RADIATIVE CORRECTIONS TO THE LOW ENERGY THEOREM FOR PSEUDOSCALAR MESON →2γ IN THE WEINBERG MODEL

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ABSTRACT

We discuss the radiative corrections to the low energy theorem for pseudoscalar meson → 2γ in the unified model of electromagnetic and weak interactions proposed by Weinberg. We show that, to the fourth order in the electric charge conpling constant,there are no radiative corrections.

1 - Introduction.

The low energy decays of the pseudoscalar mesons play an important role among the low energy theorems as they give indications of the properties of the hadronic constituents[1]. In particular, the experimental rate of the decay $\pi_0 \to 2\gamma$, in view of a theorem proved by Adler and Bardeen,[2] supports models based on triplets of integraly charged quarks. This theorem, proved in electrodynamics and in the σ - model, states that the amplitude $M(\pi_0 \to 2\gamma)$ at $k_\pi^2 = 0$ is determined, to any order in pertubation theory, by the lowest order fermion-loop diagram.

In order to prove the theorem one has first to establish that the anomaly in the chiral Ward identities is not modified by the radiative corrections. Using the n-dimensional regularization scheme, Bardeen[3] has given an argument that this scheme is sufficient to guarantee that the Ward identity is, indeed, given by the lowest order anomaly. Therefore, in the equation for the divergence of the axial vector current:

$$\partial_\mu J_\mu^{5a} = J^{5a} + D^a \tag{1}$$

where J^{5a} corresponds to the usual divergence operator (a is an internal index) we assume that D^a, given by the lowest order fermion-loop contributions, is the exact expression of the anomalous divergence. Bardeen[4] derived a minimal expression for the anomalous divergence D^a, which involves only the vector and axial vector bosons, V_μ^a and A_μ^a, and their conplings to the fermions, λ_V^a and λ_A^a. In terms of the quantities V_μ and A_μ, defined as:

$$L_{int} = \bar{\psi}\gamma_\mu(\lambda_V{}^a V_\mu{}^a + \gamma_5 \lambda_A A_\mu^a)\psi \equiv \bar{\psi}\gamma_\mu (V_\mu + \gamma_5 A_\mu)\psi \qquad (2)$$

we can write the minimal divergence D^a as follows:

$$D^a = \frac{1}{4\pi^2} \epsilon_{\mu\nu\alpha\beta} \, Tr^I \lambda_A{}^a \Big\{ \frac{1}{4} F_{\mu\nu}{}^V F_{\alpha\beta}{}^V + \frac{1}{12} F_{\mu\nu}{}^A F_{\alpha\beta}{}^A + \frac{2}{3}i A_\mu A_\nu F_{\alpha\beta}{}^V + \frac{2}{3}i F_{\mu\nu}{}^V A_\alpha A_\beta + \frac{2}{3}i A_\mu F_{\nu\alpha}{}^V A_\beta + {} - \frac{8}{3} A_\mu A_\nu A_\alpha A_\beta \Big\} \qquad (3)$$

where Tr^I stands for the trace over internal indices and $F_{\mu\nu}{}^V$ and $F_{\mu\nu}{}^A$ are given by:

$$F_{\mu\nu}{}^V = \partial_\mu V_\nu - \partial_\nu V_\mu - i\left[V_\mu, V_\nu\right] - i\left[A_\mu, A_\nu\right]$$

$$F_{\mu\nu}{}^A = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[V_\mu, A_\nu\right] - i\left[A_\mu, V_\nu\right] \qquad (4)$$

In order to verify the low energy theorem, one takes, next, the matrix element of (1) between the vacuum and two photons, with momenta k_1 and k_2 at $k_1 \cdot k_2 = 0$. As shown by Sutherland and Veltman[5], the matrix element of the left-hand side vanishes at this point, resulting that:

$$\langle \gamma (k_1, \varepsilon_1); \gamma (k_2; \varepsilon_2) | J^{5a} | o \rangle = - \langle \gamma (k_1, \varepsilon_1); \gamma (k_2, \varepsilon_2) | D^a | o \rangle \qquad (5)$$

The low energy theorem is proven if we show that there are no contributions to $\langle\gamma_1;\ \gamma_2|\ D^a\ |o\rangle$ of order e^4 and higher[6].

Here we shall verify the low energy theorem, to the fourth order in e , in the renormalizable model proposed by Weinberg.[7] In this model, the anomalies cancel between the lepton and hadron sector but, since the hadronic weak current is anomalous by itself, the pseudoscalar meson can still decay in two photons.

