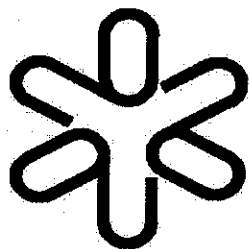


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Anomalous anapole moment of an exotic nucleus

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Abstract

Using the information on the nuclear structure of exotic neutron-rich halo nucleus ^{11}Be , we evaluate the parity violating anapole moment in its ground state. The resulting value $\kappa(^{11}\text{Be}) = 0.17$ is fifteen times bigger than the typical value of the anapole moment of a normal nucleus of the same mass, and in fact exceeds by few times anapole moments of any known neutron-odd nuclei (e.g., $\kappa(^{11}\text{Be}) > 2|\kappa(^{207}\text{Pb})|$). It is also few times bigger than the neutral current contribution.

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The nuclear anapole moment is one of the most interesting manifestations [1–9] of the spatial Parity Nonconservation (PNC) [10,11] in atomic physics. It arises from the PNC nuclear forces which create anomalous (toroidal) contribution to the electromagnetic current. The resulting PNC magnetic field can be experienced by an external lepton (e.g., atomic electron or muon in mesic atom) and can be detected in hyperfine structure atomic measurements. There were several studies of the nuclear AM [2–6]. The first calculation of the quantity in the single-particle approximation has been done by Flambaum, Khriplovich and Sushkov in Ref. [2]. Calculation of the anapole moment with accounting for residual interaction has been made by Haxton, Henley and Musolf [4]. Recently, in Refs. [6] various many-body corrections to the anapole moments (basically, the many-body contributions to the current) have been taken into account. At present, this field attracts much attention [6–9,15] as the first experimental results for the nuclear anapole in *Cs* are already available [7], [9].

Up to now, only the anapole moments of normal nuclei have been considered. The physics of “exotic” nuclei studied with radioactive nuclear beams [17–24] appears to be one of the most promising modern nuclear areas. Specific structure of exotic nuclei can offer new possibilities to probe those aspects of nuclear interactions which are not accessible with normal nuclei. The problem of the PNC effects in exotic nuclei has been addressed only recently in Ref. [12] where it was shown that the PNC mixing in halo nuclei can be considerably enhanced as compared to the case of “normal” nuclei. It is therefore interesting to examine the anapole moments of exotic nuclei.

Here, we evaluate the anapole moment of an exotic halo nucleus, focusing on the case of ^{11}Be which has been extensively studied both experimentally and theoretically [17–21]. We call the resulting anapole moment “anomalous” as it exceeds by fifteen times the average anapole moment of a normal nucleus of the same mass and is bigger than the anapole moment of any known neutron-odd nucleus. The value of the AM is even twice bigger than that of lead.

The Hamiltonian of the nucleus-lepton system can be written in the form

$$H = H_0^n + V_{res}^n + W_{PNC}^n + H_{PNC}^{n-e} + h_{PNC}^{n-e}, \quad (1)$$

where $H_0^n = \sum_i [\mathbf{p}_i^2/2m + U_S(r_i)]$ is the single particle Hamiltonian of the nucleons with momentum \vec{p} and mass m in the single-particle potential $U_S(r)$; V_{res}^n is the residual strong interaction. The operator W_{PNC}^n is the weak PNC nucleon-nucleon interaction [10]. The term H_{PNC}^{n-e} describes the interaction of the lepton with the vector potential \vec{A}_{PNC} created by the nucleus, in which we save only the PNC part,

$$H_{PNC}^{n-e} = e(\vec{\alpha}\vec{A}_{PNC}) = e(\vec{\alpha}\langle\vec{a}\rangle)\Delta(\vec{r}) \quad (2)$$

where $\vec{\alpha}$ denote the Dirac matrices [27] for the lepton and $\Delta(\vec{r})$ is a function sharply peaked in the region of the nucleus, it reduces to the δ -function on the scale of the atomic electron spatial motion, e is the proton charge, $e^2 = \frac{1}{137}$. The last term, h_{PNC}^{n-e} , is the part of the neutral current interaction contributing to the PNC nucleus-lepton forces depending on nuclear spin,

