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NUCLEAR EXCITATIONS BY ELECTRONS AND POSITRONS

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ABSTRACT

Existing measurements of the ratio σ^-/σ^+, the nuclear excitation cross sections produced by electrons and positrons, are compared with the results of distorted wave calculations. The behaviour as a function of atomic number Z and electron energy E_o is shown to be principally due to Coulomb distortion. The value of σ^-/σ^+ displays no clear structure corresponding to resonances in the photo absorption cross section, except that it should be quite sensitive to the presence of quadrupole strength.

SUMÁRIO

As medidas da razão σ^-/σ^+ entre as secções de choque para excitação nuclear produzidas por elétrons e pósitrons, existentes na literatura, são comparadas com o resultado dos cálculos em DWBA. O comportamento em função do número atomico Z e da energia E_o do elétron incidente é mostrado ser devido principalmente à distorção Coulombiana. O valor de σ^-/σ^+ não apresenta estruturas correspondentes a ressonância na secção de choque de fotoabsorção, mas é bastante sensível à presença de absorção de quadrupolo.

INTRODUCTION

It is our purpose to review available data comparing the yield of excited nuclear states produced by inelastic electron scattering, with that produced by positron scattering. We are interested to see if such data are compatible with theoretical estimates and to consider the possible advantages of such measurements, especially in view of the fact that there are now in existence several linear accelerators which could produce good positron beams.

A popular way of looking at electro excitation is to consider the field produced at the nuclear site by the passing electron and analyse this into radiation multipoles, so that the effect on the nucleus may be compared with that produced by electromagnetic radiation. In classical theory this is the well known method of Weizacker and Williams [1]), and in quantum theory is usually called the virtual photon method [2]). Under suitable conditions, which we will have to examine in detail later, it is possible to reproduce the electro-excitation cross section by calculating exactly as for photo-excitation, but substituting a virtual photon spectrum for the actual photon spectrum:

$$\sigma^{\pm}(E_o) = \int_0^{E_o} N^{\pm}(E_o, E)\sigma_\gamma(E)\ \frac{dE}{E} \qquad (1)$$

In eq(1) $\sigma_\gamma(E)$ is the photo-excitation cross section as a function of photon energy, E_o is the maximum energy present in the photon spectrum (in this case equal to the kinetic energy of the incident electron) and $N^{\pm}(E_o,E)$ is the spectrum of photons with energy E originating from the incident beam of electrons with energy E_o. The labels $\pm$ distinguish the two charge states of the electrons. Because positrons are repelled by the nucleus whereas electrons are attracted, the amplitude of a positron wave function is generally smaller in the neighborhood of the nucleus than that of an electron of compatible energy. And since in all other respects the electromagnetic interactions of electrons and positrons are identical, the number of virtual photons produced by positrons is expected to be less than that produced by electrons. The immediate conclusion is that the difference in electron and positron cross sections has little to do with nuclear physics but is a consequence solely of electrodynamics. Insofar as this is true, the measurements of the ratio of cross sections σ^-/σ^+ serve as a test of the accuracy of the distorted waves, which is perhaps not seriously in doubt, and also a test of the approximations underlying the virtual photon hypothesis, which are certainly open to question.

Should it be possible to establish the credentials of the virtual photon method, there are certain advantages in making measurements of nuclear excitation by electrons and positrons in addition to the conventional photo excitation. Among these is a possible sensitivity of the ratio σ^-/σ^+ to the presence of electric quadrupole strength (as opposed to dipole) in the nuclear excitation spectrum. We will return to this point in the conclusions.

The experiments themselves are obviously hindered by the feeble currents available with positron beams. It is possible, however, that this could be improved, particularly in view of the interest in other uses of positron beams (e.g., annihilation in flight).