2 - Fourth-order radiative corrections in the Weinberg model.

In this model, based on the group SU(2) x U(1), there is a triplet of gauge vector bosons $\vec{B}$ and a singlet B^o, who couple to the fermion doublet[8] ψ (proton and neutron) with coupling strength g and g_1, respectively:

$$L_{int} = \frac{i}{4}\, g\bar{\psi}\gamma_\mu(1+\gamma_5)\ \vec{t}.\vec{B}_\mu\ \psi\ +$$

$$+\ \frac{i}{4}g_1\ \bar{\psi}\gamma_\mu\left[\ (2\ t_o + t_3) - \gamma_5\ t_3\ \right]\ B_\mu{}^o\ \psi \qquad (6)$$

where $t_o = \frac{1}{2}$ and t_i are the isotopic matrices normalized to $[t_\ell,\ t_m] = i\ \varepsilon_{lmn}\ t_n$. We have taken the charges of the p and n quarks as 1 and o, respectively, both in order to obtain the $\pi_o\to2\gamma$ condition and, also, to yield the model anomaly free. From (6) we obtain for V_μ and A_μ, defined in (2), the expressions:

$$V_\mu = \frac{ig}{4}\ \vec{t}.\ \vec{B}_\mu + \frac{ig_1}{4}\ (2\ t_o + t_3)\ B_\mu^o \qquad (7)$$

$$A_\mu = \frac{ig}{4}\ \vec{t}.\ \vec{B}_\mu - \frac{ig_1}{4}\ t_3\ B_\mu^o$$

Inserting these relations into the expressions defining $F_{\mu\nu}{}^{V}$ and $F_{\mu\nu}{}^{A}$, we obtain effectively (since the result is to be multiplied by $\varepsilon_{\mu\nu\alpha\beta}$):

$$
\begin{aligned}
F_{\mu\nu}{}^{V} &= \frac{i}{2}\left[g\, \partial_\mu\, B_\nu{}^a t_a + i\frac{g^2}{4}\, \varepsilon_{abc}\, B_\mu{}^a\, B_\nu{}^b\, t_c + g_1\, \partial_\mu\, B_\nu{}^o (2t_o + t_3)\right] \\
F_{\mu\nu}{}^{A} &= \frac{i}{2}\left[g\, \partial_\mu\, B_\nu{}^a t_a + \frac{ig^2}{4}\, \varepsilon_{abc}\, B_\mu{}^a B_\nu{}^b t_c - g_1\, \partial_\mu\, B_\nu{}^o\, t_3\right]
\end{aligned}
\tag{8}
$$

In order to verify the low energy theorem, we will consider, for simplicity, the axial vector current:

$$
\begin{aligned}
J_\mu{}^5 &\equiv \text{sen}\, \phi\, J_\mu{}^{5,3} + \cos\phi\, J_\mu{}^{5,o} = \\
&= \overline{\psi}\, \gamma_\mu\, \gamma_5\, (\sin\phi\, t_3 + \cos\phi\, t_o)\, \psi
\end{aligned}
\tag{9}
$$

where ϕ is arbitrary. We remark that, as far as isospin is concerned, $J^{5,3}$ and $J^{5,o}$ have the quantum numbers of the π_o and χ-meson (or η-meson), respectively.

Substituting (7), (8), and (9) into (3), we obtain after a straightforward calculation the following result:

$$
\begin{aligned}
D = D^3 + D^o = \frac{-1}{192\pi^2}\, \varepsilon_{\mu\nu\alpha\beta} \Big\{ &\cos\phi \left[g^2\, \partial_\mu\, B_\nu{}^a \partial_\alpha\, B_\beta{}^a + \right. \\
&+ \left. g g_1\, \partial_\mu\, B_\nu{}^3 \partial_\alpha\, B_\beta{}^o + 4 g_1{}^2\, \partial_\mu\, B_\nu{}^o\, \partial_\alpha\, B_\beta{}^o \right] + \\
&+ \sin\phi \left[3 g g_1\, \partial_\mu\, B_\nu{}^3\, \partial_\alpha\, B_\beta{}^o + 3 g_1{}^2\, \partial_\mu\, B_\nu{}^o\, \partial_\alpha\, B_\beta{}^o \right] + \\
&+ \frac{i}{8} \cos\phi \left[3\, g^3\, \varepsilon_{abc}\, B_\mu{}^a\, B_\nu{}^b\, \partial_\alpha\, B_\beta{}^c + \right. \\
&+ \left. g^2 g_1\, \varepsilon_{ab3}\, (2\, \partial_\alpha\, B_\mu{}^a\, B_\nu{}^b\, B_\beta{}^o + B_\mu{}^a B_\nu{}^b \partial_\alpha\, B_\beta{}^o)\right] \Big\}
\end{aligned}
\tag{10}
$$

