilities

$$W_N = e^{-|\alpha^{harm}|^2}\frac{|\alpha^{harm}|^{2N}}{N!}, \qquad |\alpha^{harm}|^2 = \sum_{m=0,\pm1}|\alpha^{harm}_m|^2 = 2|\alpha^{harm}_1|^2 + |\alpha^{harm}_0|^2, \tag{6}$$

i.e., the Poisson formula with the amplitudes α^{harm}_m expressed through the modified Bessel functions. At the colliding energies sufficiently high, the longitudinal contribution $|\alpha^{harm}_0|^2$ is suppressed by a factor proportional to γ^{-2} [3]. We will work in the "transverse approximation" dropping the term $|\alpha^{harm}_0|^2$ (the results are still qualitatively valid at lower energies).

Now, we consider the nonlinear effects. Our idea is to keep the spectrum of GDR system harmonic with the Hamiltonian $H_0 = \omega N$. That is supported by the systematics of the observed DGDR energies, E_2, which yields $E_2 \simeq (1.75 - 2)\omega$ [1], so anharmonicity in the spectrum is weak. This conclusion follows also from theoretical considerations [22], [23]. The transition operators $E1^\dagger, E1$ that couple intrinsic motion to the Coulomb field can however include nonlinear effects: the expansion in terms of GDR bosons reads

$$E1^\dagger_m = d^\dagger_m + x\sum_{m_1} d^\dagger_m d^\dagger_{m_1} d_{m_1} + x_1 \sum_{m_1} d^\dagger_m d^\dagger_{m_1} d^\dagger_{m_1} + x_2 \sum_{m_1 m_2} d^\dagger_m d^\dagger_{m_1} d_{m_1} d^\dagger_{m_2} d_{m_2} + \ldots \tag{7}$$

To keep the theory treatable, the number of the nonlinear parameters x_i in (7) must be reduced. A reasonable way to do so is to save in (7) a convergent series with the leading term proportional to x, vis

$$E1^\dagger_m = d^\dagger_m + x\sum_{m_1} d^\dagger_m d^\dagger_{m_1} d_{m_1} - \frac{x^2}{2}\sum_{m_1 m_2} d^\dagger_m d^\dagger_{m_1} d_{m_1} d^\dagger_{m_2} d_{m_2} + \ldots = d^\dagger_m \left(1 + 2xN_d\right)^{1/2}, \tag{8}$$

where the single parameter $x > 0$ controls nonlinearity, and the problem reduces to the harmonic one with linear coupling when $x \to 0$. The ansatz (8) that we adopt here accounts for many higher order contributions to (7) while leading to soluble but nontrivial model.

To solve the nonlinear problem (3) with (4) and (8) we introduce the following triad of operators

$$D^- = \sqrt{\frac{1}{4x} + \frac{1}{2}N_d}(d_{-1} - d_{+1}), \quad D^0 = \frac{1}{4}\left[(d^+_{-1} - d^+_{+1})(d_{-1} - d_{+1}) + 2(1/(2x) + N_d)\right], \tag{9}$$

and D^+ the conjugate $D^+ = (D^-)^\dagger$ with $N_d \equiv d^+_{+1}d_{+1} + d^+_{-1}d_{-1}$. It is easy to check that they obey the commutation relations for the *noncompact* SU(1,1) algebra

$$\left[D^-, D^0\right] = D^-, \quad \left[D^+, D^0\right] = -D^+, \quad \left[D^-, D^+\right] = 2D^0. \tag{10}$$

The dynamics of the system can be expressed in terms of the operators $D^\pm$ and D^0 (9) only. Evolution equation (3) and its formal exact solution, the time-ordered exponential now read

$$i\frac{d}{dt}U_I(t) = 2x^{\frac{1}{2}}\left[v_1(t)e^{i\omega t}D^\dagger + v_1(t)e^{-i\omega t}D^-\right]U_I(t), \quad U_I(t) = Texp\left(-i\int\limits_{-\infty}^{t} dt'V_I(t')\right) \tag{11}$$

where (10) and $[N_d, D^\pm] = \pm D^\pm$ has been used in (3),(4) and (8). From purely mathematical viewpoint, the problem described by the last equation drops into the universality class of the systems with SU(1,1) dynamics that can be analyzed by means of generalized coherent states [24], [25]. For other algebraic approaches to scattering problems, see Ref. [26].

