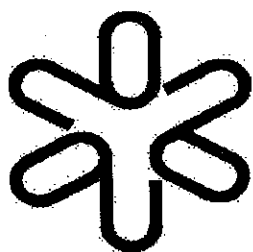


SBI/IFUSP  
BASE: 04  
SYS N°: 10804 14



Instituto de Física  
Universidade de São Paulo

**Behavior of logarithmic branch cuts in the self-energy of gluons at finite temperature**

Brandt, F. T.; Frenkel, J.

*Instituto de Física, Universidade de São Paulo, SP, Brasil*

**Publicação IF - 1375/99**

# Behavior of logarithmic branch cuts in the self-energy of gluons at finite temperature

F. T. Brandt and J. Frenkel

*Instituto de Física, Universidade de São Paulo*

*São Paulo, SP 05315-970, BRAZIL*

(June 16, 1999)

## Abstract

We give a simple argument for the cancellation of the  $\log(-k^2)$  terms ( $k$  is the gluon momentum) between the zero-temperature and the temperature-dependent parts of the thermal self-energy.

There have been many studies of thermal Green functions in gauge field theories [1-7], which show that their behavior at finite temperature is rather different from the one at zero temperature. In particular, it was recently pointed out by Weldon [8] that in QED, the logarithmic branch cut singularities cancel to one loop-order, in the thermal self-energy of the electron.

The purpose of this note is to show that in the Yang-Mills theory, a somewhat similar behavior occurs in the full gluon self-energy, which includes finite temperature effects. Of course, in this theory, the massless gluons are quite modified by these effects and the gluon propagator requires the Braaten-Pisarski resummation. Nevertheless, it is interesting to remark that, even before such a procedure is carried out, the one-loop  $\log(-k^2)$  terms cancel in the sum of the  $T = 0$  and the  $T \neq 0$  contributions to the gluon self-energy. As we shall see, this happens because the  $\log(-k^2)$  terms appear in the thermal part of the self-energy only in the combination  $\log(-k^2/T^2)$ . But one can show that the  $\log(T^2)$  contributions have the same structure as the ultraviolet divergent terms which occur at zero temperature [9].

Consequently, the  $\log(-k^2/T^2)$  terms combine directly with the  $\log(-k^2/\mu^2)$  contributions which occur at  $T = 0$  ( $\mu$  is the renormalization scale), so that the  $\log(-k^2)$  terms cancel in a simple way in the thermal self-energy of the gluon. The branch cut in the  $\log(-k^2)$  contribution at  $T = 0$  is associated with the imaginary part of the self-energy, which gives the rate of decay of a time-like virtual gluon into two real gluons. Although this contribution cancels at  $T \neq 0$ , there appear then additional, temperature-dependent logarithmic branch points. These singularities indicate processes not available at zero temperature, where particles decay or are created through scattering in the thermal bath.

To *one-loop* order, the thermal self-energy of gluons generally depends on three structure functions,  $\Pi^T$ ,  $\Pi^L$  and  $\Pi^C$  [10]

$$\Pi_{\mu\nu}^{ab}(k_0, \vec{k}) = g^2 C_G \delta^{ab} \left( \Pi^T P_{\mu\nu}^T + \Pi^L P_{\mu\nu}^L + \Pi^C P_{\mu\nu}^C \right), \quad (1)$$

where the projection operators  $P_{\mu\nu}^{T,L}$  are transverse with respect to the external four-momentum  $k^\mu$  and satisfy:  $k^i P_{i\nu}^T = 0$  and  $k^i P_{i\nu}^L \neq 0$  [6,7]. Furthermore, the projection operator  $P_{\mu\nu}^C$  can be written in the plasma rest frame as follows [10]

$$P_{\mu\nu}^C = \frac{1}{k^2} \left[ \frac{k_\nu}{|\vec{k}|} \left( k_0 k_\mu - \eta_{\mu 0} k^2 \right) + \mu \leftrightarrow \nu \right]. \quad (2)$$

Although  $\Pi^C$  vanishes at  $T = 0$  because of the Slavnov-Taylor identity, it is in general a non-vanishing function of the temperature, so that  $k^\mu \Pi_{\mu\nu} \neq 0$  for the exact self-energy.