Note that the terms quadrilinear in the fields do not contribute to the anomalous divergence D, because they appear in combinations which are symmetric in their Lorentz indices, therefore vanishing after the multiplication by the antisymmetric tensor $\varepsilon_{\mu\nu\alpha\beta}$.

In the Weinberg model, after spontaneous symmetry breaking, the physical vector bosons (of definite mass) are given as linear combinations of the gauge fields B as follows:

$$\overline{W}_\mu = \frac{1}{\sqrt{2}} (B_\mu^{\ 1} - i\, B_\mu^{\ 2}) \qquad W_\mu = \frac{1}{\sqrt{2}} (B_\mu^{\ 1} + i\, B_\mu^{\ 2})$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g_1^{\ 2}}} (g\, B_\mu^{\ 3} - g_1\, B_\mu^{\ o}) \qquad A_\mu = \frac{1}{\sqrt{g^2+g_1^{\ 2}}} (g_1 B_\mu^{\ 3} + g\, B_\mu^{\ o}) \tag{11}$$

We can also express g and g_1 in terms of the eletric charge, e_o, and the vector bosons masses, M_W and M_Z:

$$g = \frac{M_Z}{\sqrt{M_Z^{\ 2} - M_W^{\ 2}}}\, e_o \qquad g_1 = \frac{M_Z}{M_W}\, e_o \tag{12}$$

Therefore, we can express the anomalous divergence D in terms of the physical fields and parameters, obtaining:

$$\begin{aligned} D = \frac{-e_o^{\ 2}}{192\pi^2} \varepsilon_{\mu\nu\alpha\beta} \Big\{ & 2 \cos\phi \,(1+\tfrac{1}{r^2})\, \partial_\mu \overline{W}_\nu\, \partial_\alpha W_\beta + \\ & + 6(\cos\phi+\sin\phi)\partial_\mu A_\nu \partial_\alpha A_\beta + 3(\cos\phi+\sin\phi)(\tfrac{1}{r}-3r)\partial_\mu A_\nu \partial_\alpha Z_\beta + \\ & + \Big[\cos\phi\, (\tfrac{1}{r^2}+4r^2-1) + \sin\phi\,(r^2-1)\Big] \partial_\mu Z_\nu\, \partial_\alpha Z_\beta \\ & + i e_o (1+\tfrac{1}{r^2}) \cos\phi\, \partial_\beta \Big[\overline{W}_\mu W_\nu (A_\alpha + \tfrac{1}{4} (\tfrac{3}{r} - r) Z_\alpha)\Big] \Big\} \end{aligned} \tag{13}$$

with $r \equiv \sqrt{\frac{M_z^2}{M_W^2} - 1}$. We see from this expression (and using (5)],that the lowest order contributions to the low energy theorem for

$<\gamma_1;\gamma_2 \mid J^5 \mid o>$, where $J^5 = \sin\phi\, J^{5,3} + \cos\phi\, J^{5,0}$

are given by:

$$<\gamma_1;\ \gamma_2 \mid J^5 \mid o> = \frac{e_o^2}{16\pi^2}(\cos\phi + \sin\phi)\ \varepsilon_{\alpha\beta\mu\nu}\ \varepsilon_{1\alpha}\ \varepsilon_{2\beta} k_{1\mu}\ k_{2\nu} \qquad (14)$$

Clearly, the most obvious contributions of higher order in the electric conpling constant might arise from final state scattering of the two photons,as shown in Fig. 1. In this diagram, there is a photon-photon scattering between the anomalous vertex, denoted by ⊗, and the free photons.