Due to closure of the commutation relations between the operators D^+, D^- and D^0, the time-ordered exponential (11) can be represented in another equivalent form that involve ordinary operator exponentials (see, e.g., [27])

$$U_I(t) = exp\left[2\sqrt{x}\alpha(t)D^+\right]exp\left[\left[log\left(1-4x|\alpha(t)|^2\right) - i\phi(t)\right]D^0\right]exp\left[-2\sqrt{x}\alpha^*(t)D^-\right] \tag{12}$$

and some time-dependent complex number $\alpha(t)$ (star denotes complex conjugation) and real number $\phi(t)$ (phase) [24]. The unknown functions $\alpha(t)$ and $\phi(t)$ can be found from simple differential equations which relate them to the function $v_1(t)$ in the Hamiltonian $H(t)$. These equations can be restored after substituting the right hand side of Eq.(12) into the Schrödinger equation for the operator $U_I(t)$ (11) and collecting the terms which have the same operator structure. Proceeding this way, we obtain, after some algebraic manipulations, the following Riccati-type equation for the complex amplitude α:

$$i(d/dt)\alpha = v_1(t)e^{i\omega t} + 4xv_1(t)e^{-i\omega t}\alpha^2. \tag{13}$$

The phase $\phi(t)$ is given by a simple integral $\phi(t) = 8x\int\limits_{-\infty}^{t} dt_1 Re[v_1(t_1)\alpha(t_1)e^{-i\omega t_1}]$. The simple nonlinear equation (13) accounts for *all orders* of quantum perturbation theory for the problem Eqs.(3),(4),(11). From Eq.(12), we have

$$|\psi(t)\rangle = U_I(t)|0\rangle = e^{-i\phi(t)/(4x)} \left(1 - 4x|\alpha(t)|^2\right)^{1/(4x)} exp\left[2\sqrt{x}\alpha(t)D^+\right]|0\rangle.$$

It is seen from Eq.(12), that unitarity is preserved automatically within present formalism as $U_I^\dagger = U_I^{-1}$, thus $\langle\psi(t)|\psi(t)\rangle = 1$. The expression for the amplitudes $a_N(t)$ follows from (12) immediately after projection of the state $|\psi(\infty)\rangle$ onto the states with definite number of GDR phonons, N.

$$W_N = |a_N(\infty)|^2, \qquad |a_N(\infty)| = \left(1 - 4x|\bar{\alpha}(x)|^2\right)^{\frac{1}{4x}} \left(\frac{\Gamma(\frac{1}{2x}+N)}{N!\Gamma(\frac{1}{2x})}\right)^{1/2} \left(4x|\bar{\alpha}(x)|^2\right)^{N/2} \quad (14)$$

Here, the quantity $\bar{\alpha}(x)$ is the asymptotic solution to the Riccati equation (13) at $t \to \infty$ subject to the initial condition $\alpha(-\infty) = 0$. Eq.(14) is our final analytical result. The constant $x^{1/2}|\bar{\alpha}(x)|$ in (14) can be viewed as a "special function" of the two parameters, $x^{1/2}F/\omega$ and the adiabaticity parameter $\frac{\omega b}{v\gamma}$. It can be easily tabulated by solving (13). The cross sections are then obtained from the usual formula (2) with using (14).

The harmonic limit of these results corresponds to the case $x \to 0$, when the coupling to electromagnetic field via (8) becomes linear. The last term drops from Eq.(13), and $|\bar{\alpha}(x)| \to |\alpha_{\pm 1}^{harm}| = \left|-i\int_{-\infty}^{\infty} v_1(t)e^{i\omega t}dt\right| = 2\frac{F}{\omega}\left(\frac{\omega b}{v\gamma}\right)^2 K_1\left(\frac{\omega b}{v\gamma}\right)$ where K_1 is the modified Bessel function [13], [1]. The expression for W (14) reduces at $x \to 0$ to the Poisson formula (6), thus the harmonic results [13], [1] are restored.

