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EFIMOV RESONANCES IN THREE BODY-SYSTEMS

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A B S T R A C T

EFIMOV RESONANCES IN THREE BODY-SYSTEMS

On the basis of the assumption that Efimov states resonate, we construct a simple theory through which the existence of these states in three-body systems, can be detected experimentally. The concepts of compound nucleus and doorway states are utilized.

EFIMOV RESONANCES IN THREE-BODY SYSTEMS

1. INTRODUCTION

Recently there has been some discussion about the way Efimov states[(1)] manifest themselves in the three-body system[(2)] namely if these states do exist how would they be "seen" in the measured elastic cross section of, say, $n+d \rightarrow (H^3)^* \rightarrow n+d$ where for the moment we can approximate this as a prototype factory for Efimov states. H^3 ordinarily would resonate showing a peak in $d\sigma/d\Omega$ averaged over some energy interval. Since all of the Efimov states are near threshold it seems obvious that they too will resonance. One can then look at H^3 as a compound nucleus with the Efimov states as just the compound nuclear resonances that are whashed-out of the cross-section by the bad energy resolution. The other more familiar H^3 resonance is then viewed as a more favored compound nuclear state (a doorway state) that is coupled both to the continuun and to the "more complicated" Efimov states. This doorway state then escapes averaging due to the fact that its total width is larger than the energy interval whithin which the averaging is done[(3)]. One would then guess that the cross section would exhibit this intermediate structure and no Efimov resonances are seen. As we shall show soon this is not quite so as the Efimov resonances will be coupled to the continuun via the doorway state, namely ^3H resonance, in such a way that the width of the doorway will have a damping contribution exactly due to this coupling. This contribution is not insignificant since the number of Efimov resonances is assumed to be large and behaves as, under the right Efimov condition, $\frac{1}{\pi} \log \frac{|a|}{R}$ where $|a|$ is the absolute value of the two-body scattering length which is very large in this case and R is a characteristic length associated with the range of the two-body potential[1].

2. NEUTRON-DEUTERON SCATTERING

We shall discuss n-d scattering as an example. Let us assume that the Hamiltonian describing the three-body system is $H=H_o+V_{nd}$ where V_{nd} is the interaction between the neutron and the deuteron. The Schrodinger equation is then:

$$H_{nd}|\Psi_{nd}\rangle = E_{nd}|\Psi_{nd}\rangle \tag{1}$$

where $|\Psi_{nd}\rangle$ are both discrete and continuum states.

Define P such that $P|\Psi_{nd}\rangle = \Psi_h^{(+)}\Phi_d$ (2)

where Φ_d is the deuteron bound-state wave function and $Q=1-P$.

Thus the Schrodinger equation can be cast in the form of two coupled equations.

$$(E-H_{pp})\ P|\Psi_{nd}\rangle = H_{pQ}Q|\Psi_{nd}\rangle \tag{3}$$

$$(E-H_{QQ})\ Q|\Psi_{nd}\rangle = H_{Qp}P|\Psi_{nd}\rangle$$

the Q-states are states corresponding to resonances in the (n-d) system and therefore we eliminate them by solving for $P|\Psi_{nd}\rangle$. Thus it is easy to see that

$$Q|\Psi_{nd}> = \frac{1}{E-H_{QQ} - H_{Qp}G_p(E^{(+)})\ H_{pQ}}\ H_{Qp}|\phi^{(+)}_{nd}> \qquad (4)$$

where

$$G_p(E^{(+)}) \equiv \frac{1}{E - H_{pp}+i_n}$$ and $|\phi^{(+)}_{nd}>$ is a solution of

$$(E - H_{pp})|\phi^{(+)}_{nd}> = 0$$

Substituting for $Q|\Psi_{nd}>$ back in equation(3) one then gets:

$$P|\Psi_{nd}> = |\phi^{(+)}_{nd}> + G_p(E^{(+)})\ H_{pQ}\ \mathcal{G}_Q(E^{(+)})\ H_{Qp}|\phi^{(+)}_{nd}> \qquad (5)$$

where $\mathcal{G}_Q(E^{(+)})$is defined in equation(4).

The above equation(5) is the desired one for the wave function $P|\Psi_{nd}>$ where all reference to the (n-d) states other than the $P|\Psi_{nd}>$ is in $\mathcal{G}_Q(E^{(+)})$. As usual the T-matrix for the n-d scattering is then:

$$T_p(E) = t_{potential}+<\phi^{(-)}_{nd}|H_{pQ}\ \ Q(E^{(+)})\ H_{Qp}|\ \phi^{(+)}_{nd}> \qquad (6)$$

where $t_{potential}$ is the T-matrix that corresponds to $|\ \phi^{(+)}_{nd}>$ i.e. to potential scattering of n off the deuteron.