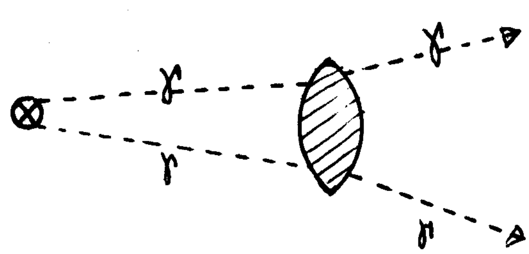

Figure 1

As a result of the antisymmetric tensor structure of the anomalous divergence term, the vertex ⊗ is proportional to $k_1 + k_2$. Also, the diagram for the scattering of photon is, itself, proportional to $k_1 \cdot k_2$ since photon gauge invariance implies that the external photons couple through their field tensors. Thus the diagram is proportional to $k_1 \cdot k_2 (k_1 + k_2)$,and is of higher order than the terms which contribute to the low energy theorem.

There are essentially two types of diagrams which give contributions of order e_o^4 to $<\gamma_1;\ \gamma_2 | D | o>$. In the diagram shown in Fig.2, the fieldstrength operators attach directly onto the external lines, without photon-photon scattering.

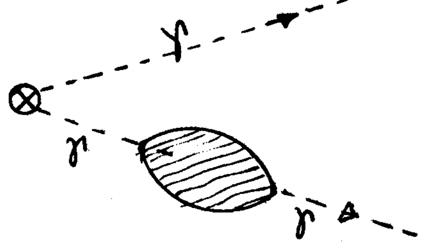

Figure 2

In the unitary gauge, the effect of the vacuum polarization diagram is to change e_0 to e, the renormalized charge. In the lowest order, the blob receives contributions from the pair of vector bosons W or from a pair of charged fermions. In the Weinberg model, there is no contribution to this diagram, in the lowest order, from the physical(massive) scalar meson, since there is no direct coupling between it and A, in this gauge. Figure 3 represents possible fourth order contributions to the low energy theorem in which charged vector bosons emerge from the anomalous vertex and interact to produce a two photon final state.

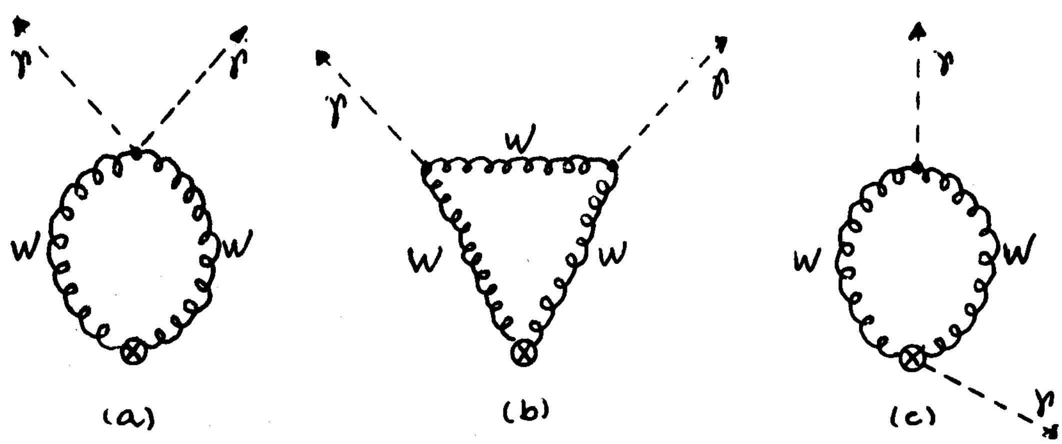

Figure 3

We remark that these diagrams do not receive contributions from the anomalous divergence D^3 (see (13)). We can understand this by observing that, if the radiative corrections are to cancel (to the order e^4), they must cancel independently for D^3 and D^0. Now, in the anomalous vertex corresponding to D^3, the only possible term, which has the isotopic structure corresponding to the third member of an isotriplet, and which yields terms bilinear in W and $\overline{W}$, is: sen $\phi\ \varepsilon_{ab3}\ \partial_\mu\ B_\nu^{\ a}\ \partial_\alpha\ B_\beta^{\ b}$ (see(3)). This term is antisymmetric with respect to the transformations $\mu \leftrightarrow \alpha$, $\nu \leftrightarrow \beta$. Since $\varepsilon_{\mu\nu\alpha\beta}$ is symmetric under these transformations, D^3 will give a vanishing contribution to the anomalous vertex for the diagrams of Fig.3a and 3b. Therefore, in order that the radiative corrections to $< \gamma_1;\ \gamma_2\ |D^3|o>$ cancel, one cannot have terms like sen $\phi\ \varepsilon_{ab3}\ B_\mu^{\ a}\ B_\nu^{\ b}\ \partial_\alpha\ B_\beta^{\ o}$ contributing to the vertex of Fig.3c, which is indeed the case (see (10)). This is due to the fact that this type of terms cancels reciprocally between the first two and the next three terms in(3), thereby verifying the low energy theorem for $<\gamma_1;\gamma_2\ |D^3|o>$.