At nonzero nonlinearity $x > 0$, the excitation probabilities W_N (14) for multiple GDR ($N > 1$) turn out to be enhanced as compared to their values in the harmonic limit W_N^{harm}, as illustrated in Fig.1. The deviation of the N-phonon excitation probabilities from their harmonic values W_N^{harm} (6) (the *enhancement factor*) is given by the ratio

$$\frac{W_N}{W_N^{harm}} = \frac{\Gamma(\frac{1}{2x}+N)}{\Gamma(\frac{1}{2x})\left(\frac{1}{2x}\right)^N} \frac{(1 - 4x|\bar{\alpha}(x)|^2)^{\frac{1}{2x}}}{e^{-2|\alpha_1^{harm}|^2}} \frac{|\bar{\alpha}(x)|^{2N}}{|\alpha_1^{harm}|^{2N}}. \quad (15)$$

The first factor in this expression reflects kinematic enhancement of the transition probabilities due to nonlinearity. The last factor in (15) results from dynamical effects caused by nonlinearity, which are incorporated in the asymptotic solution of the nonlinear equation (13). This second "dynamical factor" depends on the bombarding energy and it gives rise to additional enhancement in low energy domain.

The interesting feature of these results is that the enhancement factor in the cross section $r_2 = \frac{\sigma_2}{\sigma_2^{harm}}$ is more sensitive to the bombarding energy than to the parameters of the spectator partner. This is just what has been observed in experiments: the values of r_2^{exp} found for DGDR in ^{208}Pb projectile using different targets ^{120}Sn, ^{165}Ho, ^{208}Pb, ^{238}U [10] are close to

$$r_2(^{208}Pb) \simeq 1.33 \quad , \qquad \gamma \simeq 1.7, \tag{16}$$

bombarding energy $\varepsilon \simeq 640$ Mev/per nucleon. The same picture was found in experiments on Coulomb desintegration of ^{197}Au target using various projectiles $^{20}Ne,^{86}Kr,^{197}Au,^{209}Bi$ [9]. Nearly constant value of r_2 has been found in ^{208}Pb target [11] while scattering different projectiles at low bombarding energy $\varepsilon \simeq 60 - 100$ Mev/per nucleon. In this case,

$$r_2(^{208}Pb) \simeq 2 \quad , \qquad \gamma \simeq 1.06 - 1.10. \tag{17}$$

Within present nonlinear model, these enhancement factors would correspond to reasonably small nonlinearity parameter x

$$x(^{208}Pb) \simeq 0.16 - 0.20.$$

Below, we present the exact results for the cross sections calculated according to Eqs.(2), and (14) and with solving Eq.(13) numerically. The dependence of the enhancement factor $r_2 = \sigma_2/\sigma_2^{harm}$ for the DGDR excitation on the strength of the nonlinearity x is shown in Fig. 1 for the process $^{208}Pb +^{208}Pb$, bombarding energy $\varepsilon = 0.64GeV$/per nucleon. One sees that the enhancement factor drops to unity at small values of x (harmonic limit) and approaches the observable values at still reasonably weak nonlinearity.

