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G_v Experiment in Gravitation

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Publicação IF - 1354/99

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Abstract

G_v and G_h are the Newtonian Constant values, measured using two bodies that are attracted to each other in the laboratory, standing in the vertical and horizontal directions respectively, near the Earth's surface. (The Earth must not be one of the two referred bodies.)

$G_h = (6.67259 \pm 0.00085)\text{x}10^{-11}\text{N m}^2/\text{kg}^2$, Codata value(1).

After having computed 603 data we have measured G_v and we found $G_v = 6.41\ \text{x}10^{-11}\text{N m}^2/\text{kg}^2$. (we can't specify tolerances for this figure, see Experimental Procedure).

This experiment has been carried out in order to measure gravity absorption by using the Earth as a shield, so as to test a new theory from 1995, unknown to date, one which possesses the same main idea as the Kinetic Theory of Gravitation (11), and that is now being called The Island Effect Theory in Gravitation (12). The result we have obtained is favorable to both theories, that is, G_v is less than $G_{h.}$

PACS numbers: 06.20.Jr, 04.80.Cc, 04.80.-y, 04.50.+h

Introdution

Let G_v and G_h be the Newtonian Constant values between bodies which attract each other vertically and horizontally respectively, near the Earth's surface.

Up to now, in the experiments in which the Newton's Constant G has been measured in the laboratory, the bodies attracted to each other were usually placed in the horizontal direction, as for instance in the torsion pendulums.

Let's use the recommended current value for G_h , i.e.,

$G_h = (6.67259 \pm 0.00085)\text{x}10^{-11}\text{N m}^2/\text{kg}^2$, Codata value (1), established in 1986.

This value is based on the following results: Luther and Towler (2), Pontikis (3), and Sagitov et al (4), all of them using horizontally-placed attracting bodies for their experiments, at the laboratory.

Nowadays we face problems in order to establish the gravitational constant value. It has been measured several times, but higher accuracy measurements are mutually exclusive. See Gillies studies 1987 (5), 1997 (6) which are very rich in content and references. See also Sanders and Gillies (1997) (7), and Karagioz et al (1998) (8).

Similar Measurements

In the article "Laboratory calibration of Lacoste-Romberg-type gravimeters by using a heavy cylindrical ring" (9), in which a steel ring of mass 3200kg, possessing an inner diameter slightly larger than the width of the gravity meter to be calibrated is raised and lowered around the gravimeter fitted atop a column , Varga et al observed that the measured values for gravitational field variations differ from theoretical values when the ring is near any of the ends. Notice that in these positions the attracted bodies, i.e., the steel ring and the gravity meter sensor, stand mostly in the vertical rather than in the horizontal orientation.

According to the authors, the nature of these oscillations is not clear at present (1994).

The magnitude of oscillations up to 2 μ Gal (1 μ Gal = $1\text{x}10^{-8}$ m/s^2), both above and below gravity meter level, result in differences of up to 4 μ Gal in the 112 μ Gal range, corresponding to 3.6% of the total, and are in agreement with the difference that we had found, that is 3.9%.

Experimental Method

Our experiment consists in measuring the gravitational field of a lead rectangular prism, (which we will call $g_{l,m}$ "lead measured field", when the rectangular prism's vertical symmetry axis comprises the gravimeter sensor's center of gravity, the latter with respect to the lead prism gravitational field), and then calculating theoretically such a field, which we will call $g_{l,t}$ "lead theoretical field".

For the theoretical calculation we will use the Codata G value which was measured, as mentioned above, from the bodies which attract each other horizontally-wise in the laboratory. Then we will compare values, both the measured and the theoretical one, in order to obtain a comparison between G_v and G_h.

Theoretical value of the lead prism gravitational field $g_{l,t}$

In accordance with both General Relativity and the Newtonian Inverse Square law, a rectangular prism (Fig.1) with uniform density ρ and dimensions described by the limits $x_1 \le x \le x_2$, $y_1 \le y \le y_2$, $z_1 \le z \le z_2$, establishes a <u>vertical</u> attraction at the origin given by

$$g_{l,t} = G\rho \int_z \int_y \int_x (1/r^2).(z/r)dxdydz \qquad (1)$$

where r is the distance from the origin to an element of the rectangular prism dv. It follows then that

$$r = \sqrt{x^2 + y^2 + z^2} \quad , \qquad (2)$$

and

$$g_{l,t} = G\rho \int_{z_1}^{z_2} \int_{y_1}^{y_2} \int_{x_1}^{x_2} z/(x^2 + y^2 + z^2)^{\frac{3}{2}} dxdydz\,. \qquad (3)$$

