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CHIRAL SYMMETRY BREAKING AND THE C_K/C_π RATIO

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ABSTRACT

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It is shown that the 9 pseudoscalars are better described by a model with a $SU(3) \otimes SU(3)$ symmetry breaking Hamiltonian containing both $(3,\bar{3}) \oplus (\bar{3},3)$ and $(8,8)$ parts. The SU(3) noninvariance of the vacuum produces the η-η' mixing and $C_K \neq C_\pi$. A good value, $C_K/C_\pi = 1.25$, which dependes on the pseudoscalar masses only, is obtained.

RESUMO

QUEBRA DE SIMETRIA CHIRAL E A RAZÃO C_K/C_π

Mostra-se que os 9 pseudo escalares são melhor descritos por um modêlo com uma Hamiltoniana de quebra de $SU(3) \otimes SU(3)$ contendo partes de $(3,\bar{3}) \oplus (\bar{3},3)$ e $(8,8)$. A não invariança do vãcuo por SU(3) produz a mistura η-η' e $C_K \neq C_\pi$. Obtẽm-se um bom valor, $C_K/C_\pi = 1.25$, dependendo - apenas das massas dos pseudo escalares.

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The idea of describing the $SU(3) \otimes SU(3)$ symmetry breaking by terms in the Hamiltonian that are linear in scalar fields transforming according to representations of the group [1,2] has stimulated much work. $(3,\bar{3}) \oplus (\bar{3},3)$ has been the most often employed representation [3,4] followed by $(8,8)$ [5]. More recently, Sirlin and Weinstein[6] have argued that in order to have enough flexibility to be able to accomodate larger values of the nucleon σ term while at the same time keeping the π-π scattering lengths close to Weinberg's values and not too large $SU(3) \otimes SU(3)$ breaking, it was convenient to consider symmetry breaking terms that belong to the representation $(\bar{3},3) \oplus (3,\bar{3}) \oplus (8,8)$.

In Ref.(6), for example, the small SU(3) noninvariance of the vacuum was neglected since it would have introduced in their calculations higher order corrections only. On the other hand,in the present note, we are interested in effects that depend essentially on that lack of invariance. We contend that the SU(3) noninvariance of the vacuum is responsible both for the η-η' mixing and for $C_K \neq C_\Pi$. It is also shown that the $SU(3) \otimes SU(3)$ symmetry breaking Hamiltonian cannot be either pure $(3,\bar{3}) \oplus (\bar{3},3)$ or pure $(8,8)$ while it is more reasonable to consider, as in Ref. (6), a sum of both representations as in

$$\mathcal{H}' = M^3 \left[\frac{2}{3} \cos\phi (S_0 + \frac{\sqrt{5}}{2} c' S_8) + \sin\phi (U_0 + c U_8) \right] \qquad (1)$$

here, M is a constant with the dimension of mass, S_0 and S_8 are SU(3) singlet and octet scalar fields that belong to the (8,8) representation of SU(3) $\otimes$ SU(3) while the singlet U_0 and octet U_8 belong to $(3,\bar{3}) \oplus (\bar{3},3)$.

As usual, the current divergences can be obtained from

$$\partial_\mu A^\mu(x,t) = i \left[\mathcal{H}'(x,t), F_a^5(t) \right] \qquad (2)$$

which, using the Hamiltonian (1) leads to

$$\partial_\mu A_a^\mu = \sqrt{\frac{2}{3}}\, M^3 \left\{ \left(1 + \sqrt{\frac{3}{2}}\, c' d_{8aa}\right) \cos\phi P_a + \left[\left(1 + \sqrt{\frac{3}{2}}\, c d_{8aa}\right) V_a + c V_0 \delta_{a8} \right] \sin\phi \right\} + \text{(decuplets)} \qquad (3)$$

where the pseudoscalars P_a $[V_a]$ belong to the (8,8) $\left[(3,\bar{3}) \oplus (\bar{3},3)\right]$ representation.

For a=3, for instance, Eq.(3) gives:

$$C_\pi m_\pi^2 = \langle 0|\partial_\mu A_3^\mu|\pi^0\rangle =$$

$$= \sqrt{\frac{2}{3}}\, M^3 \left\{ \left(1 + \frac{c'}{\sqrt{2}}\right) \cos\phi \langle 0|P_3|\pi^0\rangle + \left(1 + \frac{c}{\sqrt{2}}\right) \sin\phi \langle 0|V_3|\pi^0\rangle \right\} \qquad (4)$$

where we have also used PCAC for the π-like current. From now on, we will take $c'=c$ since in that case eq. (4) suggests that the π be a mixture of two fields according to

$$\pi^o = Z^{-1/2}(\cos\phi P_3 + \sin\phi V_3) \quad , \qquad (5)$$

with Z being a sort of renormalization constant. If we treat the K-field in a similar way with the assumption that it is renormalized by the same constant Z, we will have

$$C_\pi \; M_\pi^2 = Z^{1/2}\sqrt{\frac{2}{3}}\; M^3\left(1 + \frac{c}{\sqrt{2}}\right) ,$$

$$C_K \; M_K^2 = Z^{1/2}\sqrt{\frac{2}{3}}\; M^3\left(1 - \frac{c}{2\sqrt{2}}\right) . \qquad (6)$$