$$h_{PNC}^{n-e} = \kappa_{nc} \frac{G [1/2 - (-1)^{j+l+1/2}(j+1/2)]}{\sqrt{2} j(j+1)} (\vec{j}\vec{\alpha})\Delta(\vec{r}), \quad (3)$$

where $\kappa_{nc} \equiv (5/8)(1 - 4\sin^2\theta)$ with θ the Weinberg angle. The vector $\langle\vec{a}\rangle$ is the expectation value of the anapole moment operator

$$\vec{a} = -\pi \int d^3r r^2 \vec{J} \quad (4)$$

in the nuclear ground state, where \vec{J} is the nuclear electromagnetic current. Its is convenient to define the ‘‘anapole moment’’, κ , rewriting Eq.(2) according to [3]

$$H_{PNC}^{n-e} = e(\vec{\alpha}\langle\vec{a}\rangle)\Delta(\vec{r}) \equiv \kappa \frac{G (-1)^{j+l+1/2}(j+1/2)}{\sqrt{2} j(j+1)} (\vec{j}\vec{\alpha})\Delta(\vec{r}), \quad (5)$$

where \vec{j} is the nuclear spin in the ground state which coincides with the angular momentum of the external nucleon if one works in the single-particle approximation; where $G = 10^{-5}m^{-2}$ is the Fermi constant and m is the nucleon mass. The factors depending on j and on the orbital angular l of the external nucleon absorb the spin-angular dependence of the anapole

expectation value $\langle \vec{a} \rangle$, and the anapole moment κ chosen in this way contains merely the nuclear structure information.

In the single-particle approximation, the anapole moment operator (4) is the sum of the spin- and orbital terms [3]

$$\vec{a} = \frac{\pi e}{m} \sum_i \left(\mu_i \vec{r}_i \times \vec{\sigma}_i + \frac{q_i}{2} \{ \vec{p}_i, r_i^2 \} \right). \quad (6)$$

where $\vec{\sigma}$ are the spin Pauli matrices, μ are the nucleon magnetic moments [$+2.79$ for proton and -1.91 for neutron], q measures nucleon charge [$q = 1(0)$ for protons (neutrons)] and $\{, \}$ denotes anticommutator. The expectation value of (6) in any eigenstate of the nuclear Hamiltonian, $H_0^n + V_{res}^n$ is zero unless parity violating forces W_{PNC}^n are taken into account. As a result of the PNC weak interaction W_{PNC}^n in the Hamiltonian (1), a nuclear state of definite parity $|\psi\rangle$, acquires very small admixtures of wrong parity configurations $|\bar{\psi}_n\rangle$. This can be accounted for by using the first order of perturbation theory with respect to W_{PNC}^n . Thus the expectation value of the anapole moment operator \vec{a} in the state $|\tilde{\psi}\rangle$ with energy E containing the PNC admixtures is

$$\langle \tilde{\psi} | \vec{a} | \tilde{\psi} \rangle = \sum_n \left(\frac{\langle \psi | W_{PNC} | \bar{\psi}_n \rangle}{E - E_n} \langle \bar{\psi}_n | \vec{a} | \psi \rangle - \langle \psi | \vec{a} | \bar{\psi}_n \rangle \frac{\langle \bar{\psi}_n | W_{PNC} | \psi \rangle}{E_n - E} \right) \quad (7)$$

where sum runs over the opposite parity states $|\bar{\psi}_n\rangle$. In a finite nucleus, a nucleon experiences the combined action of the two-body PNC forces W_{PNC} [10] from other nucleons, which can be modeled [11] by the effective one-body PNC weak potential w_{PNC}

$$w_{PNC} = g \frac{G}{2\sqrt{2}m} \{ (\vec{\sigma} \vec{p}), \rho \}, \quad (8)$$