The Plouff (10) numerical solution yields

$$g_{l,t} = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\mu_{ijk}[a-b-c] \quad , \tag{4}$$

where

$$\mu_{ijk} = (-1)^i(-1)^j(-1)^k \quad , \tag{5}$$

$$a = z_k \arctan(x_i y_j / z_k R_{ijk}) \quad , \tag{6}$$

$$R_{ijk} = \sqrt{x_i^2 + y_j^2 + z_k^2} \quad , \tag{7}$$

$$b = x_i \ln(R_{ijk} + y_j) \quad , \text{ and} \tag{8}$$

$$c = y_j \ln(R_{ijk} + x_i) \quad . \tag{9}$$

See example software at Appendix A, which provides a C++ subroutine to calculate equation 4.

Measured value of lead prism gravitational field $g_{l,m}$

We have placed the gravimeter over a glass table in the laboratory (Fig.2). The lead rectangular prism could move to either below the gravity meter or far away from it, so that it wouldn't interfere with the gravitational field value indicated by the gravity meter.

Calling g' the value read when the lead prism is centered under the gravity meter, and calling g the value read without the lead prism, then the difference

g'- g = $g_{l,m}$ is the measured lead prism gravitational field.(We are using the superposition principle here).

Experimental Procedure

In order to illustrate the experimental procedure let's consider the data collected on February 4, 1998 at the Institute of Physics of the University of São Paulo(IFUSP).

Dr.Wladimir Shukowsky (from IAGUSP - Institute of Astronomy and Geophysics of the University of São Paulo) and I placed the LCR-G996 (Lacoste-Romberg) gravity meter, calibrated by the manufacturer, on a (8.0$\pm$0.1)mm-thick glass table, free from vibrations.

The lead rectangular prism on a wooden cart, which could move along rails, was initially (3.475 $\pm$ 0.002)m far from the position which it would occupy later, centered under the gravity meter, so that its vertical symmetry axis comprised the gravimeter sensor's center of gravity. At this starting position the lead prism <u>vertical</u> contribution was 0.10 μ Gal, and the gravimeter precision was 1 μ Gal, so that it could be considered negligible.

The rectangular prism mass is (1306.2 ± 1.3)kg, and such a prism is made up of 15 layers with 8 lead bricks each, totalling 120 bricks. We would like to stress that each brick has been weighed separately.

At 9 a.m.(local daylight saving time) the gravimeter was leveled, turned on and unlocked. At 9:12 a.m. (11:12 a.m. Greenwich time) we made the first reading, obtaining 0.233 μ Gal. This value was multiplied by the gravity meter correction factor 1012.139, leading to 235.8 μ Gal, and then the lunisolar tide -93.11 μ Gal (Longman Model with Lat. -23.5587deg and Long.-46.7330deg) was subtracted, obtaining 329 μ Gal for the Earth's gravitational field (This constitutes a small fraction of the Earth's gravitational field which is about $9.8\text{x}10^8$ μ Gal).

Having kept the lead prism far away, other 25 readings were made (table 1).

At 10:13a.m. the lead prism was set and centered (i.e., with its vertical axis of symmetry comprising the gravimeter sensor's center of gravity) under the gravimeter and 9 other readings were made, corresponding now to the resulting value from each measurement added to the gravitational fields of both the lead prism and the Earth. Then the lead prism was removed again and the procedure went on according to table 1.

Having measured the values, which totalled 69 on that day, we plotted the graphics on figures 4, 5, and 6.

Within the first half hour after turning on the gravimeter it isn't advisable to make any readings, because the gravimeter needs that amount of time for relaxation and also for it to stabilize, especially regarding its internal temperature.

Considering the data from table 1, and starting from the gravimeter stabilization phase, i.e., from 9:34 a.m., we obtained the graphics of figures 5 and 6.

What we accomplished on that day was the following:

Gravitational field for the lead prism:

measured: $g_{l,m} = (46.2 \pm 0.3)\ \mu$ Gal ,

theoretical: $g_{l,t} = 48.2\ \mu$ Gal ,

difference: 4.1% with respect to 48.2 μ Gal.

We can't obtain the associated error for the theoretically-obtained value 48.2 μ Gal. Let us try to understand why.

On that day, the distance between the gravimeter sensor's center of gravity and the upper base of the rectangular prism was $z_1 = 12.2$cm.

For the theoretical value calculation on that day, we used equation 4 by means of the attached C++ subroutine of the Appendix, at the values:

$x_1 = (-0.201 \pm 0.001)$m ; $x_2 = (0.201 \pm 0.001)$m ;

$y_1 = (-0.201 \pm 0.001)$m ; $y_2 = (0{,}201 \pm 0.001)$m ;

$z_1 = 0.122$m ; $z_2 = 0.867$m;

$\rho = (10.885 \pm 0.055)\mathrm{x}10^3 \mathrm{kg/m}^3$ (average lead density).