On the other hand the commutators between the axial charges and the divergences lead to relations of the type

$$C_\pi^2 \; M_\pi^2 = \frac{2}{3} M^3 (1 + \frac{c}{\sqrt{2}}) \left\{ \frac{3}{2} \left(<S_o> + \sqrt{\frac{2}{5}} <S_8> \right) \cos\phi + \right.$$

$$\left. + \left(<U_o> + \frac{1}{\sqrt{2}} <U_8> \right) \sin\phi \right\} \qquad (7)$$

which together with the first of Eqs.(6) yields

$$C_\pi = \sqrt{\frac{2}{3}}\; Z^{-1/2} \left\{ \frac{3}{2} \left(<S_o> + \sqrt{\frac{2}{5}} <S_8> \right) \cos\phi + \right.$$

$$\left. + \left(<U_o> + \frac{1}{\sqrt{2}} <U_8> \right) \sin\phi \right\}$$

$$= \sqrt{\frac{2}{3}}\; Z^{-1/2}\left(\lambda_o + \frac{\lambda_8}{\sqrt{2}}\right) = \sqrt{\frac{2}{3}}\, Z^{-1/2}\lambda_o\left(1 + \frac{\lambda}{\sqrt{2}}\right) \qquad (8)$$

where we have introduced

$$\lambda_o \equiv \frac{3}{2} \langle S_0 \rangle \cos\phi + \langle U_0 \rangle \sin\phi \quad ,$$

$$\lambda_8 \equiv \frac{3}{\sqrt{5}} \langle S_8 \rangle \cos\phi + \langle U_8 \rangle \sin\phi \quad ,$$

$$\lambda \equiv \frac{\lambda_8}{\lambda_o} \quad . \tag{9}$$

In similar fashion the decay constant of the K can be obtained as

$$C_K = \sqrt{\frac{2}{3}}\, Z^{-1/2} \lambda_o \left(1 - \frac{\lambda}{2\sqrt{2}}\right) \tag{10}$$

giving for the ratio

$$\frac{C_K}{C_\Pi} = 1 - \frac{3}{2} \frac{\lambda}{(\sqrt{2} + \lambda)} \tag{11}$$

Using now Eqs.(6),(8) and (10) one can write for the masses

$$M_\Pi^2 = M_o^2 \left(1 + \frac{c}{\sqrt{2}}\right)\left(1 + \frac{\lambda}{\sqrt{2}}\right)^{-1} ,$$

$$M_K^2 = M_o^2 \left(1 - \frac{c}{2\sqrt{2}}\right)\left(1 - \frac{\lambda}{2\sqrt{2}}\right)^{-1} , \tag{12}$$

$$M_o^2 \equiv \frac{M^3}{Z \lambda_o}$$

As we can see, M_Π^2, M_K^2, C_Π and C_K are independent of the angle ϕ, i.e., a pure (8,8) model or a pure $(3,\bar{3}) \oplus (\bar{3},3)$ or any mixture of both will give the same relations among these quantities.

In order to proceed we need some information on the η-η' system which can be obtained by looking at the A_μ^8 and A_μ^0 currents. We take the vanishing of the comutator

$$\left[F_8^5, \partial_\mu A_0^\mu(8,8) \right] = 0 \quad , \qquad (13)$$

as an indication that (8,8) symmetry breaking leads to no η-η' mixing. If that is the case, the divergence

$$\partial_\mu A_8^\mu = \sqrt{\frac{2}{3}}\, M^3 \left\{ \left(1 - \frac{c}{\sqrt{2}}\right)\left(P_8\cos\phi + V_8\sin\phi\right) + cV_0\sin\phi \right\} \qquad (14)$$

would not contain η' when ϕ=0. The simplest way of enforcing that fact is to assume that

$$\eta = Z^{-1/2}\,(P_8\cos\phi + V_8\sin\phi) \ ,$$

$$\eta' = Z_0^{-1/2} V_0 \qquad (15)$$

Z is the same constant that renormalizes the Π and K fields while for η' we use a different constant Z_0. The good results to be obtained will provide a further justification of the assignment of fields given by the Eqs.(15) .