We now turn to the calculation of $<\gamma_1;\ \gamma_2|D^o|o>$, to the order e^4, which will be done in the unitary gauge using the n-dimensional regularization procedure[(9)]. In this scheme, the integrals are defined by continuation in a n-dimensional space-time. In order to calculate the contributions from the diagrams of Fig.3 to $<\gamma_1;\ \gamma_2|D^o|o>$, we need to know the vertices involving $\overline{W}$ W A and $\overline{W}$ W A A. These vertices can be obtained, in the Weinberg model, from the interaction Lagrangean between the photon field A and the massive vector bosons fields, W,$\overline{W}$:

$$\begin{aligned} L_{int} = & \ ie\ A_\mu\ (\ \partial_\mu\ \overline{W}_\nu\ W_\nu - \overline{W}_\nu\ \partial_\mu\ W_\nu + \overline{W}_\nu\ \partial_\nu\ W_\mu + \\ & - \partial_\nu\ \overline{W}_\mu\ \ W_\nu\ ) + i\ e\ \overline{W}_\mu\ W_\nu\ \ (\partial_\nu\ A_\mu - \partial_\mu\ A_\nu) + \\ & + e^2 \overline{W}_\mu\ W_\nu\ \ (\ \partial_{\mu\nu}\ A^2 - A_\mu\ A_\nu\ ) \end{aligned} \qquad (15)$$

From (13) and (15), we obtain for the diagram shown in Fig.3a:

$$I^{a} = \frac{\cos\phi}{192\pi^2}\left(1+\frac{1}{r^2}\right)e^4\,\varepsilon_{1\alpha}\,\varepsilon_{2\beta}\,J^{a}_{\alpha\beta} \qquad (16)$$

where

$$J^{a}_{\alpha\beta} = \frac{\varepsilon_{\mu\nu\sigma\tau}\,k_\tau}{(2\pi)^n}\int d^nQ\,Q_\sigma\,\frac{1}{Q^2+M_W^2}\cdot\frac{1}{(Q+k)^2+M_W^2}\,\times$$

$$\times\left(\delta_{\mu\alpha}+\frac{Q_\mu Q_\alpha}{M_W^2}\right)\cdot\left(\delta_{\nu\beta}+\frac{(Q+k)_\nu(Q+k)_\beta}{M_W^2}\right)\cdot\left(\delta_{\mu\nu}\,\delta_{\alpha\beta}-\delta_{\mu\alpha}\,\delta_{\nu\beta}\right)$$

with $k=k_1+k_2$.
We remark that $J^{a}_{\alpha\beta}$ depends only on the four vector k, and, therefore, it must vanish identically, since it is not possible to form a pseudotensor involving only a vector.

Making use of this argument, we can write the contributions resulting from the diagrams of Fig.3b and 3c respectively as:

$$I^{b} = \frac{\cos\phi}{192\,\pi^2}\left(1+\frac{1}{r^2}\right)e^4\,\varepsilon_{1\alpha}\,\varepsilon_{2\beta}\,J^{b}_{\alpha\beta} \qquad (17)$$

where

$$J^{b}_{\alpha\beta} = \frac{\varepsilon_{\rho\gamma\xi\omega}\,k_{1\omega}}{(2\pi)^n}\int d^nQ\,Q_\xi\,\frac{1}{(Q^2+M_W^2)^2}\cdot\frac{1}{(Q-k_2)^2+M_W^2}\,\times$$

$$\times\left(\delta_{\gamma\sigma}+\frac{Q_\gamma\,Q_\sigma}{M_W^2}\right)\cdot\left(\delta_{\rho\mu}+\frac{Q_\rho\,Q_\mu}{M_W^2}\right)\cdot\left(\delta_{\tau\nu}+\frac{(Q-k_2)_\tau(Q-k_2)_\nu}{M_W^2}\right)\times$$

$$\times\left[\delta_{\mu\nu}\,(-2Q+k_2)_\alpha+\delta_{\nu\alpha}(Q-2k_2)_\mu+\delta_{\alpha\mu}(Q+k_2)_\nu\right]\times$$

