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" ON THE CALCULATION OF NUCLEAR MOMENT OF INERTIA WITH GENERALIZED PAIRING"

A. KAMIMURA

Pelletron Laboratory

Instituto de Física

Universidade de São Paulo

BRASIL

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ABSTRACT

A generalization of Beliaev's expression for the nuclear moment of inertia is obtained self-consistently with generalized pairing. In a particular case it coincides with Beliaev's expression.

RESUMO

Obtemos uma generalização da expressão do momento de inércia nuclear de Beliaev com emparelhamento generalizado, tratado de maneira auto consistente. Ela coincide com a expressão de Beliaev num caso particular.

The nuclear moment of inertia was calculated, in a not so recent work by S.T.Beliaev (1) taking account the nucleon pairing in the framework of the BCS model. The expression is:

$$I = \sum_{\alpha\beta} < \alpha \mid jx \mid \beta > (U_\beta V_\alpha - V_\beta U_\alpha) \frac{f\beta\alpha}{\omega} \qquad (1)$$

where:

(α, β) single particle-states

jx = angular momentum operator related to the rotation X axis

v, (u) = particle occupation (non occupation) probabilities

$f\beta\alpha$ = coefficients that could be obtained self-consistently.

ω = nuclear angular velocity.

We generalized this result, treating the pairing by means of the general Bogoliubov's canonical transformation:

$$b_i^+ = \sum_\alpha (A_\alpha^i C_\alpha^+ + B_\alpha^i C_\alpha)$$

$$b_i = \sum_\alpha (B_\alpha^{i*} C_\alpha^+ + A_\alpha^{i*} C_\alpha) \qquad (6)$$

with $\mid \psi_o >$ as the vacuum states for the b's operator, such that:

$$b_i \mid \psi_o > = o$$

C^{+}_{α} (C_{α}) is the creation (annihilation) operator of a nucleon in the state α. The A's and B's coefficients in the more general form,are complex, allowing generalized pairing at T=0 and T=1[3].

We obtain the moment of inertia searching for the lowest state of the system, with a fixed average value of the angular momentum about the axis of rotation X. For this purpose we add the term H_{ω} to the general Hamiltonian H, such that

$$H' = H + H_{\omega} \qquad (7)$$

where

$$H = \sum_{\alpha\beta} < \alpha | \; T \; |\beta> C^{+}_{\alpha} \; C_{\beta} + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} <\alpha\beta|v|\gamma\delta> C^{+}_{\alpha} \; C^{+}_{\beta} \; C_{\delta} \; C_{\gamma} \qquad (8)$$

$$H_{\omega} = -\omega \sum_{\alpha\beta} < \alpha | \; jx \; | \; \beta > C^{+}_{\alpha} \; C_{\beta} \qquad (9)$$

Calculating the average values of H' in the vacuum state $|\psi_{0}>$, we get:

$$<H'> = \sum_{\alpha\beta} <\alpha| \; T + \frac{1}{2} V - \lambda_{p} - \lambda_{n} - \omega Jx \; |\beta> \rho\alpha\beta + \frac{1}{2} \Delta_{\alpha\beta} \; \kappa^{*}_{\beta\alpha} \qquad (10)$$

where:

$\lambda_{p(n)}$ is the Lagrange multiplier, to reproduce correctly the expectation value of the number of protons (neutrons).

T = kinetic energy

V = self-consistent potential with correction due to rotations[4] through the density matrix $\rho\gamma\delta$

$$\langle\alpha|V|\beta\rangle = \sum_{\gamma\delta} (\alpha\gamma|\ v\ |\beta\delta)\ \rho\gamma\delta \tag{11}$$

$\rho\alpha\beta$ = density matrix defined by:

$$\rho\alpha\beta = \langle\psi_o|\ C^+_\alpha\ \ C_\beta|\ \psi_o\rangle = \sum_i B^{i*}_\alpha\ B^i_\beta \tag{12}$$

$\kappa_{\delta\gamma}$ = pairing density matrix, given by:

$$\kappa_{\delta\gamma} = \langle\ \psi_o|\ C_\delta\ C_\gamma|\ \psi_o\rangle = \sum_i A^i_\delta\ B^{i*}_\gamma \tag{13}$$

$\Delta_{\alpha\beta}$ = pairing potential:

$$\Delta_{\alpha\beta} = \frac{1}{2} \sum_{\gamma\delta} (\alpha\beta|v|\gamma\delta)\ \kappa_{\delta\gamma} \tag{14}$$

We used to solve (10) the well known method[2] that lead us to the HFB non linear equations with self-consistent solution;

$$\begin{aligned} &\sum_\beta\{\langle\alpha|\ \Gamma\ |\beta\rangle\ A^i_\beta + \Delta_{\alpha\beta}\ B^i_\beta\} = E_i\ A^i_\alpha \\ &\sum_\beta\{\Delta^*_{\alpha\beta}\ A^i_\beta + \langle\alpha|\ \Gamma^*|\beta\rangle B^i{}_\beta\} = -E_i\ B^i_\alpha \end{aligned} \tag{15}$$

with $\Gamma = (\ T - \lambda_p - \lambda_n - \omega Jx + V\ )$. (16)

We get the moment of inertia by varying $\langle H' \rangle$ with respect to B_{α}^{i*}:

$$\frac{\partial \langle H' \rangle}{\partial B_{\alpha}^{i*}} = 0 + \sum_{\beta} \langle \alpha | T + \frac{1}{2} V - \lambda_p - \lambda_n | \beta \rangle B_{\beta}^{i} + \frac{1}{4} \sum_{\delta} (\alpha\beta | v | \gamma\delta) \times$$

$$A_{\delta}^{i} \, \delta_{\alpha\gamma} \, \kappa_{\beta\alpha}^{*} = \omega \sum_{\beta} \langle \alpha | jx | \beta \rangle B_{\beta}^{i} \tag{17}$$

Substituting (17) into $\langle H \rangle$:

$$\langle H \rangle = \frac{1}{4} \sum_{\substack{\gamma\delta \\ \gamma \neq \alpha}} (\alpha\beta | v | \gamma\delta) \sum_{i} A_{\delta}^{i} B_{\gamma}^{i*} \kappa_{\beta\alpha}^{*} + \omega \sum_{\alpha\beta} \langle \alpha | jx | \beta \rangle \rho\alpha\beta \tag{18}$$

or

$$\langle H \rangle = W_o + \frac{1}{2} \omega^2 \left(2 \sum_{\alpha\beta} \langle \alpha | jx | \beta \rangle \frac{\rho\alpha\beta}{\omega} \right) \tag{19}$$

where W_o is the ω independent term of (18).

We can identify the round bracket in (19) as the moment of inertia. This is obtained as seen, following a self-consistent Cranking model where the density matrix is constructed, taking account both the generalized pairing and the nucleon rotation, in a iterative process.

We see that the moment of inertia:

$$I = 2 \sum_{\alpha\beta} \langle \alpha | jx | \beta \rangle \frac{\rho\alpha\beta}{\omega} \tag{20}$$

is the same Beliaev's expression [1] in one particular canonical transformation. In fact the term

$$(U_{\beta} V_{\alpha} - V_{\beta} \; U_{\alpha}) \;\; f\beta\alpha$$

corresponds to the expected value of the $C^{+}_{\alpha} C_{\beta}$ operator in the vacuum of this quasiparticles or in other words to the density matrix $\rho\alpha\beta$ for this case.

The apparent simplicity of equation (20) involves beyond the well known hard numerical solution of HFB equations, the treatment of the rotation part with one more Lagrange multiplier.

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