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General structure of the graviton self-energy

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The graviton self-energy at finite temperature depends on fourteen structure functions. We show that, in the absence of tadpoles, the gauge invariance of the effective action imposes three non-linear relations among these functions. The consequences of such constraints, which must be satisfied by the thermal graviton self-energy to all orders, are explicitly verified in general linear gauges to one loop order.

The non-linear relation imposed by gauge invariance on the thermal self-energy of gluons, has been recently discussed by Weldon in an interesting paper [1]. He proved that in QCD, the Slavnov-Taylor identities [2,3] require a non-linear constraint among the structure functions which occur at finite temperature. In this brief report, we show that a similar behavior occurs in the gauge theory of gravity. In this case, local gauge invariance leads to three non-linear relations which restrict the form of the thermal self-energy of gravitons.

The Einstein theory of gravity is described by the Lagrangian density [4-6]

$$\mathcal{L} = \frac{2}{\kappa^2} \sqrt{-g} R, \quad (1)$$

where $\kappa^2 = 32\pi G$, G is the Newton constant and R is the Ricci scalar. The graviton field $h_{\mu\nu}$ can be defined in terms of the metric tensor as

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \quad (2)$$

Using this parametrization, the Einstein action will be invariant under the gauge transformation [7]

$$\begin{aligned} \delta h_{\mu\nu} &= [\delta_\mu^\lambda \partial_\nu + \delta_\nu^\lambda \partial_\mu + \kappa (h_\mu^\lambda \partial_\nu + h_\nu^\lambda \partial_\mu + \partial^\lambda h_{\mu\nu})] \xi_\lambda \\ &\equiv G_{\mu\nu}^{(0)\lambda} \xi_\lambda, \end{aligned} \quad (3)$$

where ξ_λ is an infinitesimal gauge parameter.

In this gauge theory, the corresponding identities which occur at finite temperature, differ from those at $T = 0$ because of the appearance of one-particle graviton functions (tadpoles). Their thermal contribution is purely leading, being proportional to $T^{2(n+1)}$ at the n -loop order. Hence, we may assume that the tadpoles are important, in the Ward identities, only for the leading thermal contributions to the graviton self-energy. (To one-loop order, for example, the tadpoles can be neglected for the purpose of studying the sub-leading T^2 , $\log(T)$ and $T = 0$ contributions).

At finite temperature, the graviton self-energy may depend on the four-velocity u_α of the plasma, so that

it can be a linear combination of the 14 independent tensors given in Table I. This contains three traceless tensors $T_{\alpha\beta,\mu\nu}^A$, $T_{\alpha\beta,\mu\nu}^B$ and $T_{\alpha\beta,\mu\nu}^C$, which are transverse with respect to the wave 4-vector k_μ . They are also, respectively, completely transverse, partially transverse and longitudinal with respect to the spatial component \vec{k} [8]. These tensors depend individually on the plasma four-velocity, but their sum is a Lorentz covariant tensor which is independent of u_α

$$\begin{aligned} (T^A + T^B + T^C)_{\alpha\beta,\mu\nu} &= \\ \frac{1}{2} (P_{\alpha\mu} P_{\beta\nu} + P_{\alpha\nu} P_{\beta\mu}) - \frac{1}{3} P_{\alpha\beta} P_{\mu\nu}, \end{aligned} \quad (4)$$

where $P_{\alpha\beta} = \eta_{\alpha\beta} - k_\alpha k_\beta / k^2$.

In terms of this basis, which is convenient for our purpose, the graviton self-energy can be parametrized as

$$\begin{aligned} \Pi_{\alpha\beta,\mu\nu} &= \Pi_A T_{\alpha\beta,\mu\nu}^A + \Pi_B T_{\alpha\beta,\mu\nu}^B + \Pi_C T_{\alpha\beta,\mu\nu}^C \\ &+ \sum_{i=4}^{14} \Pi_i T_{\alpha\beta,\mu\nu}^i \end{aligned} \quad (5)$$

$$\begin{aligned} T_{\alpha\beta,\mu\nu}^{1,2,3} &= T_{\alpha\beta,\mu\nu}^{A,B,C} \\ T_{\alpha\beta,\mu\nu}^4 &= \eta_{\alpha\beta} \eta_{\mu\nu} \\ T_{\alpha\beta,\mu\nu}^5 &= u_\mu u_\nu \eta_{\alpha\beta} + u_\alpha u_\beta \eta_{\mu\nu} \\ T_{\alpha\beta,\mu\nu}^6 &= [u_\beta (k_\nu \eta_{\alpha\mu} + k_\mu \eta_{\alpha\nu}) + k_\beta (u_\nu \eta_{\alpha\mu} + u_\mu \eta_{\alpha\nu}) + \\ &u_\alpha (k_\nu \eta_{\beta\mu} + k_\mu \eta_{\beta\nu}) + k_\alpha (u_\nu \eta_{\beta\mu} + u_\mu \eta_{\beta\nu})] / k \cdot u \\ T_{\alpha\beta,\mu\nu}^7 &= [k_\nu u_\alpha u_\beta u_\mu + k_\mu u_\alpha u_\beta u_\nu + \\ &k_\beta u_\alpha u_\mu u_\nu + k_\alpha u_\beta u_\mu u_\nu] / k \cdot u \\ T_{\alpha\beta,\mu\nu}^8 &= [k_\beta k_\nu \eta_{\alpha\mu} + k_\beta k_\mu \eta_{\alpha\nu} + k_\alpha k_\nu \eta_{\beta\mu} + k_\alpha k_\mu \eta_{\beta\nu}] / k^2 \\ T_{\alpha\beta,\mu\nu}^9 &= [k_\mu k_\nu u_\alpha u_\beta + k_\alpha k_\beta u_\mu u_\nu] / k^2 \\ T_{\alpha\beta,\mu\nu}^{10} &= (k_\beta u_\alpha + k_\alpha u_\beta) (k_\nu u_\mu + k_\mu u_\nu) / (k \cdot u)^2 \\ T_{\alpha\beta,\mu\nu}^{11} &= [k_\beta k_\mu k_\nu u_\alpha + k_\alpha k_\mu k_\nu u_\beta + \\ &k_\alpha k_\beta k_\nu u_\mu + k_\alpha k_\beta k_\mu u_\nu] / (k^2 k \cdot u) \\ T_{\alpha\beta,\mu\nu}^{12} &= (k_\alpha k_\beta k_\mu k_\nu) / k^4 \\ T_{\alpha\beta,\mu\nu}^{13} &= (k_\mu k_\nu \eta_{\alpha\beta} + k_\alpha k_\beta \eta_{\mu\nu}) / k^2 \\ T_{\alpha\beta,\mu\nu}^{14} &= [(k_\nu u_\mu + k_\mu u_\nu) \eta_{\alpha\beta} + (k_\beta u_\alpha + k_\alpha u_\beta) \eta_{\mu\nu}] / (k \cdot u) \end{aligned}$$

TABLE I. A basis of 14 independent tensors.