Experiments on the ratio σ^-/σ^+

Experiments on the disintegration of nuclei which employed both positrons and electrons were first performed in 1965 by Herring *et al* 3) on ^{12}C, ^{63}Cu, ^{107}Ag and ^{181}Ta. Using electrons and positrons of various energies up to 32 MeV, the ratio of the cross sections was obtained by comparing the activation induced by the two projectiles. Apart from corrections necessary to allow for excitation via positron annihilation gamma rays, most sources of error are substantially eliminated by the act of taking the ratio of two measurements, which have been made under virtually identical conditions.

The cross section ratio showed no recognizable dependence on energy, but a systematic increase with the atomic number of the target, Z (at a fixed energy of 27 MeV), was clearly seen. The dependence upon Z was in qualitative agreement with the result anticipated from distortion considerations. Moreover the ratio should approach unity at high energy, but over the limited range of energy available, the lack of a recognizable trend in this direction was not particularly significant. Calculations then available predicted only that there should be no difference between electron and positron cross sections [2,4]). Later measurements by Grishaev et al [5]) found a value of σ^-/σ^+ consistent with unity for the two nuclei ^{181}Ta and ^{238}U. However these measurements, being at relatively high energy (the lowest was 70 MeV), cannot be held to be irreconcilable with those of Herring et al. One further point was added by the measurements of Charlesworth et al[6]) who looked at fission with a ^{238}U target. The measured ratio at 20 MeV, 1.70 ±.10,(as ultimately corrected [7])) was greater than a simple linear extrapolation of the results of ref.[3]) to $Z = 92$. The lower energy of this measurement (20 MeV as compared with 27 MeV) might account in part for this higher ratio.

Most recently Kuhl and Kneissl [8]) have carried out a variety of measurements, repeating, among other things, the 27 MeV measurements on ^{12}C, ^{63}Cu and ^{107}Ag of ref.[3]). Kneissl et al [9]) also measured the cross sections for both electron and positron induced fission of ^{238}U from 15 to 40 MeV. There appears to be no discrepancy between these measurements and those of Herring et al and Charlesworth et al. These sets of data are shown in fig(1) displaying the Z dependence of σ^-/σ^+ and in figs (2) and (3) where the energy dependence for ^{107}Ag and ^{238}U is shown.

Comparison with Calculations

We found, in the previous section, no evidence that any of the data is inconsistent with any other, and we now seek confirmation that the Coulomb distortion effect on the passing electrons and positrons can account for the observations. We have used the distorted wave calculation described by Gargaro and Onley 10). There is a second Born approximation calculation, due to Cutler 11), which is far simpler to calculate but does not appear to give the same results, although the methods certainly agree in terms of the direction and general magnitude of the distortion effect.

It may be helpful to review the approximations which are inherent in the virtual photon approximation. Of the radiation which originates from the electron, bremsstrahlung is that which escapes and can be detected at a distant point, for example by exciting a nuclear transition, and virtual radiation is that which is absorbed by the same nucleus from which the electron is scattering. Virtual radiation, in contrast to real radiation, is not a plane wave. Or, to put it in terms of the multipole decomposition, the spectrum does not contain the different multipole components in equal amounts, whereas the plane wave does (by virtue of the way the components are normalized). Labelling the multipoles by λ, which is E and M for electric or magnetic, and by L for the angular momentum, we may decompose the photo-absorption cross section in the form

$$\sigma_\gamma(E) = \sum_{\lambda L} \sigma_\gamma^{\lambda L}(E) \qquad (2)$$

The electro-excitation cross section, in a more general form than eq.(1), is then

$$\sigma^{\pm}(E_o) = \int_0^{E_o} \sum_{\lambda L} N_{\pm}^{\lambda L}(E_o,E)\, \sigma_\gamma^{\lambda L}(E) \frac{dE}{E} \qquad (3)$$

For plane wave photons $N(E_o,E)$ is independent of λL and eq(3) immediately reduces to eq.(1). For virtual radiation, if it is possible to assert that one multipole is dominant (usually $E1$), then the sum is irrelevant and again eq.(3) reduces to eq.(1). In such cases the electro disintegration result should be exactly predictable from the photo-disintegration cross section. If this is not the case, then electro-excitation can conceivably be used to detect quadrupole and other components in the absorption cross section.