We discuss now the dependence of the enhancement factor on bombarding energy. Deviations of r_2 from the straight line $\frac{\Gamma(\frac{1}{2x}+2)}{\Gamma(\frac{1}{2x})(\frac{1}{2x})^2} = 1 + 2x$ (cf. Eq.(15)) occur at both low and high energies. At $\gamma \to 1$, one can solve (13) within an adiabatic perturbation theory to see that $|\alpha| > |\alpha_1^{harm}|$. Thus, $r_2 > 1 + 2x$. At higher energies, by contrast, the dynamical nonlinear effects tend to reduce the magnitude of $|\alpha|$. One can see from (13) that at $\gamma \gg 1$, $|\alpha|/|\alpha_1^{harm}| \simeq \frac{tanh(2\sqrt{x}|\alpha_1^{harm}|)}{(2\sqrt{x}|\alpha_1^{harm}|)} < 1$, and thus $r_2 < 1 + 2x$. To sum up, the enhancement factor for the DGDR excitation cross section, $r_2 = \sigma_2/\sigma_2^{harm}$ drops from $2 - 2.5$ (for low bombarding energies $\varepsilon \sim 100MeV$ per nucleon) to $1.2 - 1.3$ (for $\varepsilon \sim 640 - 700MeV$ per nucleon) while fixed value of nonlinearity x is used. In Fig.2., we plotted the value of the enhancement factor calculated numerically for the case of $^{208}Pb + ^{208}Pb$ process as a function of the relativistic factor γ. The magnitude of nonlinearity is kept fixed $x = 0.19$. One sees that reasonably small nonlinearity reproduces correctly the observable value of the enhancement factor and its energy dependence.

To conclude, we presented here a simple model that accounts for the nonlinear effects in the transition probabilities for the excitation of multi-phonon Giant Dipole Resonances in Coulomb excitation via relativistic heavy ion collisions. The model is based on the group theoretical properties of the boson operators. It allows to construct the solution for the dynamics of the multi-phonon excitation within coupled-channel approach in terms of the generalized coherent states of the corresponding algebras. The harmonic phonon model appears to be a limiting case of the present model when the nonlinearity parameter x goes to zero. The model enjoys the main advantages of the harmonic case (unrestricted multiphonon basis, preservation of unitarity and analytical results in nonperturbative domain). Therefore, this soluble model can be viewed as a natural nonlinear extension of the harmonic phonon model.

The Double GDR excitation probabilities and cross sections are found enhanced by the factors which agree with experiment for reasonably weak nonlinearity x. This can be viewed as a hint that the discrepancy between the measured cross-sections of double GDR and the harmonic phonon calculations can be resolved within present nonlinear model by means of using an appropriate value of the nonlinear parameter x for a given nucleus. The enhancement factor drops as the bombarding energy grows. This is consistent with the data and gives results similar to those recently obtained in a possibly different context, with a theory based on the concept of fluctuations (damping) and the Brink-Axel mechanism [16], [17], [28], [29]. It would be certainly worthwhile to establish possible connections between the two approaches.

The work has been supported by FAPESP (Fundacao de Amparo a Pesquisa do Estado de Sao Paulo).

REFERENCES

[1] H. Emling, Prog. Part. Nucl. Phys. **33**,729 (1994).

[2] C. A. Bertulani, L. F. Canto, M. S. Hussein, A. F. R. de Toledo Piza, Phys.Rev. **C53**,334 (1996).

[3] G. Baur and C.A. Bertulani, Phys. Lett. **B174**,23 (1986); Phys. Rep. **163**,299 (1988); T. Aumann, C.A. Bertulani and K. Sümmerer, Phys. Rev. **C51**,416 (1995).

[4] P. Chomaz and N. Frascaria, Phys. Rep. **252**, 275 (1995).

[5] P.G. Hansen, Nucl. Phys. **A553**,89c (1993); C.A. Bertulani, L.F. Canto and M.S. Hussein, Phys. Rep. **226**,282 (1993).

[6] W.J. Llope and P. Braun-Munzinger, Phys. Rev. **C41**, 2644 (1990).

[7] R. Schmidt et al. Phys. Rev. Lett. **70**,1767 (1993).

[8] J.L.Ritman et al. Phys. Rev. Lett. **70**, 533, 2659 (1993).

[9] T.Aumann et al. Phys. Rev. **C47**,1728 (1993).

[10] K.Boretzky et al., Phys. Lett. **B384**,30 (1996).

[11] J.R. Beene, Nucl. Phys. **A569**,163c (1994).