We will discuss here, for definiteness, the retarded thermal self-energy of the gluon, which is obtained by the analytic continuation  $k_0 \rightarrow k_0 + i\epsilon$ . (A rather similar analysis can be made in the case of time-ordered self-energy, following the approach presented in reference [11]). In order to illustrate in a simple way the mechanism of the cancellation of the  $\log(-k^2)$  contributions, let us first consider the special case of the *Feynman gauge*, where  $\Pi^C$  vanishes even at finite temperature. Then,  $\Pi^T$  and  $\Pi^L$  can be expressed in the plasma rest frame in terms of linear combinations of  $\Pi_\mu^\mu$  and  $\Pi_{00}$ . After performing the integration over the internal energies  $q_0$ ,  $\Pi_\mu^\mu$  and  $\Pi_{00}$  can be written as an integral over internal on-shell momenta  $q = (|\vec{q}|, \vec{q})$ , as follows

$$\Pi_{\mu}^{\mu ab} = g^2 C_G \delta^{ab} \left( \frac{T^2}{3} - 10k^2 I_0 \right) \quad (3)$$

and

$$\Pi_{00}^{ab} = 2g^2 C_G \delta^{ab} |\vec{k}|^2 (I_0 + 4I_1). \quad (4)$$

where ( $x$  is the cosine of the angle between  $\vec{k}$  and  $\vec{q}$ ).

$$I_{0,1} = \frac{\mu^\epsilon}{(2\pi)^{3-\epsilon}} \int \frac{d^{3-\epsilon}\vec{q}}{2|\vec{q}|} \left( \frac{1}{k^2 + 2q \cdot k} + \frac{1}{k^2 - 2q \cdot k} \right) \left[ \frac{\vec{q}^2}{k^2} (1 - x^2) \right]^{0,1} \left[ \frac{1}{2} + N \left( \frac{|\vec{q}|}{T} \right) \right]. \quad (5)$$

The two terms in the last square bracket are associated respectively with the  $T = 0$  and the  $T \neq 0$  contributions ( $N$  is the Bose-Einstein distribution).

In order to express the integrations in (5) in terms of known functions, it is convenient to define the variable

$$K(x) = \frac{1}{4\pi i} \frac{k^2}{k_0 - |\vec{k}|x}. \quad (6)$$

Then it is straightforward to show that

$$I_0 = \frac{i\pi}{|\vec{k}|} \left( \frac{1}{4\pi} \right)^{\frac{3-\epsilon}{2}} \frac{\mu^\epsilon}{\Gamma\left(\frac{3-\epsilon}{2}\right)} \int_{K_-}^{K_+} dK \int_0^\infty d|\vec{q}| \frac{|\vec{q}|^{1-\epsilon}}{|\vec{q}|^2 + (2\pi K)^2} \left[ \frac{1}{2} + N \left( \frac{|\vec{q}|}{T} \right) \right], \quad (7)$$

where

$$K_{\pm} \equiv K(\pm 1) = \frac{k_0 \pm |\vec{k}|}{4\pi i}. \quad (8)$$

The above form shows that the integrals appearing in the calculation of the gluon self-energy can be naturally expressed in terms of the quantities  $K_{\pm}$  (which are proportional to the light-cone momenta  $k_0 \pm k_3$ , if one chooses, for example, the third axis along  $\vec{k}$ ).

The  $|\vec{q}|$  integration of  $T = 0$  part of  $I_0$ , gives

$$\frac{i}{8\pi|\vec{k}|} \int_{K_-}^{K_+} dK \left[ \frac{1}{\epsilon} - \log \frac{2\sqrt{\pi}K}{\mu} - \frac{\gamma}{2} + 1 \right]. \quad (9)$$

Using the fact that  $\text{Re}K(x) > 0$ , the  $|\vec{q}|$  integration of the  $T \neq 0$  part of  $I_0$  (where we may set  $\epsilon = 0$ ), yields the result [12]

$$\frac{i}{8\pi|\vec{k}|} \int_{K_-}^{K_+} dK \left[ \frac{T}{2K} + \log \frac{K}{T} - T \frac{d}{dK} \log \Gamma \left( 1 + \frac{K}{T} \right) \right], \quad (10)$$

where the logarithm of the gamma function is analytic when  $K \rightarrow 0$ . Then, the approximation

$$N \left( \frac{|\vec{q}|}{T} \right) = \frac{1}{\exp(|\vec{q}|/T) - 1} \simeq \theta(T - |\vec{q}|) \left( \frac{T}{|\vec{q}|} - \frac{1}{2} \right), \quad (11)$$

would simply lead, after performing the  $|\vec{q}|$  integration in equation (7), to the first two terms in the exact expression (10). As far as the  $\log(K)$  contribution to the  $T \neq 0$  part is concerned, one may effectively replace in Eq. (7), for small  $|\vec{q}|$ ,  $N(|\vec{q}|/T)$  by  $-1/2$ . Consequently, this contribution will cancel the  $\log(K)$  term associated with the  $T = 0$  part of  $I_0$  (this cancellation can also be explicitly verified from Eqs. (9) and (10)).