In the distorted wave treatment of ref. 10), as with the simple Born approximation expression for the virtual photon distribution 2), the finite size of the nucleus is eliminated by taking the limit of zero nuclear radius. This is a necessary step in order to make the nuclear matrix elements which govern the two processes (electro-excitation and photo-excitation) identical, so that $\sigma_{\gamma}^{\lambda L}$ is the same quantity in the two processes. Attempts to correct for finite nuclear size have been made in the first Born approximation 12) but are not strictly consistent. To correct for finite size it is necessary to include the form factors for both elastic and inelastic scattering and this requires at least second Born approximation such as in ref. 11). In this approximation it appears that the finite size correction should diminish the Coulomb correction, and thus we may anticipate that our results may over estimate the ratio σ^{-}/σ^{+} somewhat.

We examine now the virtual photon distribution calculated using distorted waves, supposing for the present that we have a strictly dipole absorption process. The calculations presented in ref. 10) show only the comparison of distorted wave calculations for electrons for various values of Z, with the plane wave result (which is equivalent to having $Z = 1$). For positrons it is merely necessary to reverse the sign of the interaction, and hence these calculations are equivalent making Z negative. In fig. 3 we show the virtual photon spectra for several values of Z, positive and negative. We note the Z dependence is far from linear which casts some doubt on the value of a second Born approximation calculation which, for inelastic processes, is correct only to first order in αZ. There is also a noticeable change in the shape of the spectrum as one goes from $Z = -92$ to $Z = 92$, i.e. positrons as compared with electrons on a uranium target. Since results for the yield involve an integration over a product of the spectrum with the photo-disintegration cross-section, and the latter has generally only broad structure, the change in overall magnitude of the spectrum is the dominant feature. We therefore expect, and indeed obtain, a smooth progession for the ratio of the total cross sections σ^-/σ^+ as a function of Z. Of course in any attempt to unfold the integral of eq(1) to obtain σ_γ from $\sigma^\pm$, the shape of the spectrum would become extremely important.

To calculate the ratio of the cross sections σ^-/σ^+ we must use measured values of σ_γ. Experiments using "monoenergetic" photons from positron annihilation in flight, have been performed for most of the nuclei under consideration; ref.13) for ^{12}C, ref.14) for ^{63}Cu, ref.15) for ^{107}Ag, ref.16) for ^{181}Ta and ref.17,18) for ^{238}U. The virtual photon spectrum is calculated using VIRFO 1, which is a version of the program used in ref.10). As this is unfortunately very slow, especially for small values of the photon energy, a few points were calculated for each spectrum and by a best fit to these we obtained an analytical expression, which is a function of E_o, E

and Z, for the E1 virtual photon spectra(details in Appendix and ref.[19]). The spectra shown in figs. 3 and 4 and the integrals of eq.(1) were calculated using expressions (1A) and (2A) from the Appendix. For the case of ^{107}Ag, Fig.4 shows the spectra for different incident electron energies. Fig.2 shows the calculated result for σ^-/σ^+ compared with the experiments of refs.[3,8]) which, as remarked earlier, show no clear behaviour as a function of energy. The scatter of the experimental points is too great to reveal the trend predicted by the calculation, namely a very slow approach to unity beyond 15 MeV. The mean value of the points, which is very significantly greater than unity (1.26±.06) is just what one would expect from the prediction. The steep slope below 15 MeV occurs where the two cross sections are very small and only the tip of the spectrum overlaps the giant resonance, being difficult to realize. The reduction of the ratio σ^-/σ^+ with increasing energy is more apparent with the electrofission data of ref.[9]), and comparison with the present calculation is shown in fig.5. In this case, naturally the ratio is still further from unity, but the value indicated by the experimental points is still lower than the calculation; this could be anticipated since no finite size correction has been made.