The nuclear core density $\rho = \sum_{occ} |\psi_{occ}|^2$ in (8) reflects the coherent contribution from all the occupied nucleon orbitals. The dimensionless constants g for proton and neutron are $g_p = 4.5 \pm 2$, $g_n = 1 \pm 1.5$. These widely used values [11,2,3,14] correspond to the best values [10] of the microscopic parameters in the DDH Hamiltonian [10]. They are found in reasonable agreement with the bulk experimental data on PNC including the compound nuclear experiments by TRIPLE group [13] and anapole moments of stable nuclei [9].

The basic specific properties of the halo nuclei are determined by the fact of existence of loosely bound nucleon in addition to the core composed by the rest of the nucleons [19].

In one-body halo nuclei like ^{11}Be , the ground state is particularly simple: it can be represented as direct product of the single-particle wave function of the external neutron, ψ_{halo} , and the wave function of the core. The spin-saturated core does not contribute to (7). The residual interaction V_{res}^n in (1) can be neglected as the many-body effects related to the core excitations are generically weak in such nuclei [25]. As a result of the relatively heavy core for $A \simeq 10$, difference between the center of mass coordinate and the center of core coordinate can also be neglected. The problem with the Hamiltonian (1) and (8) is reduced to a single-particle problem for the external nucleon. For the nucleus with the external neutron, as is the case for the halo nucleus ^{11}Be , the orbital part of the anapole operator (6) does not contribute. Using the reduced matrix elements of $\langle l', j, m | \vec{r} \times \vec{\sigma} | l, j, m \rangle = i(-1)^{j+l+1/2}(j+1/2)\sqrt{\frac{2j+1}{j(j+1)}}$, $l' = l \pm 1$, the expression for the anapole moment in terms of the radial wave functions R_{nlj} is

$$\kappa = -\frac{2\pi\mu_n e^2 g_n}{m^2} \sum_{n'l'j} \frac{\int_0^\infty r^2 dr R_{n'l'j} \left[\rho \left(\frac{dR_{nlj}}{dr} + \frac{(l-j)(2l+1-j)}{r} R_{nlj} \right) + \frac{d\rho}{2dr} R_{nlj} \right]}{E_{nlj} - E_{n'l'j}} \int_0^\infty r^3 dr R_{n'l'j} R_{nlj} \quad (9)$$

The effective potential that binds external neutron is rather shallow yielding small single-neutron separation energy, so one can expect small energy spacing between the opposite parity states. The PNC effect in (9) can therefore be considerably magnified [12]. The nucleus ^{11}Be has the only bound excited state, $1p_{1/2}$, above the ground state $2s_{1/2}$ [17,18], [20], [21] (the well known "inversion of levels"). As a result of the small energy separation between these levels of opposite parities which is known experimentally,

$$|\Delta E_{\text{HALO}}| = E_{p_{1/2}} - E_{s_{1/2}} = 0.32 \text{ MeV}, \quad (10)$$

one can save the only $1p_{1/2}$ term in the expression (9) for the anapole moment κ of the ground state $2s_{1/2}$. The form of the single-particle wave functions of halo states can be deduced from their basic properties [21] and their quantum numbers [18]. The results of the Hartree-Fock calculations which reproduce the main halo properties (e.g., mean square

radii) are also available [18]. We use the following ansatz for the model wave functions of the 2s and the excited 1p halo states:

$$R_{2s}(r) = \frac{2^{3/2}a^2(1 - (r/a)^2)\exp(-r/r_0)}{r_0^{3/2}\sqrt{45r_0^4 + 2a^4 - 12a^2r_0^2}}, \quad R_{1p}(r) = \frac{2}{\sqrt{3}}r_1^{-5/2}\exp(-r/r_1), \quad (11)$$