[12] K. Alder and A. Winther, Coulomb Excitation (Academic Press, New York, 1965).

[13] A. Winther and K. Alder, Nucl. Phys. **A319**,518 (1979).

[14] L.F. Canto, A. Romanelli, M.S. Hussein and A.F.R. de Toledo Piza, Phys. Rev. Lett. **72**,2147 (1994).

[15] J. Norbury and G. Baur, Phys. Rev. **C48**,1915 (1993).

[16] B.V.Carlson, L.F.Canto, S. Cruz-Barrios, M. S. Hussein, and A. F. R. de Toledo Piza, Phys.Rev.Lett., to appear.

[17] B.V.Carlson, M. S. Hussein, and A. F. R. de Toledo Piza, Phys. Lett. **B431**,249 (1998).

[18] C.A. Bertulani and V. Zelevinsky, Phys. Rev. Lett. **71**, 967 (1993).

[19] C. Volpe, F. Catara, Ph. Chomaz, *et al.*, Nucl. Phys. **A 589**,521 (1995).

[20] C. Volpe, P. Chomaz, M.V.Andres, *et al.* Nucl. Phys. **A 599**,C347 (1996).

[21] P.F. Bortignon and C.H.Dasso, Phys. Rev. **C56**, 574 (1997).

[22] F.Catara, Ph. Chomaz and N. Van Giai, Phys. Lett. **B233**,6 (1989).

[23] G. F. Bertsch and H. Feldmeier, Phys.Rev. **C56**,839 (1997).

[24] A.Perelomov, *Generalized Coherent States & Their Applications*, Springer,Berlin, 1986.

[25] A.M. Perelomov, V.S. Popov, ZhETF **56**, 1375 (1969); A.M. Perelomov, Teor. Mat. Fyz. **1**,360 (1969).

[26] J.N. Ginocchio and G. Wenes, in: Relations between Structure & Reactions in Nuclear Physics, Ed. D.H.Feng, M.Vallieres and B.H.Wildenthal, (World Scientific, Singapore, 1987) p.53; F.Iachello, *ibid*, p. 165; Y.Alhassid, F.Gürsey and F.Iachello, Phys. Rev. Lett. **50**,873 (1983).

[27] H. Kleinert, Fortschr. Phys. 26: **11**,565 (1978).

[28] D.M. Brink, J. Neto and H.A. Weidenmüller, Phys. Lett. **B80**, 170 (1979).

[29] B.V.Carlson, M. S. Hussein, and A. F. R. de Toledo Piza, subm. to Phys. Lett. **B**.

Figure Captions

Fig.1.

N- phonon excitation probabilities W_N as compared to W_N^{harm} in the harmonic limit as functions of the phonon number N (schematic plot). One sees that while both W_N and W_N^{harm} are decreasing rapidly as N increases, their ratio W_N/W_N^{harm} is bigger than unity at $N \geq 2$. It is seen that $W_0 < W_0^{harm}$, as unitarity implies that $\sum_{N=0}^{\infty} W_N = \sum_{N=0}^{\infty} W_N^{harm} = 1$.