Now, what most interferes with the theoretical value error (48.2 μ Gal) is the z_1 value, and in order to calculate this coordinate we make use of a 4cm distance, measured from the outer surface of the lower base of the gravimeter box to the gravimeter sensor's center of gravity, with respect to the gravitational field from the lead prism, and we can't estimate tolerances for this.

To quote Varga et al (our reference number 9): *At the Institute of Geophysics and Planetary Physics, University of California in Los Angeles, an attempt dated from 1974 to calibrate LCR-gravimeters suspended in a frame by approaching rectangular lead masses was abandoned because of this problem*

If this distance would be exactly 4cm then our final result would have been $G_v = (6.41 \pm 0.07) \times 10^{-11} N\, m^2/kg^2$, corresponding to the difference $(3.9 \pm 1.0)\%$ between G_v and G_h.

In order to minimize the problem which is caused by such 4cm uncertainty, we have performed measurements for several days, from February 2, 1998 to August 6, 1998, varying z_1 values considerably from 9.5cm to 34.3cm (table 2), and we kept on obtaining the same difference between both theoretical and measured values for the lead prism gravitational field, which shows us that the referred value of 4cm from inside the gravimeter fits well here.

Other z_1 values corresponding to measurements from other days are in table 2, as well as other results including the final one.

Main error sources

(1)z_1 is the most important one as we have already stated.

(2)The microseismic noise is another very important error source. Having computed 603 readings distributed along different days, we have significantly minimized this problem. For example, on August 4,1998 the readings oscillated considerably (from 100 μ Gal to 900 μ Gal) due to an earthquake that took place in Ecuador. On that day it wasn't possible to make any readings.

(3)The lunisolar influence was overridden as mentioned above.

(4)The table over which the gravimeter is placed mustn't oscillate. On December 10, 1997 we performed some measurements that had to be disconsidered, because the table the gravimeter was on had a frequency vibration of its own, imperceptible without instrumentation, though revealed by the gravimeter. Without such vibrations, we came to extremely accurate measurements.

(5)Instrumental drift can be easily removed.

(6)Human presence didn't produce any effects which we could measure.

(7)We didn't take into account possible electric or magnetic fields, as the gravity meter LCR-G996 does not suffer any interference from these kinds of field.

(8)Temperature effects weren't measured either because should these effects be detected, they would have been quite below our accuracy levels.(See Gillies 1987).

(9)Another source of error could be the horizontal position of the rectangular prism with respect to the gravimeter sensor, when the prism is under the gravimeter. However, around a 3cm radius, the greatest variation for the lead gravitational field is 0.20 μ Gal, and gravimeter precision is 1 μ Gal. Furthermore, the cart that transports the lead prism moves over rails, and should there be any horizontal displacement at the prism position it would have been less than 2mm.

Conclusions

In table 2 we have the results for seven different days of measurements as well as our final result.

If the distance referred to above is exactly 4cm at the insides of the gravimeter then our results shall be followed by the tolerances $(3.9 \pm 1.0)\%$ between G_v and G_h, or more appropriately

$G_v = (0.961 \pm 0.010)\, G_h$, which yields

$G_v = (6.41 \pm 0.07) \times 10^{-11} \mathrm{N\, m^2/kg^2}$, with several very important consequences.

This experiment was carried out in order to test the new Island Effect Theory in Gravitation (12), that has the same central idea as that of the Kinetic Theory of Gravitation (11), and the result was favorable to these theories, i.e. $G_v < G_h$.

When the two bodies which are attracted to each other are in the vertical direction in the laboratory, it follows from these theories that we are using the Earth as a shield to the waves which cause the gravitational attraction between the referred bodies.

When G_h is measured according to these theories, there is also absorption imposed by the Earth, but such absorption is lower than when G_v is measured, and so we can conclude that despite G_h being greater than G_v, Newton's Constant G values greater than G_h should exist also. And this is shown in free fall experiments in which the Earth is one of the attracting bodies. (Schwarz et al 1998 and Hsui 1987) (13).

References and Notes

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14. We are grateful to Prof. Marta Mantovani for the gravimeter loan, to Mr. José Augusto Nasser (UNIP's director), and also to Prof. Cesar Lattes for all of them having encouraged us to perform this experiment. It is our desire to give many thanks to those who have greatly contributed to this work, especially Dr. Wladimir Shukowsky, whose theoretical support have been invaluable. Mr. Sergio S.Jabur,

Dr.Carlos E.I. Carneiro, Prof. Andre K.Assis, Dr. Roberto de A.Martins, Danilo Macsoud, Mr. Eduardo M.Leite, Prof. Heraldo Barbui, Prof. Roberto Mallet, Mr.Olivaldo Pereira, Prof. Giorgio Moscatti, Mrs. Sueli M.do Amaral, and Mr. Wram R.de C.Accorsi. Sponsored by UNIP (Universidade Paulista - Paulista University).