The projection operator P is defined in the following manner:

$$H_{pp} = pHp = |\Psi_{nd}>\Psi_n^{(+)}\Phi_d\ H\Phi_d\ \Psi_n^{(+)}\ <\Psi_{nd}|$$

$$\therefore\ (E-H_{pp})|\phi_{nd}^{(+)}> = \{E-|\Psi_{nd}>\Psi_n^{(+)}\Phi_d H\Phi_d\Psi_n^{(+)}\Phi_d\}|\phi_{nd}^{(+)}> = 0$$

$$\text{or}\quad (E-h_n)\ \Psi_n^{\dagger} = 0$$

$$\text{where:}\ h_n \equiv \frac{k_n^2}{2m} + V_{nd}$$

$|\phi_{nd}^{(+)}>$ describes potential scattering i.e. scattering of a neutron from a sum of two potential wells. This problem is doable in principle. Once we get $|\phi_{nd}^{(+)}>$ we can construct $T_{pp}(E)$ including the resonance contribution of Efimov states:

$$T_{pp}^{R}(E) = \ell^{2i\delta} < \phi_{nd}|H_{pQ}\mathscr{G}_Q(E^{(+)})\ H_{ap}|\ \phi_{nd}> \qquad (7)$$

Assuming one has many nonoverlapping resonances then the above can be written as a series of Breit-Wigner terms:

$$T_{pp}^{R}(E) = \ell^{2i\delta} \sum_{Q} \frac{|<\phi_{nd}|H|Q>|^2}{E-E_Q-\Delta_Q(E) + i\ \frac{\Gamma a(E)}{2}} \qquad (8)$$

where one is reminded that only discretet states in $Q|\Psi>$ are taken into account since we are dealing with elastic scattering only. Let us assume that one of the resonating states is more important than the others and we would like to extract it from the formula given above i.e. we would like to isolate it from the rest. It is easy to show that the expressions for T becomes in this case[3].

$$T = T_{pot} + T_D + T_{Q'} \tag{9}$$

where pot refers to potential, D for doorway state and Q' to the rest of the compound nuclear states namely Q'=Q-D. The different T-operators T_D and $T_{Q'}$ have the following forms:

$$T_D = < \phi_{nd}^{(-)} | H_{pD} \frac{1}{E^{(+)} - H_{DD} - W_{DD}} H_{Dp} | \phi_{nd}^{(+)} >$$

$$T_{Q'} = < \phi_{nd}^{(-)} | H_{pD} \frac{1}{E^{(+)} - H_{DD} - W_{DD}} H_{DQ'} \times$$

$$\frac{1}{E^{(+)} - H_{Q'Q'} - H_{Q'D}(E - H_{DD} - W_{DD})^{-1} H_{DQ'}} H_{Q'D} \times$$

$$\frac{1}{E^{(+)} - H_{DD} - W_{DD}} H_{DP} | \phi_{nd}^{(+)} >$$

One can then write the energy-averaged T-matrix:

$$<T>_I = t_{pot} + < \phi_{nd}^{(-)} | H_{pD} \frac{1}{E^{(+)} - H_{DD} - W_{DD} - W_{DD}} H_{Dp} | \phi_{nd}^{(+)} > ;$$

$$W_{DD} = D(\Delta\uparrow - \frac{i}{2} \Gamma\uparrow) D$$

$$W_{DD} = D(\Delta\downarrow - \frac{i}{2} \Gamma\downarrow) D$$

The presence of compound nuclear states that are not observed is nevertheless manifested through the nonvarishing of $\Delta\downarrow$ and $\Gamma\downarrow$. These are given by (for an isolated doorway):

$$\Gamma\downarrow = \mathrm{Im}\sum_{Q'} \frac{|\langle\phi_{Q'}H_{Q'D}\psi_D\rangle|^2}{(E-E_{Q'})^2 + \frac{1}{4}I^2} \approx \frac{2\pi}{\Delta E}\sum_{Q'}|\langle\phi_{Q'}H_{Q'D}\Psi_D\rangle|^2$$

$$\Delta\downarrow = \mathrm{Real}\sum_{Q'} \frac{|\langle\phi_{Q'}H_{Q'D}\psi_D\rangle|^2}{E-E_{Q'} + \frac{1}{2}iI} \tag{11}$$

The sum extends over all resonating Efimov states. If $|\langle\phi_{Q'}H_{Q'D}\Psi_D\rangle|^2 \sim \Lambda$ = constant ,then:

$$\Gamma\downarrow \approx \frac{2\pi}{\Delta E}\,\frac{\Lambda}{\pi}\,\log\frac{|a|}{R} \equiv \Gamma_E \tag{12}$$

$$\Delta\downarrow \approx \text{constant} \equiv \Delta_E$$

Thus the presence of Efimov resonances could be "seen" through the average T-matrix

$$\langle T\rangle_I = t_{pot} + \frac{\langle\phi^{(-)}_{nd}|H_{pD}|D\rangle\langle D|H_{Dp}|\phi^{(+)}_{nd}\rangle}{E-E_D-(\Delta_D+\Delta_E)+\frac{i}{2}(\Gamma_D+\Gamma_E)} \tag{13}$$

Γ_D is the width corresponding to the coupling of Ψ_D to the continuun i.e. the normal width whereas the extra term corresponds to the contribution of Efimov resonances.

3. DISCUSSION AND CONCLUSIONS

This seems to be the only place and way that these pathological states enter in the cross section. It would then be of interest to check the data of n+d elastic scattering and compare the cross section with a calculated width $\Gamma\downarrow$ using a model for the H^3 bound state wave function Ψ_D, since the total width, namely $\Gamma\downarrow + \Gamma_E$ is finite it is doubtful that the number of resonating Efimov states is very large. This is probably due to the fact that the estimate for N_E above is made under extreme conditions namely equal masses, very simple forces, no mention of coulomb interaction (which is not important is our example anyway)as well as the fact that the binding energy of the deuteron is not zero but rather 2.2 Mev! All of these definitely affect the estimate for N_E. An indirect "measurement" of at least part of N_E is the one proposed above.

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