A display of σ^-/σ^+ as a function of the charge of the target nucleus is shown in fig. 1 for all experiments [3,8,9]) examined here. The predicted behaviour is also shown and is slightly steeper function of Z, but the scatter of the measurements renders this a somewhat tentative conclusion.

Conclusions

We have examined all available data on the ratio of the disintegration cross sections for positrons and electrons, and compared with the results expected by combining a distorted wave calculation of the virtual photon spectrum with the measured photo-disintegration cross sections. There is good agreement between experiment and theory with perhaps a consistent tendency to overstimate the ratio σ^-/σ^+ for heavy nuclei. This is consistent with the estimate that finite size corrections would reduce this ratio, and would of course be most noticeable for heavy nuclei.

We would like to comment on the use of the ratios σ^-/σ^+ to detect the presence of multipole admixtures. It is clear from ref.$^{10)}$ that the Coulomb correction expected for quadrupole and magnetic dipole levels is much greater than for electric dipole. We show in fig. (6) the expected quadrupole virtual photon distributions for 10 MeV electrons and positrons scattering from ^{238}U. Comparing the ratios of $E1^-/E1^+$ and $E2^-/E2^+$ spectra, we see that the ratio σ^-/σ^+expected for a pure quadrupole resonance, would be much greater (~one order of magnitude) than the ratio for a pure dipole resonance. Of course a completely isolated quadrupole resonance is not what one expects to encounter, but it is nevertheless tempting to assert that presence of quadrupole components (or possibly magnetic dipole components) would show up as an enhancement of the ratio σ^-/σ^+ when compared with the value predicted on the basis of a pure dipole cross section. To test this we have considered an imaginary photo cross section for Z=92 composed of a quadrupole resonance situated on the fringe of a larger dipole resonance. Such a situation is shown in fig.7, along with the total photo cross section.

The photo cross section shows some evidence of two bumps but, of course, does not distinguish the multipolarities. Using the curve we may predict the ratio σ^-/σ^+ on the assumption that the entire cross section is due to electric dipole absorption; this would produce the lower curve indicated as (a) in fig.(7). If the smaller peak was a quadrupole resonance, observations of the ratio σ^-/σ^+ should follow the upper curve indicated as (b) in fig.(7); this discrepancy is then the proposed indicator of quadrupole strength. Notice that the behaviour of σ^-/σ^+, although not very dramatic, is unmistakeable. No resonable dipole spectrum could produce such a rise with energy. The observation was made in the introduction that σ^-/σ^+ may be largely independent of the details of the shape of nuclear spectrum. This is true provided only one multipole component is present, but the fact can also be exploited to detect fairly small contaminants of quadrupole strength. Measurements of σ^-/σ^+ would not locate resonances at all precisely as they display no clear structure corresponding to the energy at which the supposed quadrupole resonance occurs (in this respect we differ with the conclusions of ref.[9]), but such measurements should determine quite sensitively the relative strength of quadrupole absorption integrated over the energy region spanned by the virtual photon spectrum.

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APPENDIX

The DWBA calculations for the virtual photon spectra were carried out with a modified version of VIRFO 1. The original version used the IBM scientific subrotine TEAS to project the partial waves sum N_n given in expression (18) of ref. $^{10)}$. We substituted TEAS by two subrotines which project respectively a lower and an upper limit for the sum. The difference between both projections is defined as the uncertainty of the result and the average value as the actual limit. The uncertainty is negligible near the tip of the spectrum but it increases as the virtual photon energy decreases. Except for a few cases, it was possible to obtain the results within a few percent uncertainty using a maximum of 40 partial waves.