Fig.1. A rectangular prism and the coordinates system.

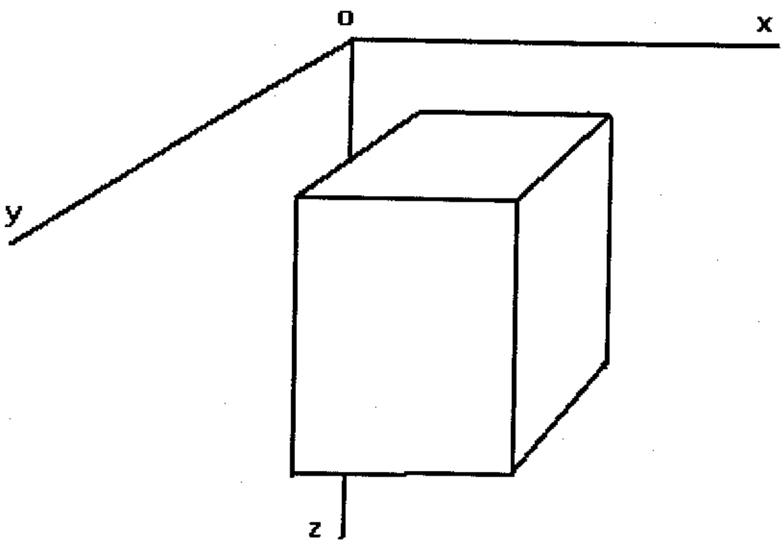
o
x
y
z

Fig.2. Gravity meter "A" placed on a glass table free from vibrations, and the lead rectangular prism at positions "B' and "C" respectively, which can move over a wooden cart on rails. When the lead prism is at position "B" its vertical axis of symmetry comprises the gravimeter sensor's center of gravity, and at position "C" the vertical gravitational field contribution from the lead prism isn't indicated by the gravity meter.

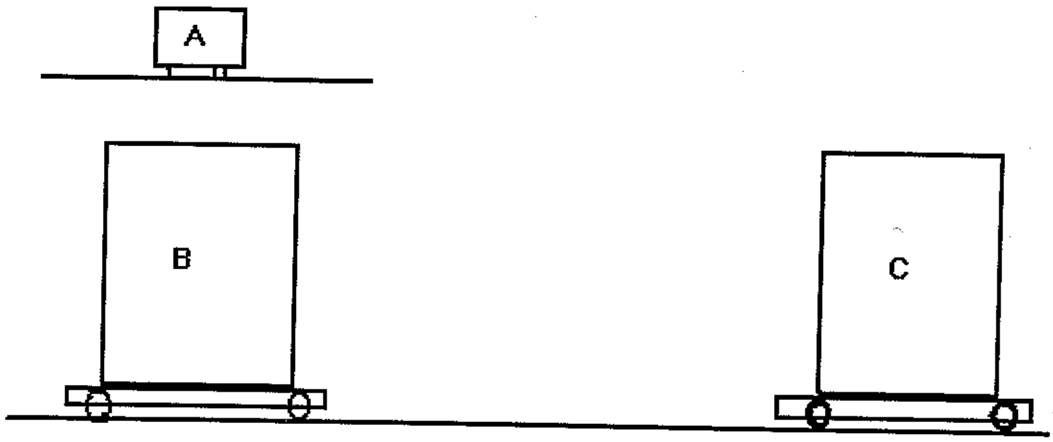
A
B
C

Fig.3.The lead rectangular prism on that day (Oct.6, 1998),comprising less than fifteen (only ten) layers of lead bricks over a wooden cart on rails, the gravity meter LCR-G996 on a vibration-free glass table, and the readings being done.

Fig.4. Shows all non-lead readings made on Feb.4, 1998, including the ones that took place within the first half an hour in which the gravimeter was stabilizing.