By itself, the  $K$ -integration of the  $\log(K/T)$  term in Eq. (10) gives the contribution

$$\begin{aligned} \frac{i}{16\pi|\vec{k}|} \left[ (K_+ + K_-) \log \frac{K_+}{K_-} + (K_+ - K_-) \log \frac{K_+ K_-}{T^2} - 2(K_+ - K_-) \right] = \\ \frac{1}{32\pi^2} \left[ \frac{k_0}{|\vec{k}|} \log \frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|} + \log \frac{-k^2}{16\pi^2 T^2} - 2 \right]. \end{aligned} \quad (12)$$

The emergence of the  $\log(-k^2)$  term in the special combination  $\log(-k^2/T^2)$ , is a direct consequence of the fact that the integrand in Eq. (10) depends only on the dimensionless ratio  $K/T$ . Similarly, the  $\log(K/\mu)$  term in Eq. (9) yields a contribution which, apart from sign, can be obtained from Eq. (12) by the replacement  $T \rightarrow \mu$ . Consequently, the  $\log(-k^2)$  terms will cancel between the zero-temperature and the temperature-dependent contributions, leaving a net factor proportional to  $\log(\mu^2/T^2)$ . After calculating the contributions from the first and third terms in Eq. (10), we obtain the following result for the temperature-dependent part of  $I_0$ :

$$I_0(T) = \frac{1}{32\pi^2} \log \frac{\mu^2}{T^2} + \frac{iT}{16\pi|\vec{k}|} \log \frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|} + \frac{T}{8\pi i|\vec{k}|} \log \frac{\Gamma(1 + K_+/T)}{\Gamma(1 + K_-/T)}. \quad (13)$$

Next, consider the  $I_1$  integral which can be written as:

$$\begin{aligned} I_1 = \frac{i\pi}{|\vec{k}|^3} \left( \frac{1}{4\pi} \right)^{\frac{3-\epsilon}{2}} \frac{\mu^\epsilon}{\Gamma\left(\frac{3-\epsilon}{2}\right)} \int_{K_-}^{K_+} dK \left[ \frac{k^2}{(4\pi K)^2} - \frac{i k_0}{2\pi K} - 1 \right] \\ \times \int_0^\infty d|\vec{q}| |\vec{q}|^{1-\epsilon} \left[ 1 - \frac{(2\pi K)^2}{|\vec{q}|^2 + (2\pi K)^2} \right] \left[ \frac{1}{2} + N \left( \frac{|\vec{q}|}{T} \right) \right]. \end{aligned} \quad (14)$$

Note that the  $T = 0$  contribution, associated with the factor of 1 in the second square bracket, would apparently lead to a quadratically divergent integral, which however vanishes in the dimensional regularization scheme. On the other hand, this factor yields a leading thermal contribution which is quadratic in the temperature

$$I_1^{\text{lead}}(T) = \frac{T^2}{24|\vec{k}|^2} \left( 1 - \frac{k_0}{2|\vec{k}|} \log \frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|} \right) \quad (15)$$

The  $|\vec{q}|$ -integration of the second term in the second square bracket of Eq. (14) is identical to the one which occurs in  $I_0$ , so that it gives analogous  $\log(K)$  contributions which cancel between the  $T = 0$  and thermal parts. As we have seen, only such contributions would give rise, after the  $K$ -integration, to individual  $\log(-k^2)$  terms. It is possible to evaluate exactly all other temperature-dependent contributions to  $I_1$ , in terms of logarithmic functions and of Riemann's zeta functions with arguments  $(1 + K_{\pm}/T)$ , which are analytic when  $K_{\pm} \rightarrow 0$  [11]. Since the complete expression is rather involved, we indicate here, for simplicity, only the logarithmic temperature-dependent contributions to  $I_1$ :

$$I_1^{\text{log}}(T) = -\frac{1}{192\pi^2} \log \frac{\mu^2}{T^2} - \frac{iTk^2}{64\pi|\vec{k}|^3} \log \frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|}. \quad (16)$$