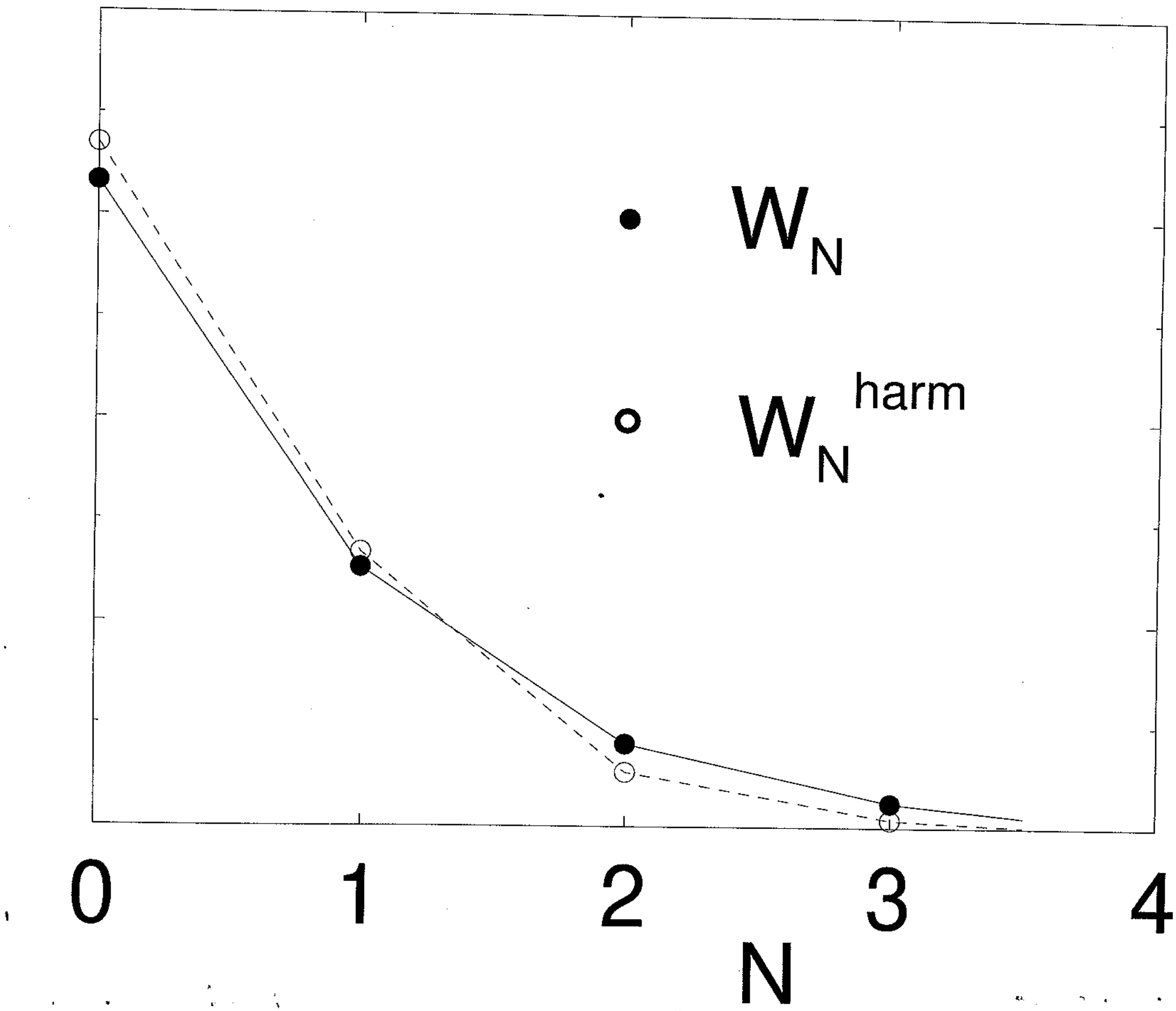
Fig.2.