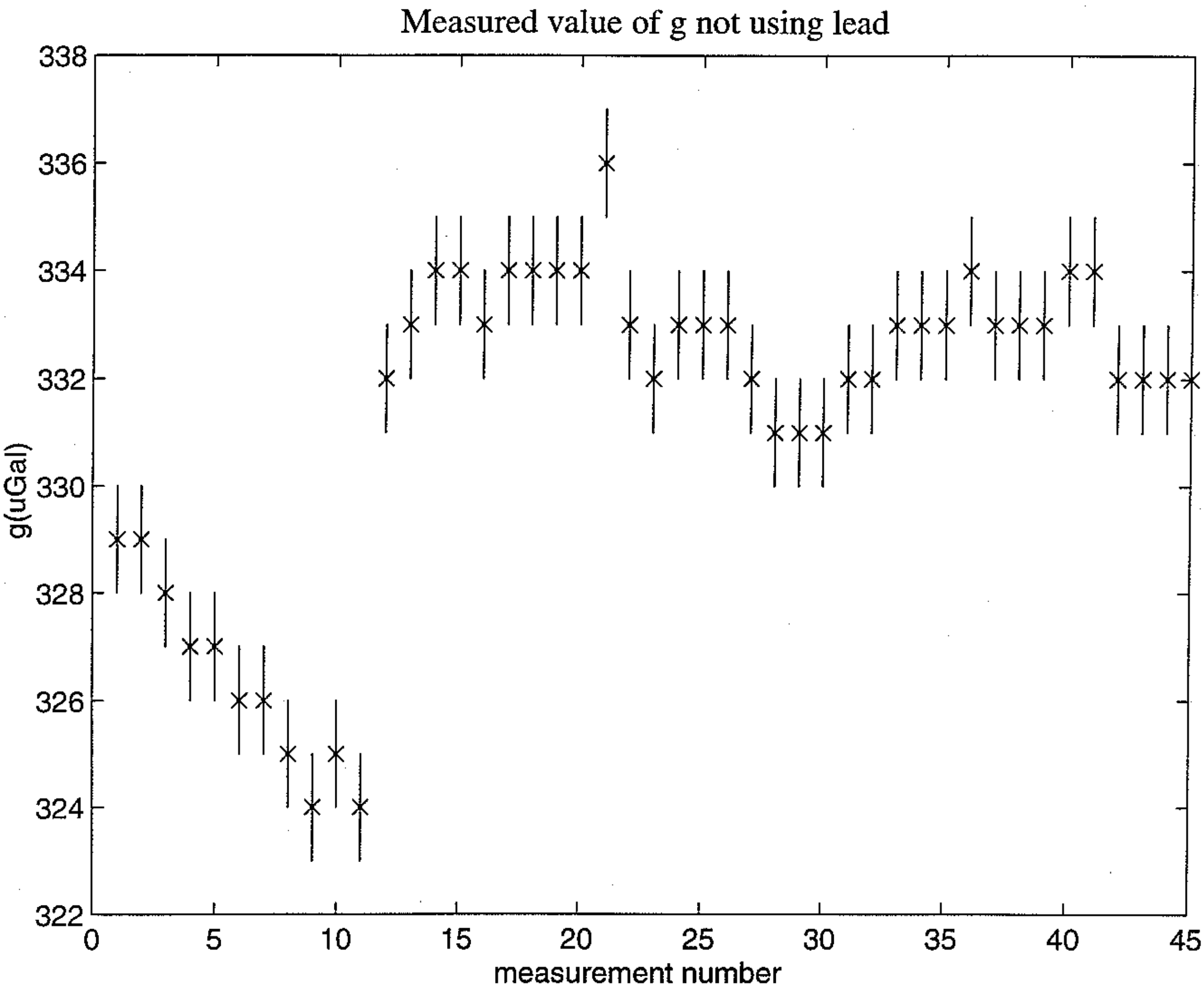
Measured value of g not using lead
g(uGal)
338
336
334
332
330
328
326
324
322
0
5
10
15
20
25
30
35
40
45
measurement number

Fig.5. Readings made on Feb.4, 1998, not using lead, and excluding the ones that took place within the first half an hour in which the gravimeter was stabilizing. The result of these readings is $g = (332.9 \pm 0.2)\times10^{-8}$ m/s^2.