The analytical expressions obtained by a best fit to the calculated points, as function of $E1$, E and Z for $E1$ virtual photon spectra, are the following:

$$N_{-}^{E1}(E_1,E,Z) = N_{\mathrm{PW}}^{E1}(E_1,E) + E\Big[1.29\times10^{-5}\exp(1.245Z^{1/3} - .052\,E_1)\Big](E_1 + m_e)/(E_2 + m_e) \qquad (1A)$$

$$N_{+}^{E1}(E_1,E,Z) = N_{\mathrm{PW}}^{E1}(E_1,E) - 3\times10^{-5}\exp(.675Z^{1/3} - -.06135E_1)\,(E_2 - m_e) - \mathrm{T}\,(E_2,Z) \qquad (2A)$$

where N_{PW}^{E1} is the plane wave calculation for the virtual photon spectra:

$$N_{E1}^{\mathrm{PW}}(E_1,E) = (\alpha/\pi)\Big\{\Big[(E_1^2 + E_2^2)/(E_1^2 - m_e^2)\Big]\log\Big[(E_1E_2 + (E_1^2 - m_e)^{1/2}\mathrm{x}(E_2^2 - m_e^2)^{1/2} - m_e^2)/\,m_eE\Big] - 2\Big[(E_2^2 - - m_e^2)\,/(E_1^2 - m_e^2)\Big]^{1/2}\Big\} \qquad (3A)$$

and: $E_1 = E_o + m_e$ = total electron energy

$E_2 = E_1 - E$ = final electron energy

m_e = electron's rest energy.

The function $\mathrm{T}(E_2,Z)$ in expression (2A) acts only near the tip of the positron virtual photon spectra and is given by:

$$\mathrm{T}(E_2,Z) = 1.85\times10^{-4}\exp(.206Z^{1/2}) - 4.568\times10^{-4}(E_2 - m_e)$$

for $m_e < E_2 \leq .405\exp(.206Z^{1/2})$ (4A)

and:

$$\mathrm{T}(E_2,Z) = 0 \text{ for } E_2 > .405\exp(.206Z^{1/2}) \qquad (5A)$$

The analytical expression (1A) and (1B) when compared with a few points of several spectra calculated with VIRFO 1 showed agreement in general better than 6%.

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FIGURE CAPTIONS

Fig. 1 - Z dependence of σ^-/σ^+ for E_0 = 27 MeV, showing the DWBA calculation and experimental data.For Z = 92 the points (ref.9) were taken in the vicinity of 27 MeV.

Fig. 2 - Energy dependence of σ^-/σ^+ for Z=47 showing the DWBA calculation and experimental data.