The values of the parameters r_0 , a and r_1 must be chosen to fit the density distributions [18] and the mean square radii

$$\langle r_{2s}^2 \rangle = r_0^2 \frac{6(45r_0^4 + 2a^4 - 12a^2r_0^2)}{105r_0^4 + a^4 - 15a^2r_0^2}, \quad \langle r_{1p}^2 \rangle = \frac{15}{2}r_1^2. \quad (12)$$

The value of $a = 2fm$ in the wave function of the 2s state is determined by the position of the node [18], [21] which can be extracted, e.g., from the neutron scattering experiments [21]. The core nucleon density $\rho_c(r)$ has been taken according to Ref. [18]

$$\rho_c(r) = \rho_0 e^{-(r/R_c)^2}, \quad \rho_0 = 0.2fm^{-3}, \quad R_c = 2fm, \quad (13)$$

as shown on Fig.1. Evaluation of (9) with the wave functions (11) and the core density (13) gives the expression for the anapole moment in terms of the parameters:

$$\begin{aligned} \kappa &= \frac{\pi\mu e^2 g_n \rho_0}{m^2 |\Delta E|} \frac{4R_c^{10} a^2 [a^2(r_0 + r_1)^2 - 30r_0^2 r_1^2]}{r_0^5 r_1^7 (45r_0^4 + 2a^4 - 12a^2 r_0^2)} \times \\ &\times \left\{ 3I_2(y) - \left[3 \left(\frac{R_c}{a} \right)^2 + 1 \right] I_4(y) + \left(\frac{R_c}{a} \right)^2 I_6(y) - \frac{R_c}{r_1} \left[I_3(y) - \left(\frac{R_c}{a} \right)^2 I_5(y) \right] \right\} \quad (14) \end{aligned}$$

where $y = \frac{R_c(r_0+r_1)}{2r_0r_1}$ and the functions $I_n(y) = (-1)^n \frac{\sqrt{\pi}}{2^{n+1}} \frac{d^n}{dy^n} e^{y^2} \text{erfc}(y)$, are given in terms of the error function $\text{erfc}(y) = 1 - \frac{2}{\sqrt{\pi}} \int_0^y dt \exp(-t^2/2)$.

The results for the densities calculated with the optimal values of the parameters, $r_0 = 1.45$ fm, $r_1 = 1.80$ fm, are shown in Fig.1. One sees good agreement with the Hartree-Fock calculations [18]. The values of the halo radii given by (12), $\sqrt{\langle r_{2s}^2 \rangle} = 5.9fm$ and $\sqrt{\langle r_{1p}^2 \rangle} = 4.9fm$ are close to the values of Ref. [18] $6.5fm$ and $5.9fm$ which agree with experimental matter radii.

With the above values of the parameters, we finally obtain from (9) the resulting value of the anapole moment κ

$$\kappa(^{11}\text{Be}) = 0.17g_n = 0.17 \quad (\text{for } g_n \simeq 1). \quad (15)$$

It is few times bigger than the contribution from neutral current $-\kappa_{nc} = -0.05$, thus the nuclear spin-dependent PNC interaction of a lepton with the halo nucleus is dominated by the anapole moment contribution, as in heavy nuclei.

To appreciate how big the value $\kappa(^{11}\text{Be})$ is, one can compare (15) to the anapole moment of the normal spherical nucleus with odd neutron which is given by [3]:

$$\kappa_{norm} = \frac{9}{10} \frac{g_n e^2 \mu_n}{m r_0} A^{2/3}, \quad (16)$$

where $r_0 = 1.2\text{fm}$ is the nucleon radius. Resulting from the PNC toroidal electromagnetic currents, the anapole moment grows fast ($\propto A^{2/3}$) as the size of the system increased [2,3]. For this reason, the anapole moments of *normal* light nuclei give only a small correction to the neutral current lepton-nucleus PNC interaction (see Fig. 2). From (15) and (16), we find the ratio of the anapole moment to its value in a nucleus with the same A (enhancement factor):