In a general gauge,  $\Pi_{\mu}^{\mu}$  and  $\Pi_{00}$  will have a similar behavior (in particular, the leading  $T^2$  contribution is gauge independent). In this case, the thermal contributions to  $\Pi_C = k_{\mu}\Pi_0^{\mu}/|\vec{k}|$  are non-vanishing, and can be written as [13]

$$\Pi_C = \frac{(1-\xi)}{(2\pi)^3|\vec{k}|} \int \frac{d^3\vec{q}}{|\vec{q}|} \left[ \left( \frac{k^2}{k^2 + 2k \cdot q} + \frac{1}{2} \frac{d}{dq_0} \frac{k \cdot q}{q_0} \right) \frac{k \cdot q k_0 - k^2 q_0}{k^2 + 2k \cdot q} + q \leftrightarrow -q \right]_{q^2=0} N \left( \frac{|\vec{q}|}{T} \right), \quad (17)$$

where  $\xi$  is the gauge parameter ( $\xi = 1$  in the Feynman gauge) and the derivative  $d/dq_0$  acts on all terms at its right. Performing the  $|\vec{q}|$  integration, the terms involving  $\log(K)$  factors turn out to be proportional to

$$\int_{K_-}^{K_+} dK \log K \left[ 8\pi i K - 3k_0 - \frac{k^2 k_0}{16\pi^2 K^2} \right]. \quad (18)$$

However, the coefficient of the  $\log(-k^2)$  term, which is obtained after the  $K$ -integration is performed, actually vanishes:

$$\left(2\pi i K_+^2 - \frac{3}{2} k_0 K_+ - \frac{k^2 k_0}{32\pi^2 K_+}\right) - \left(2\pi i K_-^2 - \frac{3}{2} k_0 K_- - \frac{k^2 k_0}{32\pi^2 K_-}\right) = 0 \quad (19)$$

Thus, the full self-energy of the gluon, which includes the thermal effects, does not contain  $\log(-k^2)$  contributions. Essentially, these effects replace the zero-temperature  $\log(-k^2/\mu^2)$  term by a  $\log(T^2/\mu^2)$  contribution. Although this correspondence seems plausible, it is not so obvious. For instance, it would not hold if the thermal contributions would involve individual terms like  $\log(k_0^2/T^2)$ ,  $\log(|\vec{k}|^2/k^2)$ , etc. To discard this possibility, it is necessary to show that the  $\log(-k^2)$  and  $\log(T^2)$  contributions appear in the thermal part only in the combination  $\log(-k^2/T^2)$ . Furthermore, in order to explain the cancellation of the  $\log(-k^2)$  terms between the zero-temperature and the temperature-dependent parts, one must also argue [9] that the  $\log(T)$  dependence of the self-energy is simply related to its ultraviolet behavior at zero-temperature. Here, these properties of the thermal gluon self-energy have been explicitly verified to one-loop order.

#### ACKNOWLEDGMENTS

We would like to thank CNPq (Brazil) for a grant and Prof. J. C. Taylor for a helpful correspondence.

## REFERENCES

- [1] H. A. Weldon, Phys. Rev. **D26**, 1394 (1982).
- [2] E. Braaten and R. D. Pisarski, Nucl. Phys. **B337**, 569 (1990); **B339**, 310 (1990); Phys. Rev. **D45**, 1827 (1992).
- [3] J. Frenkel and J. C. Taylor, Nucl. Phys. **B334**, 199 (1990); **B374**, 156 (1992).
- [4] R. Baier, G. Kunstatter, and D. Schiff, Phys. Rev. **D45**, 4381 (1992), *ibid* R. Kobes, G. Kunstatter, and K. Mak, Phys. Rev. **D45**, 4632 (1992).
- [5] J.-P. Blaizot and E. Iancu, Phys. Rev. **D55**, 973 (1997); **56**, 7877 (1997).
- [6] J. I. Kapusta, *Finite Temperature Field Theory* (Cambridge University Press, Cambridge, England, 1989).
- [7] M. L. Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1996).
- [8] H. A. Weldon, Phys. Rev. **D59**, 065002 (1999).
- [9] F. T. Brandt and J. Frenkel, Phys. Rev. **D55**, 7808 (1997).
- [10] H. A. Weldon, Annals Phys. **271**, 141 (1999).
- [11] A. P. de Almeida, J. Frenkel, and J. C. Taylor, Phys. Rev. **D45**, 2081 (1992).
- [12] I. S. Gradshteyn and M. Ryzhik, *Tables of Integral Series and Products* (Academic, New York, 1980).
- [13] F. T. Brandt and J. Frenkel, Phys. Rev. **D56**, 2453 (1997).