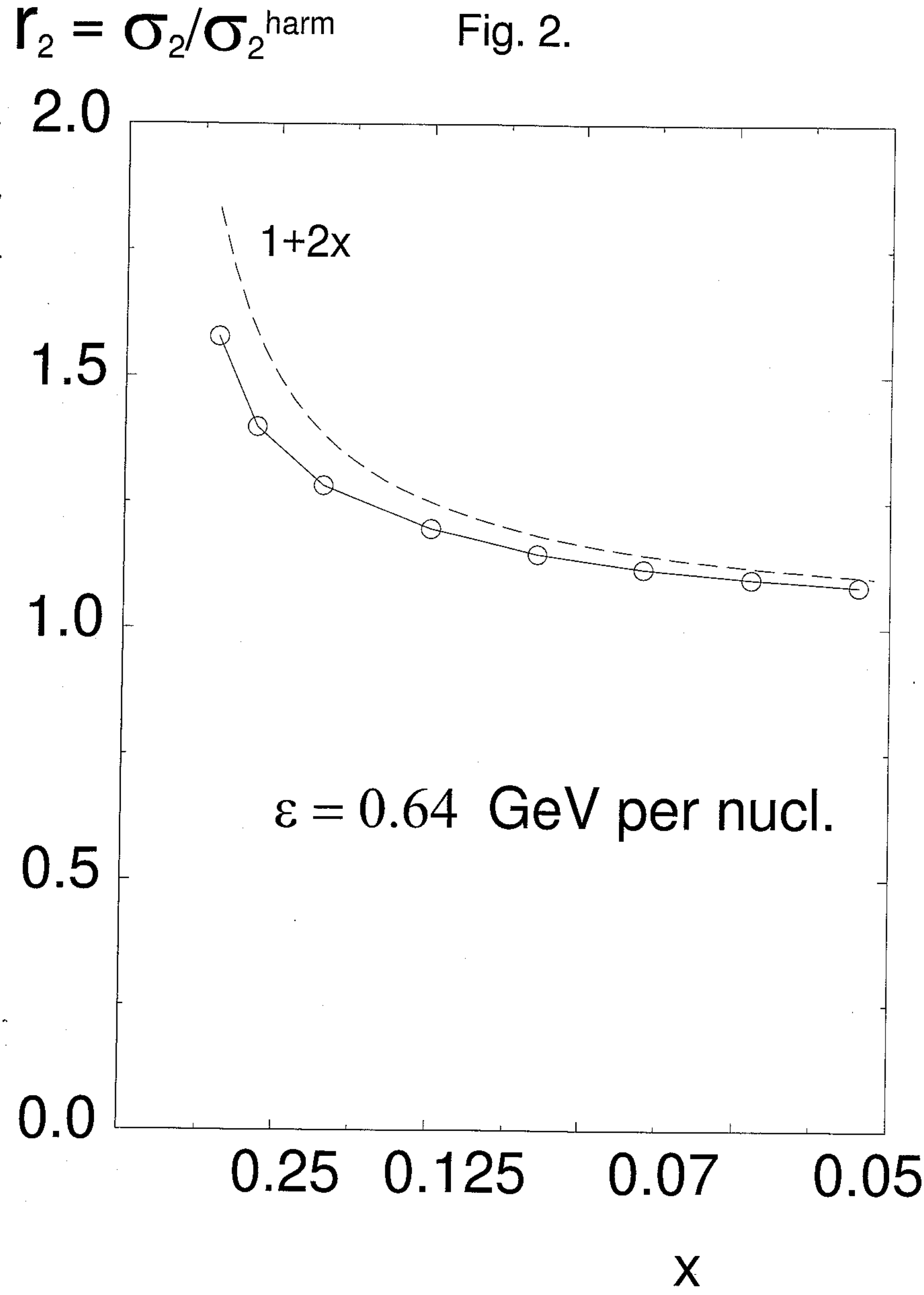
The cross section enhancement factor $r_2 = \sigma_2/\sigma_2^{harm}$ for the Double GDR excitation in ${}^{208}Pb + {}^{208}Pb$ process at bombarding energy $\varepsilon = 640MeV/$per nucleon as a function of the nonlinearity parameter x (circles, solid curve is to guide the eye). The value $1+2x$ is shown by the dashed curve.

Fig.3.

The cross section enhancement factor $r_2 = \sigma_2/\sigma_2^{harm}$ for the Double GDR excitation in the process ${}^{208}Pb + {}^{208}Pb$, as a function of relativistic factor γ (circles, solid curve is to guide the eye). The value of the nonlinear parameter x is kept to be equal to $x = 0.19$. The constant $1+2x$ is shown by dashed line.

W_N *vs.* W_N^{harm} Fig.1





$r_2 = \sigma_2/\sigma_2^{harm}$
Fig. 2.
2.0
1.5
1.0
0.5
0.0
1+2x
$\varepsilon = 0.64$ GeV per nucl.
0.25
0.125
0.07
0.05
x

Fig. 3.