$$\times\left[\delta_{\sigma\tau}\,(-2Q+k_2)_\beta+\delta_{\tau\beta}(Q-2k_2)_\sigma+\delta_{\beta\sigma}(Q+k_2)_\tau\right]$$

and

$$I^{c}= \frac{\cos\phi}{192\pi^2} \left(1+\frac{1}{r^2} \right) e^4 \, \varepsilon_{1\alpha} \, \varepsilon_{2\beta} \, J^{c}_{\alpha\beta} \tag{18}$$

where

$$J^{c}_{\alpha\beta} = \frac{\varepsilon_{\nu\mu\alpha\omega} \, k_{1\omega}}{(2\pi)^n} \int d^nQ \, \frac{1}{Q^2+M_W^2} \cdot \frac{1}{(Q-k_2)^2+M_W^2} \, x$$

$$x \, (\delta_{\mu\sigma}+ \frac{Q_\mu \, Q_\sigma}{M_W^2}) \cdot (\delta_{\nu\tau}+\frac{(Q-k_2)\,\nu \;\; (Q-k_2)\tau}{M_W^2}) \, x$$

$$x \left[\delta_{\sigma\tau}(-2Q+k_2)\beta+\delta_{\tau\beta} \, (Q-2k_2)\sigma \, + \, \delta_{\beta\sigma}(Q+k_2)\tau \right]$$

We will calculate these integrals, keeping only the leading terms of order $k_1 \cdot k_2$. After a straightforward **calculation** we obtain:

$$J^{b}_{\alpha\beta} = -\varepsilon_{\alpha\beta\mu\nu} \, k_{1\mu} \, k_{2\nu} \frac{1}{(2\pi)^n} \, \frac{2}{n} \int d^nQ \, \frac{1}{Q^2+M_W^2} \, Q^2 \left(9+\frac{Q^2}{M_W^2} \right) \tag{19}$$

$$J^{c}_{\alpha\beta} = \varepsilon_{\alpha\beta\mu\nu} \, k_{1\mu} \, k_{2\nu} \frac{1}{(2\pi)^n} \int d^nQ \, \frac{1}{Q^2+M_W^2} \left[3-(1-\frac{4}{n}) \, \frac{Q^2}{M_W^2} \right] \tag{20}$$

We observe that $J^{b}_{\alpha\beta}$ diverges quadratically, while, for $n \to 4$, $J^{c}_{\alpha\beta}$ diverges logarithmically. However, in the n-dimensional regularization scheme the divergences manifest themselves as poles of the gamma-function, $\Gamma(\ell-\frac{n}{2})$, where ℓ integer ≤ 2, and are therefore related by the recurrence property, $\Gamma(x+1) = x \, \Gamma(x)$. Performing the Q integration, we obtain:

$$J^b_{\alpha\beta} = -\varepsilon_{\alpha\beta\mu\nu}\, k_{1\mu}\, k_{2\nu} \frac{1}{(2\pi)^n} \frac{i\pi^{n/2}}{(M_W^{\ 2})^{2-n/2}} \times$$

$$\times\left[\frac{16}{n}\ \Gamma(2-\tfrac{n}{2}) + \Gamma(1-\tfrac{n}{2})\right] = \tag{21}$$

$$= \varepsilon_{\alpha\beta\mu\nu}\, k_{1\mu}\, k_{2\nu} \frac{1}{(2\pi)^n} \frac{i\pi^{n/2}}{(M_W^{\ 2})^{2-n/2}} \frac{7n-16}{n(1-\frac{n}{2})}\, \Gamma(2-\tfrac{n}{2})$$

and

$$J^c_{\alpha\beta} = \varepsilon_{\alpha\beta\mu\nu}\, k_{1\mu}\, k_{2\nu} \frac{1}{(2\pi)^n} \frac{i\pi^{n/2}}{(M_W^{\ 2})^{2-n/2}}\, 3\, \Gamma(2-\tfrac{n}{2}) \tag{22}$$

We find, therefore, that in the n-dimensional regularization, when n=4, $J^b_{\alpha\beta}$ and $J^c_{\alpha\beta}$ are equal and opposite, so that:

$$I = I^a + I^b + I^c = 0 \tag{23}$$

To summarize, we have shown that in the Weinberg model, up to fourth order in the electric charge coupling constant, there are no radiative corrections to the low energy theorem for $<\gamma_1;\ \gamma_2\, |J^5|\, o>$ due to final state scattering of the gauge vector bosons.

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