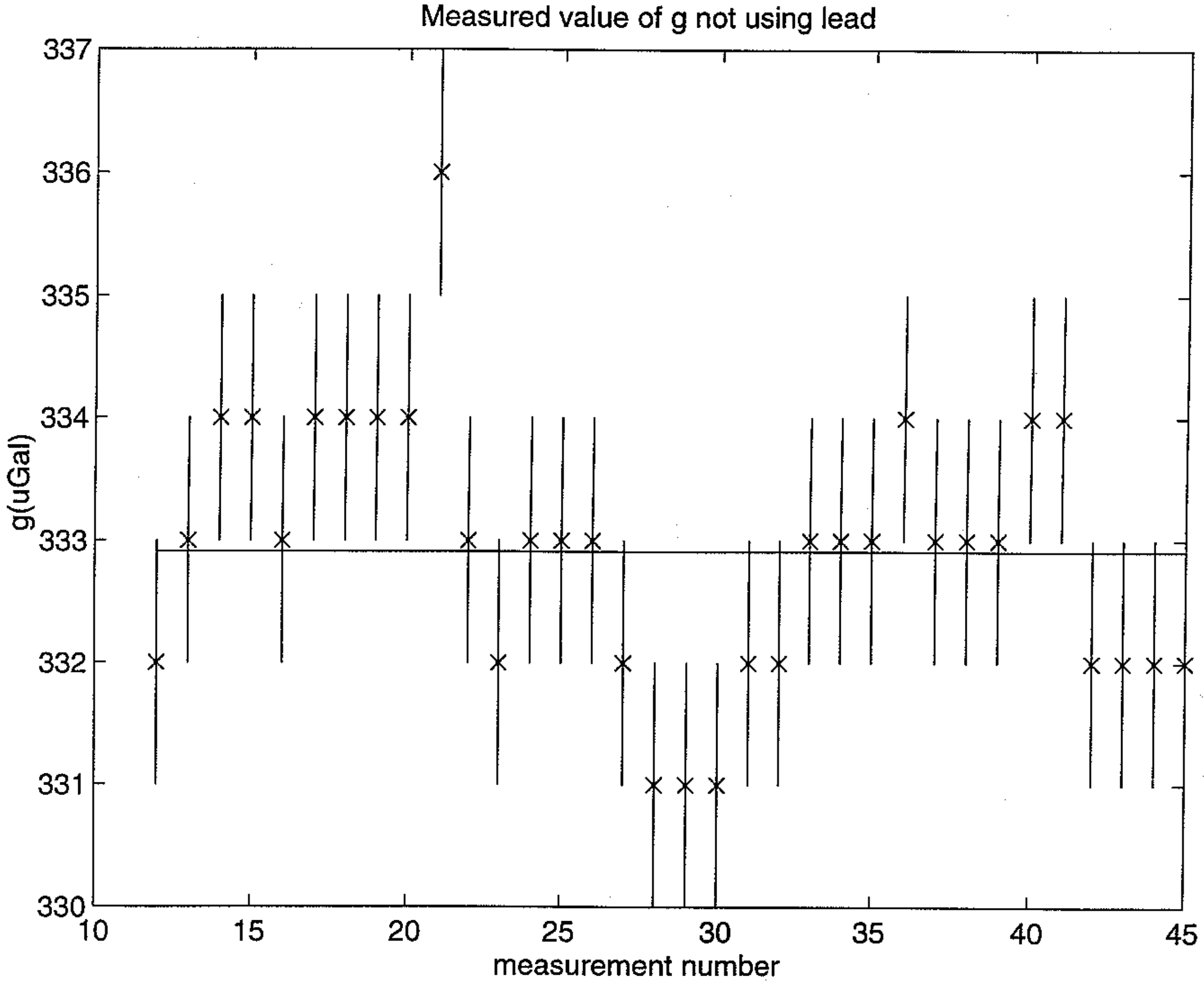
Measured value of g not using lead
g(uGal)
337
336
335
334
333
332
331
330
10
15
20
25
30
35
40
45
measurement number

Fig.6. All the readings made using lead on Feb.4, 1998.

The result of these readings is $g' = (379.1 \pm 0.2) \times 10^{-8}\ m/s^2$.

