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Input-Output Analysis of Pricing and Inflation

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Abstract

The present paper intends to show through input-output analysis, for the simple case of an isolated ideal economy that inflation is intrinsic to economic process and that it is acerbated by intermediation and taxing.

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1 Introduction

Input-output theory [Leontieff (1975)] is founded in two vectorial equations that relate, both through the technical coefficients matrix, four economic variables; one relating *output* with *input* and the other relating *prices* with *costs.* Two additional equations are then required to complete the system of equations.

In the present paper it is shown, for the simple case of an isolated economy that when one of the Leontieff's additional equations is replaced by a phenomenological law that reflects actual pricing, a new system of equations results that explain inflation as a natural consequence of this law.

1.1 Fundamental Identities

The technical-coefficients matrix $\mathbf{Q}(n \times n)$ — whose coefficients q_{kj}, represent the amount of input k needed to produce one unity of output j — establishes the relationship between input $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}'$ and output $\mathbf{z} = \{z_1, z_2, \ldots, z_n\}'$ of an economy

$$\mathbf{x} = \mathbf{Q} \times \mathbf{z}. \tag{1}$$

If $\mathbf{c} = \{c_1, c_2, \ldots, c_n\}'$ represents the column vector of input production costs and $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}'$ the corresponding column vector of output prices, then

$$\mathbf{c} = \mathbf{Q}' \times \mathbf{p} \tag{2}$$

where $\mathbf{Q}'$ is the transpose of $\mathbf{Q}$.

Leontieff's Additional Rules. From the fundamental relations (1) and (2), different theories can be derived, depending on the additional hypothesis introduced to complete the system of equations. Leontieff introduced two new *known* variables, the *output surplus*

$$\mathbf{y} = \mathbf{z} - \mathbf{x}, \tag{3}$$

that allows the elimination of $\mathbf{x}$ in (1)

$$\mathbf{z} = (\mathbf{I} - \mathbf{Q})^{-1} \times \mathbf{y}, \tag{4}$$

where $(\mathbf{I} - \mathbf{Q})^{-1}$ is the Leontieff's inverse matrix and the *value added to a unity of output* $\mathbf{v}$ through the identity

$$\mathbf{p} - \mathbf{c} = \mathbf{v}, \tag{5}$$

that allows the elimination of $\mathbf{c}$ in (2)

$$\mathbf{p} = (\mathbf{I} - \mathbf{Q}')^{-1} \times \mathbf{v}. \tag{6}$$

2 The Re-Marking Law

The *re-marking law* that rules the dynamics of prices is the formal expression for the rationale suppliers usually invoke in inflationary economies to justify price mark-ups.

If the suppliers of outputs $1, 2, \ldots, n$, are mutually independent, i.e., if no economic agent can affect its input prices, then the price of an output at a given instant $t+1$ is a function of its cost at the previous instant t

$$\mathbf{p}(t+1) = h(\mathbf{c}(t), t), \tag{7}$$

where the function h characterizes the pricing practice in the economy.

2.1 The Re-Marking Law in Input-Output Theory

In this section we will replace Leontieff's hypothesis (5) by the re-marking law (7).

Substituting (2) in (7), it follows

$$\mathbf{p}(t+1) = h(\mathbf{Q}' \times \mathbf{p}(t), t). \tag{8}$$

In real economies, $\frac{\partial h}{\partial t} \neq 0$, i.e., not only prices, but the pricing mechanisms themselves, vary with time. Although not realistic, it is illustrative to analyse the cases in which $\frac{\partial h}{\partial t} = 0$. With this simplification, (8) can be rewritten as

$$\mathbf{p}(t+1) = h(\mathbf{Q}' \times \mathbf{p}(t)). \tag{9}$$

Iterative equations of the form (9) have been thoroughly studied in relation to fractals. It is known that their solutions may present, even in simple cases of the function h, highly complex behaviours. Nevertheless, for the purpose of finding the simplest model for an "ideal" economy, it is instructive to study the simplest case of the function h in order to show the possible influences of the re-marking law on price evolution.

In the following, we consider two simple examples of pricing mechanisms: the *Cost-Bound* and the *Percentage Mark-up.* These two ideal examples are indicative of the transitive nature of price mechanisms that give rise to *intrinsic inflation* in real economies.

2.2 Cost-Bound Pricing

Let us consider an ideal economy where each supplier establishes the prices of its output at a given instant, as equal to its cost at the previous instant. The re-marking law, in this case, is then expressed by the identity

$$\mathbf{p}(t+1) = \mathbf{c}(t), \tag{10}$$

i.e., h is the *identity* function. Economies whose pricing is given by equation (10) will be called cost-bound economies. Combining (2) and (10), we have the finite differential equation

$$\mathbf{p}(t+1) = \mathbf{Q}' \times \mathbf{p}(t). \tag{11}$$

2.3 Percentage Mark-up Pricing

In a more usual pricing practice, economic agents create profits by adding, to the input cost of any product or service k, not the cost of their contribution to the production chain, but an arbitrary percentage l_k of the cost value c_k of their inputs to form their output prices. Conventional taxing practice also adds an arbitrary percentage i_k to the cost, so that the combined price of k becomes

$$p_k(t+1) = (1+l_k)(1+i_k)\, c_k(t) = r_k c_k(t). \tag{12}$$

Denoting by $\mathbf{R} = \{r_1, r_2, \ldots, r_n\}$ the diagonal matrix of elements r_k, equation (12) can be rewritten as

$$\mathbf{p}\,(t+1) = \mathbf{R}{\times}\mathbf{c}\,(t)\,. \tag{13}$$

Economies whose pricing practice are described by identity (13) are called *price mark-up* economies.

Combining (2) and (13) it follows the equation

$$\mathbf{p}\,(t+1) = \mathbf{R} \times \mathbf{Q}' \times \mathbf{p}\,(t) = \mathbf{S}{\times}\mathbf{p}\,(t)\,. \tag{14}$$

that defines $\mathbf{S}$.

2.4 Inflation

Both equations (11) and (14) cab be represented by the common equation

$$\mathbf{p}\,(t+1) = \mathbf{M} \times \mathbf{p}\,(t)\,, \tag{15}$$

where $\mathbf{M}$ stands for $\mathbf{Q}'$ or $\mathbf{S}$ and whose solution is known to be

$$\mathbf{p}(t+\tau) = \mathbf{M}^{\tau} \times \mathbf{p}(t). \tag{16}$$

If $\mu(\mathbf{M})$ is the only dominant eigenvalue of $\mathbf{M}$ and ρ its corresponding eigenvector, then (15) can be expressed asymptotically, for $t \gg 0$, by the identity

$$\rho\,(t+1) = \mu(\mathrm{M}) \cdot \rho\,(t)\,,$$

i.e., inflation tends to the dominant eigenvalue $\mu(\mathbf{M})$.

Since $\mu(\mathbf{R}) > 1$, and $\mu(\mathbf{S}) = \mu(\mathbf{R}) \cdot \mu(\mathbf{Q}')$, it follows $\mu(\mathbf{S}) > \mu(\mathbf{Q}')$, i.e., percentage mark-up pricing practice always aggravates intrinsic inflation.

References

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