The η-η' mixing can be parametrized in terms of the angle Θ defined by

$$C_8 M_\eta^2 \cos\Theta \equiv \langle 0|\partial_\mu A_8^\mu|\eta\rangle = \sqrt{\frac{2}{3}}\, M^3 Z^{-1/2}\left(1 - \frac{c}{\sqrt{2}}\right) \qquad (16)$$

and

$$-C_8 M_{\eta'}^2 \sin\Theta \equiv \langle 0|\partial_\mu A_0^\mu|\eta'\rangle = \sqrt{\frac{2}{3}}\, M^3\, Z_0^{-1/2} c \sin\phi\ , \qquad (17)$$

showing that ϕ and Θ are related by

$$\tan\Theta = -\frac{M_\eta^2}{M_{\eta'}^2}\ \frac{c}{1 - \frac{c}{\sqrt{2}}}\left(\frac{Z}{Z_0}\right)^{1/2} \sin\phi\ . \qquad (18)$$

From the vacuum expectation value of $\left[F_8^5, \partial_\mu A_8^\mu\right]$, using Eqs.(14) and (15) one can obtain

$$C_8^2\, M_\eta^2 \cos^2\Theta = \frac{2}{3} M^3 \lambda_0 \left(1 - \frac{c}{\sqrt{2}}\right)\left(1 - \frac{\lambda}{\sqrt{2}}\right) \qquad (19)$$

and

$$C_8^2\, M_{\eta'}^2 \sin^2\Theta = \frac{2}{3} M^3 c \sin\phi \langle U_8\rangle \qquad (20)$$

from which another expression for $\tan\theta$ follows,

$$\tan^2\theta = \frac{M_\eta^2}{M_{\eta'}^2} \frac{c}{1-\frac{c}{\sqrt{2}}} \frac{\langle U_8\rangle}{\lambda_o\left(1-\frac{\lambda}{\sqrt{2}}\right)} \sin\phi \,. \tag{21}$$

This equation, when divided by Eq.(18), yields

$$\tan\Theta = -\left(\frac{Z_o}{Z}\right)^{1/2} \frac{\langle U_8\rangle}{\lambda_o\left(1-\frac{\lambda}{\sqrt{2}}\right)} \,. \tag{22}$$

Eqs.(18) and (22) show that the parameters of the model must be related by

$$\frac{Z_o}{Z} \frac{\langle U_8\rangle}{\lambda_o} = \frac{M_\eta^2}{M_{\eta'}^2} \frac{c\left(1-\frac{\lambda}{\sqrt{2}}\right)}{\left(1-\frac{c}{\sqrt{2}}\right)} \sin\phi = \frac{M_o^2}{M_{\eta'}^2} c\sin\phi, \tag{23}$$

where we have used an expression for the η mass that follows from Eqs.(16) and (19) and reads

$$M_\eta^2 = M_o^2 \frac{1-\frac{c}{\sqrt{2}}}{1-\frac{\lambda}{\sqrt{2}}} \,. \tag{24}$$

This,together with the π and K masses as given by Eq.(12), allow us to express λ in terms of the masses as

$$\lambda = -\sqrt{2}\,\frac{\frac{1}{3}\left(4M_K^2 - M_\pi^2\right) - M_\eta^2}{M_\eta^2 - \frac{1}{3}\left(2M_K^2 + M_\pi^2\right)} \qquad (25)$$

which, when introduced into Eq.(11) gives

$$\frac{C_K}{C_\pi} = 1 + \frac{\left(4M_K^2 - M_\pi^2\right) - 3M_\eta^2}{4\left(M_\eta^2 - M_K^2\right)} = \frac{1}{4}\,\frac{M_\eta^2 - M_\pi^2}{\left(M_\eta^2 - M_K^2\right)} = 1.25\ , \qquad (26)$$

numerical value obtained with the masses listed in the Particle Properties Tables (7). That expression for C_K/C_π is our main result, in very good agreement with the experimental estimate (8).

$$\frac{C_K}{C_\pi}\ \frac{1}{f_+(0)} = 1.27 \pm 0.03\ , \qquad (27)$$

since $f_+(0)$ is expected to be very close to one.

We should also mention that numerically one has

$$\left(\frac{Z_o}{Z}\right)^{1/2} \frac{\langle U_8 \rangle}{\lambda_o} \simeq \lambda$$

$$(-0.20) \simeq (-0.21) \qquad (28)$$

If this relation, that we were not able to prove, holds, the mixing angle would be simply given by

$$\tan\Theta = - \frac{\lambda}{1 - \frac{\lambda}{\sqrt{2}}} \tag{29}$$

To summarize we can say that the present model based on a Hamiltonian where the symmetry is broken according to the representations $(3,\bar{3})\oplus(\bar{3},3)\oplus(8,8)$ leads to Π and K fields that are mixtures of $(3,\bar{3})\oplus(\bar{3},3)$ plus $(8,8)$ fields. The simple ansatz that the η is also such a mixture [Eq.(15)] allows us to relate the η-η' mixing angle to the symmetry breaking parameters [Eq.(22) or the more appealing Eq.(29)]. The model also yields the expression (24) for C_K/C_Π in terms of the Π, K and η masses only and in good agreement with experiment.

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