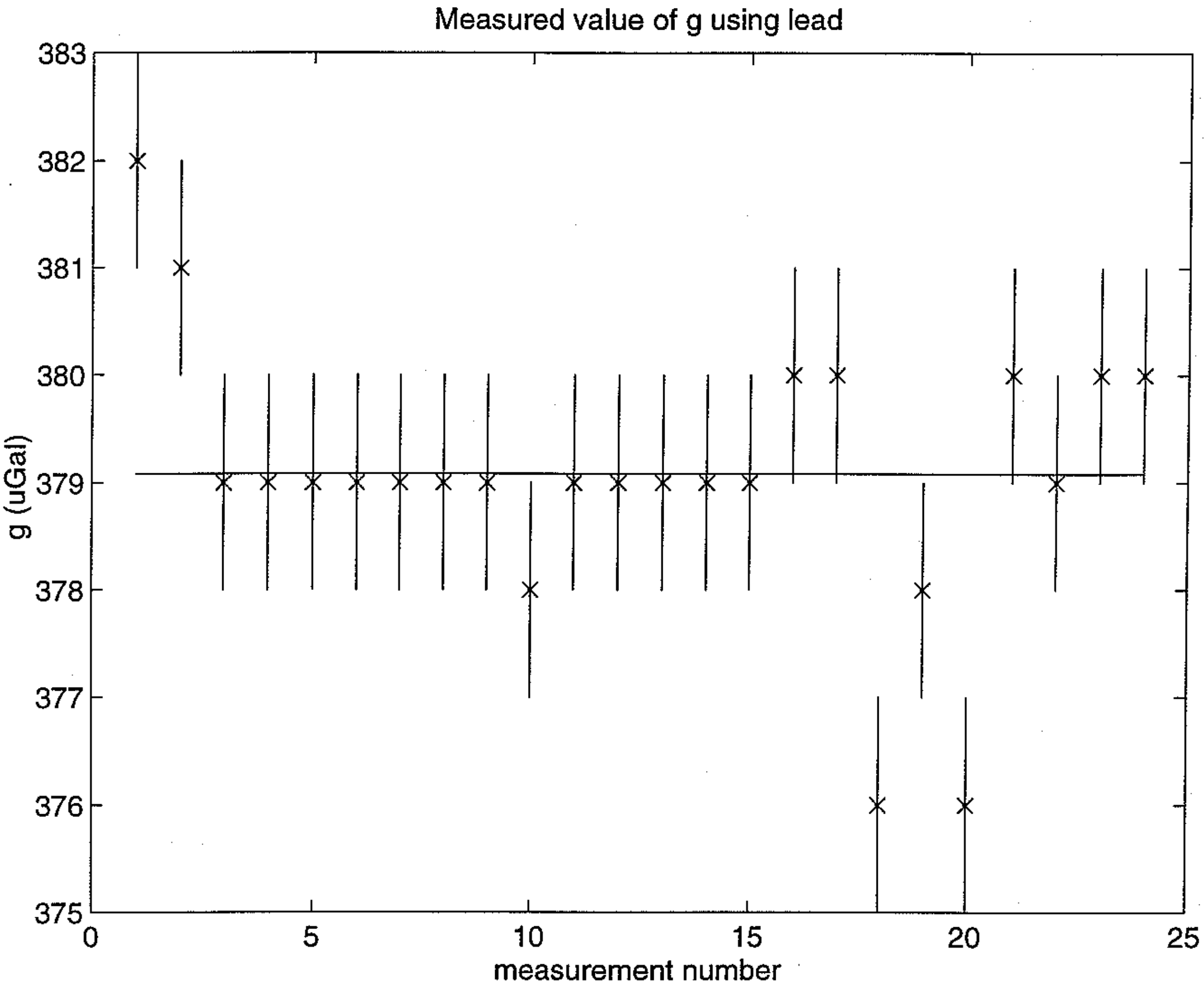
Measured value of g using lead
g (uGal)
measurement number
383
382
381
380
379
378
377
376
375
0
5
10
15
20
25

Table 1. All readings made on February 4, 1998. (1 μ Gal = $1x10^{-8}$ m/s^2).

Local time	Reading	Lunisolar tide(μ Gal)	Results(μ Gal) Not using lead	Using lead
	A	B	C	C
09:12:00am	0.233	-93.11	329	
09:13:00am	0.233	-92.86	329	
09:14:00am	0.233	-92.62	328	
09:15:00am	0.232	-92.37	327	
09:17:00am	0.232	-91.86	327	
09:19:00am	0.232	-91.34	326	
09:21:00am	0.232	-90.81	326	
09:23:00am	0.232	-90.27	325	
09:26:00am	0.232	-89.44	324	
09:28:00am	0.233	-88.87	325	
09:30:00am	0.233	-88.29	324	
09:34:00am	0.242	-87.10	332	

09:37:00am	0.244	-86.19	333
09:39:00am	0.245	-85.57	334
09:41:00am	0.246	-84.93	334
09:43:00am	0.246	-84.29	333
09:45:00am	0.247	-83.64	334
09:47:00am	0.248	-82.98	334
09:53:00am	0.250	-80.95	334
09:57:00am	0.251	-79.55	334
09:59:00am	0.254	-78.84	336
10:02:00am	0.252	-77.76	333
10:03:00am	0.252	-77.39	332
10:07:00am	0.254	-75.92	333
10:10:00am	0.255	-74.79	333
10:13:00am	0,256	-73.65	333

THE LEAD PRISM WAS PLACED UNDER THE GRAVITY METER

10:15:00am	0.305	-72.88	382
10:17:00am	0.305	-72.10	381
10:18:00am	0.304	-71.71	379
10:21:00am	0.305	-70.53	379

10:23:00am	0.306	-69.73	379
10:25:00am	0.306	-68.93	379
10:26:00am	0.307	-68.53	379
10:28:00am	0.308	-67.72	379
10:30:00am	0.308	-66.90	379