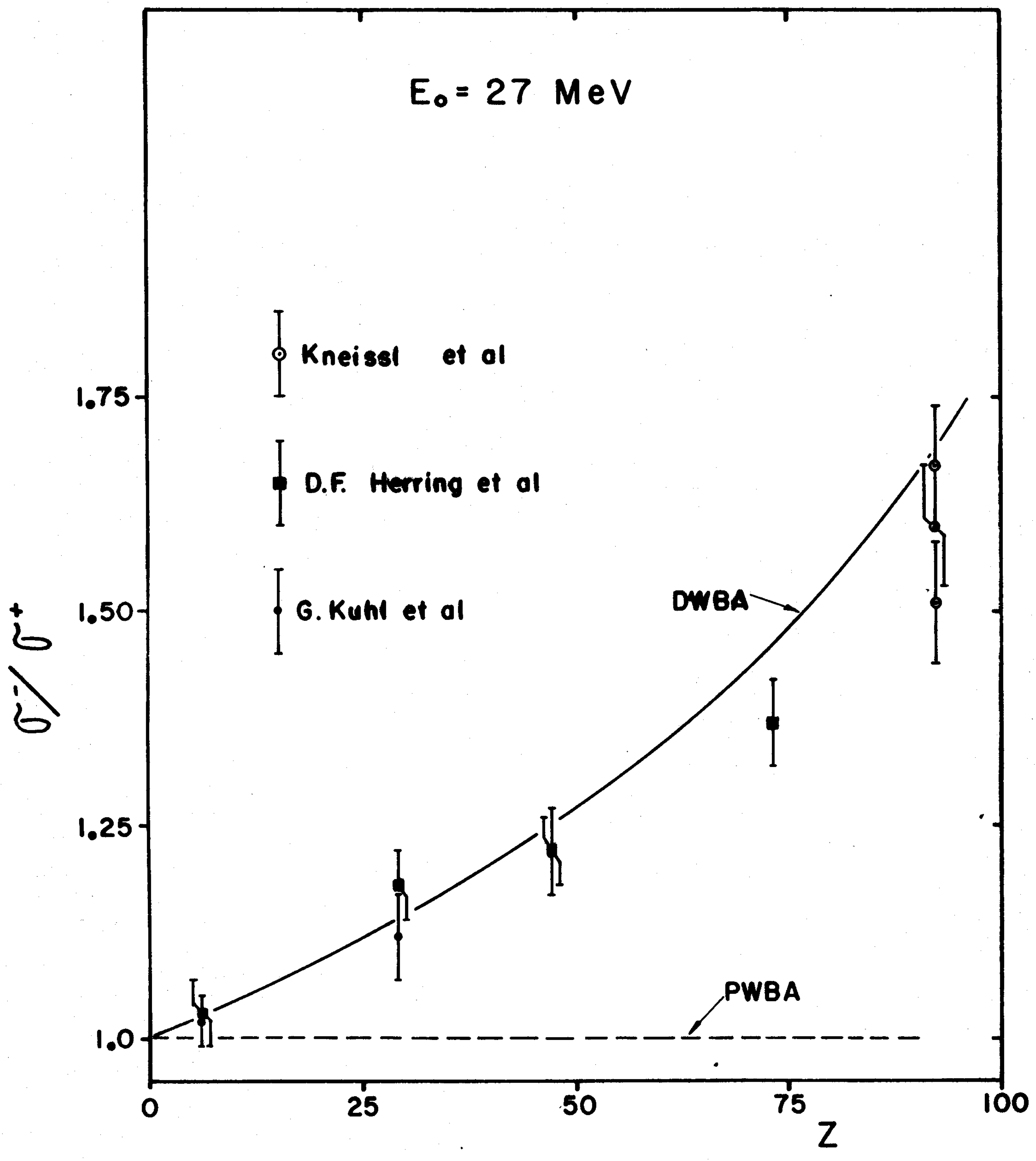
Fig. 3 - $E1$ virtual photon spectra for different Z and E_0 = 27 MeV. Z refers to electrons and negative Z to positrons.

Fig. 4 - Energy dependence of the $E1$ virtual photon spectra, for positrons and electrons calculated in DWBA; Z=47. The PWBA calculation is also shown for comparison.

Fig. 5 - Experimental data showing the energy dependence of σ^-/σ^+ for the electrofission of ^{238}U. Full curve and dashed curve are respectively, the DWBA and PWBA predictions.

Fig. 6 - $E1$ and $E2$ virtual photon spectra for Z=92 and E_0= 9.5 MeV, calculated in DWBA. The $\pm$ labels refer to the two charge states of the electron. The PWBA $E1$ and $E2$ spectra are also shown for comparison.

Fig. 7 - Energy and multipolarity dependence of σ^-/σ^+ for an hypothetical (γ,x) cross section in Z=92. Curve (a): total photo cross section is $E1$. Curve (b): smaller resonance is $E2$.



$E_0 = 27$ MeV
Kneissl et al
D.F. Herring et al
G. Kuhl et al
DWBA
PWBA
σ^-/σ^+
1.75
1.50
1.25
1.0
0
25
50
75
100
Z