$$R_{halo} = \frac{\kappa(^{11}\text{Be})}{\kappa_{norm}} = 15. \quad (17)$$

In fact, the anapole moment (15) exceeds few times the anapole moments of any known odd nucleus, as seen in Fig.2. For example, the $\kappa(^{11}\text{Be})$ is two times bigger than the anapole moment of nucleus as heavy as lead [3],

$$\kappa(^{207}\text{Pb}) = -0.08g_n$$

The remarkable enhancement factor (17) in (15) comes from two features of the halo structure: a) enhancement of the PNC mixing in the halo ground state and b) enhancement of the matrix elements of the anapole operator in halo states

$$R_{HALO} \sim \frac{\omega}{\Delta E} \left(\frac{w_{halo}}{w_{norm}} \right) \left(\frac{r_{halo}}{r_{norm}} \right) \quad (18)$$

The parity violating effect originates from the weak interaction of the external halo neutron with the core nucleons in the nuclear interior. As a result, the neutron halo cloud surrounding the nucleus acquires the wrong parity admixtures. Those give rise to the anomalous

PNC toroidal currents in the nuclear exterior (the halo region) which results in additional enhancement of the anapole moment (the last factor in (18)). As is seen from Table I, the results are stable with respect to variation of the details of the halo structure. We have neglected here the contribution from the core excitation to the ^{11}Be ground state [28] which is found however to be rather small, $\approx 16\%$. [29]. The inclusion of the structure of the core in the calculations is in progress.

This curious "halo anomaly" is quite interesting in a number of respects. First, search for sources of enhancement in anapole moments has been always important from the experimental viewpoint. Possibilities offered by the normal nuclei are rather limited here. The most promising case of deformed nuclei, where one can find close levels of opposite parity near the ground state, does not offer any enhancement because of the suppression in the matrix elements of \vec{a} [3]. In this respect, the anomalies in anapole moments of exotic nuclei like ^{11}Be seem to give unexpected opportunity. Secondly, the anapole moment of neutron-rich nuclei is determined by the neutron weak constant g_n only. Usually, sensitivity of experiments to the value of this constant is "spoiled" by relatively large value of the proton weak constant g_p , in Eq.(8). The large enhancement of the anapole moment in neutron halo nuclei provides therefore an unique opportunity to test the isospin structure of the weak potential (8) which is at present of great interest [15].

One should note that the nucleus ^{11}Be has a rather long life-time (13.81 sec). This makes therefore possible the atomic measurements of the hyperfine structure effects in traps planned for the future ISOL facility where the anapole moment can be detected.

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TABLES

TABLE I. Dependence of $\kappa(^{11}\text{Be})$ on the halo parameters r_0 and r_1 in Eq.(14); the ratios to (15) are given. The central entry in the table corresponds to the optimal values. It is seen that variations in r_0 and r_1 do not affect (15) any considerably.

	$r_0 = 1.35$	$r_0 = 1.40$	$r_0 = 1.45$	$r_0 = 1.50$	$r_0 = 1.55$
$r_1 = 1.70$	1.26	1.15	1.05	0.96	0.87
$r_1 = 1.75$	1.23	1.12	1.03	0.94	0.85
$r_1 = 1.80$	1.19	1.09	1.00	0.91	0.83
$r_1 = 1.85$	1.16	1.06	0.97	0.89	0.81
$r_1 = 1.90$	1.12	1.03	0.95	0.87	0.80

Figure Captions

Fig.1. Densities $R^2(r)$ for the halo states $2s$ and $1p$ as function of r and the core density $\rho_c(r)$ calculated from Eqs.(11,13) (solid lines). The Hartree-Fock results for the same quantities [18] are given by the dashed line, the dotted line and the dotted-dashed line, respectively.

Fig.2. "Halo anomaly" in ^{11}Be : the value $\kappa(^{11}\text{Be})$ (circle) as compared to the absolute values of the anapole moments of normal neutron-odd nuclei (solid curve) and the neutral current contribution $\kappa_{nc} = 0.05$ (dashed curve).

Fig. 1.