THE LEAD PRISM WAS TAKEN AWAY

10:34:00am	0.2635	-65.25	332
10:37:00am	0.264	-64.01	331
10:39:00am	0.265	-63.17	331
10:41:00am	0.2655	-62.33	331
10:43:00am	0.2675	-61.48	332
10:45:00am	0.2685	-60.63	332
10:50:00am	0.2715	-58.49	333
10:52:00am	0.2725	-57.62	333
10:59:00am	0.2755	-54.58	333
11:01:00am	0.2765	-53.70	334
11:03:00am	0.277	-52.83	333
11:05:00am	0.278	-51.95	333

THE LEAD PRISM WAS PLACED UNDER THE GRAVITY METER

11:09:00am	0.324	-50.18	378
11:10:30am	0.3255	-49.52	379
11:12:00am	0.326	-48,85	379
11:16:00am	0.3275	-47.08	379
11:18:00am	0.3285	-46.19	379
11:21:00am	0.3305	-44.86	379
11:28:00am	0.3345	-41.74	380
11:30:00am	0.335	-40.85	380

THE LEAD PRISM WAS TAKEN AWAY

11:33:00am	0.290	-39.52	333
11:35:00am	0.2915	-38.63	334
11:44:00am	0.296	-34.66	334
12:04:00pm	0.3025	-26.00	332
12:06:00pm	0.303	-25.15	332
12:08:00pm	0.304	-24.31	332
12:13:00pm	0.306	-22.22	332

THE LEAD PRISM WAS PLACED UNDER THE GRAVITY METER

12:18:00pm	0.352	-20.16	376
12:20:00pm	0.354	-19.34	378
12:22:00pm	0.3535	-18.53	376
12:25:00pm	0.358	-17.33	380
12:26:00pm	0.358	-16.93	379
12:28:00pm	0.359	-16.14	380
12:31:00pm	0.360	-15.74	380

Table 2. General Results. Column A shows the difference between measured and theoretical values for the Newtonian Constant G, and in all of them the measured value was less than the theoretical one. Column B shows the distance from the upper base of the rectangular lead prism to the gravitmeter's sensor center of gravity. Column C shows the number of lead brick layers that constitute the rectangular prism.

	A	B	C
	(100ΔG/G)	(cm)	(layers)
February 4,1998	4.1	$z_1 = 12.2$	15
July 30,1998	2.7	$z_1 = 9.5$	15
July 31,1998	5.4	$z_1 = 14.5$	15
October 5,1998	5.1	$z_1 = 24.3$	12
October 6,1998	3.2	$z_1 = 34.3$	10
October 6,1998	3.4	$z_1 = 29.3$	11
October 6,1998	3.7	$z_1 = 24.3$	12
Final Result	(3.9 ± 1.0)		

APPENDIX A

C++ SUBROUTINE

```
#include <iostream.h>
#include <math.h>
int main(void)
{
      int i,j,k,m;
      double gpb,dens,tens,x[3],y[3],z[3],Rijk,a,b,c,d[9];
      double soma = 0;
      const double G = 6.67259e-11 ;
      int pc = 2;

    while(pc == 2)
    {
      cout << "give x[1] meter value"  << "\n";
      cin >> x[1];
      cout << "give x[2] meter value"  << "\n";
      cin >> x[2];
      cout << "give y[1] meter value"  << "\n";
      cin >> y[1];
      cout << "give y[2] meter value"  << "\n";
      cin >> y[2];
      cout << "give z[1] meter value"  << "\n";
      cin >> z[1];
      cout << "give z[2] meter value"  << "\n";
      cin >> z[2];
      cout << "give the rectangular prism density"   << "\n";
      cout << "in kilogram by cubic meter"   << "\n";
      cin >> dens;

      m=0;
      for(i=1; i<3; i++)
      for(j=1; j<3; j++)
      for(k=1; k<3; k++)
      {
         m++;
         if((i+j+k)%2 == 0)
            tens = 1;
         else
            tens = -1;

         Rijk = sqrt(x[i]*x[i] + y[j]*y[j] + z[k]*z[k]);
         a = z[k] * atan(x[i]*y[j]/(z[k]*Rijk));
         b = x[i]* log(Rijk + y[j]);
         c = y[j]* log(Rijk + x[i]);
         d[m] = tens*(a-b-c);
         soma = soma + d[m];
      }
         gpb = 1e8 * G * dens * soma;
         cout << "gpb = " << gpb << " microGal" << "\n";
         soma = 0;
         cout <<"stop(1) or continue(2)"  << "\n";
```

```
        cin >> pc;
    }
        return(0);
}
```