Fig.-1

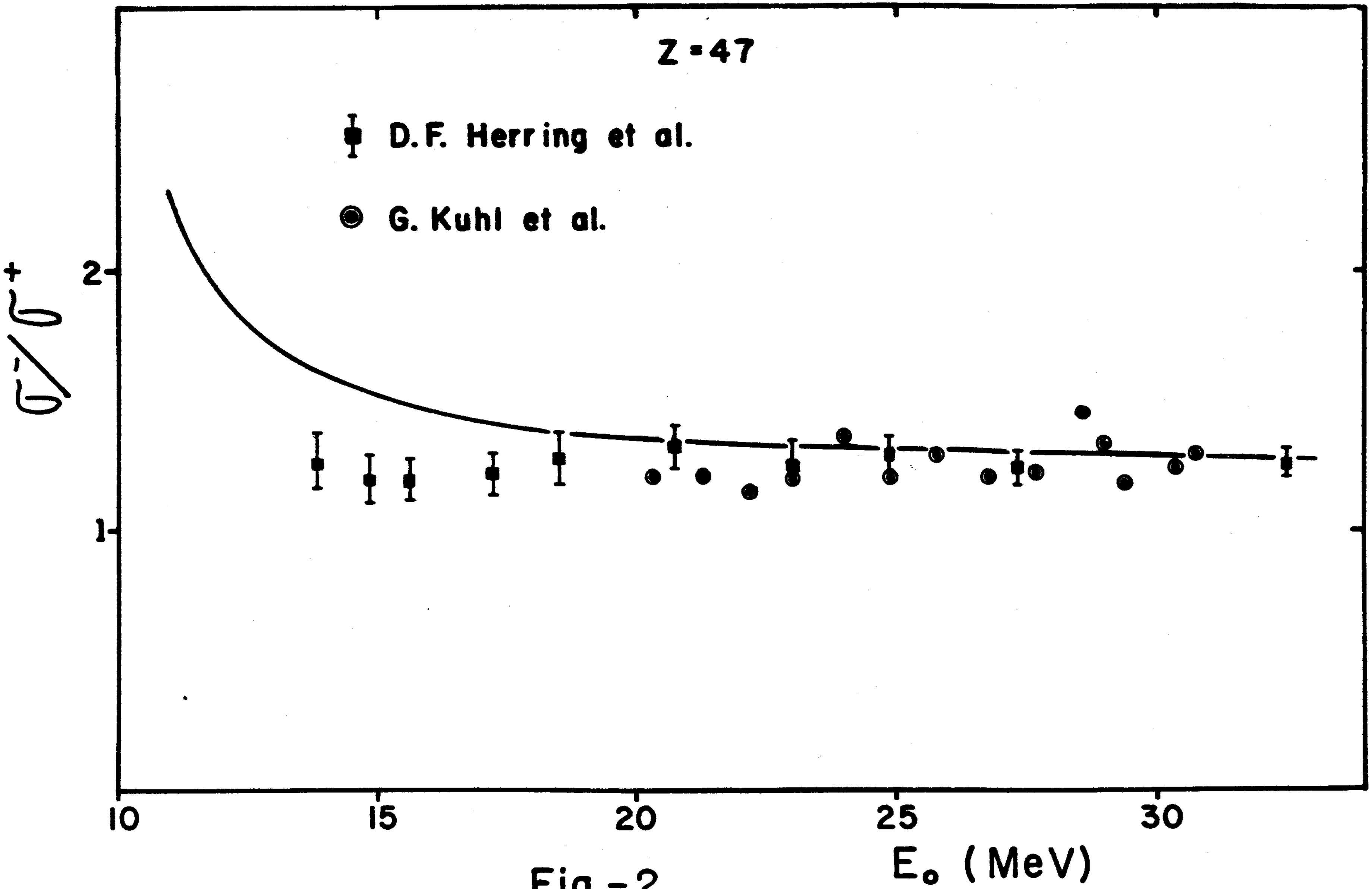

Z = 47
D.F. Herring et al.
G. Kuhl et al.
σ^- / σ^+
2
1
10
15
20
25
30
E_o (MeV)