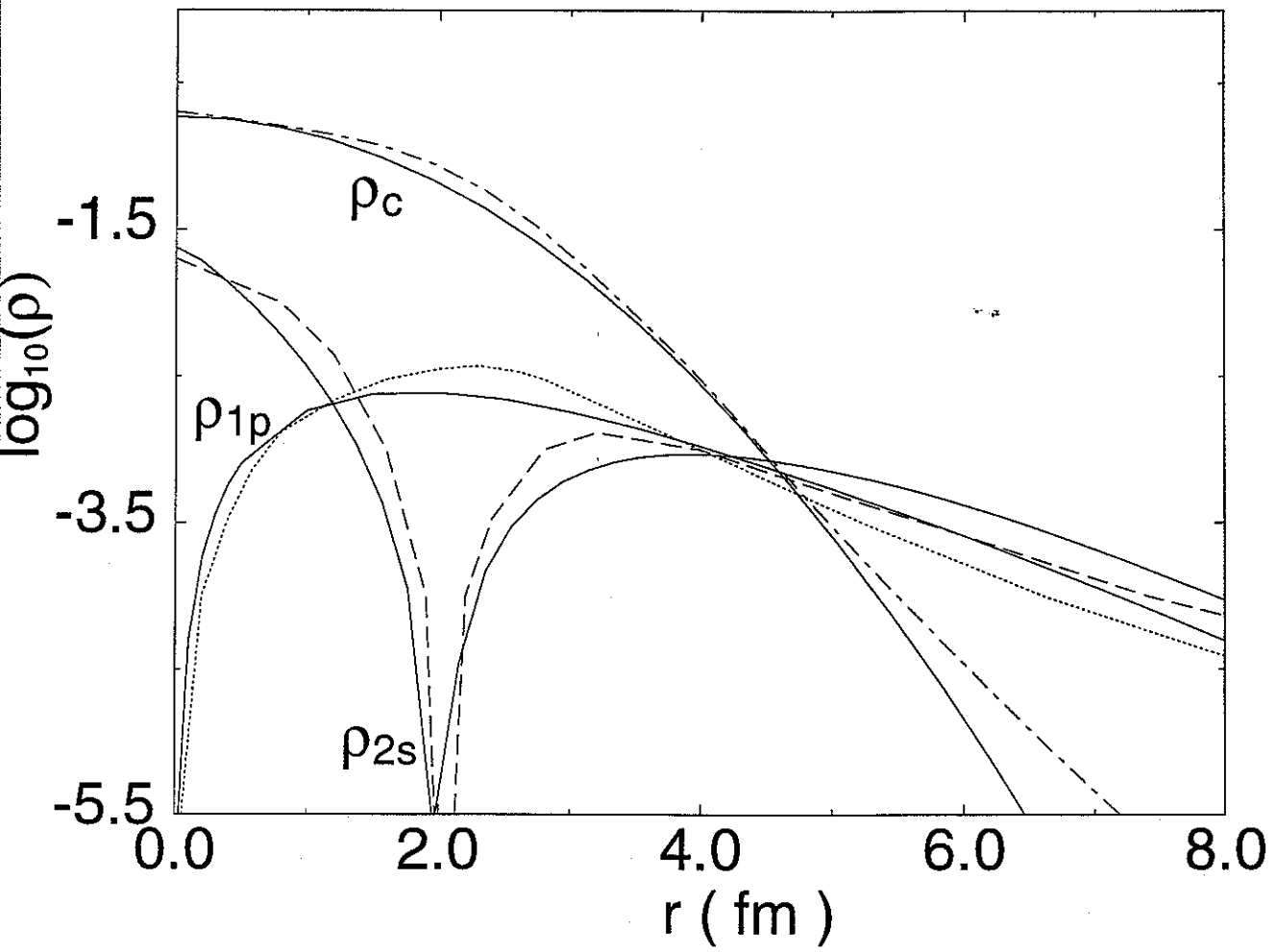


Fig. 2.