Fig.-2

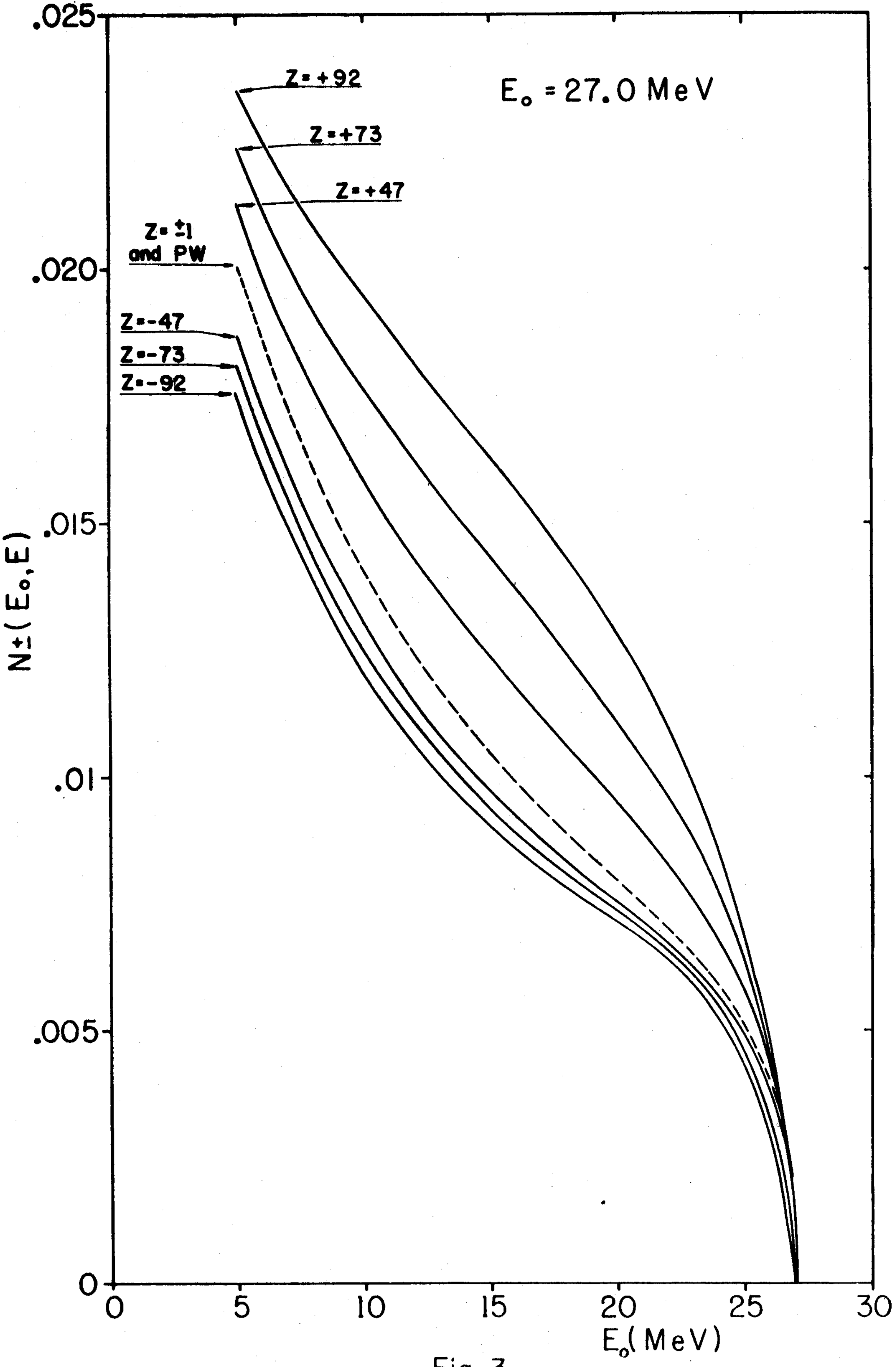

.025
.020
.015
.01
.005
0
N±(E₀, E)
0
5
10
15
20
25
30
E₀(MeV)
E₀ = 27.0 MeV
Z = +92
Z = +73
Z = +47
Z = ±1 and PW
Z = -47
Z = -73
Z = -92

Fig.-3

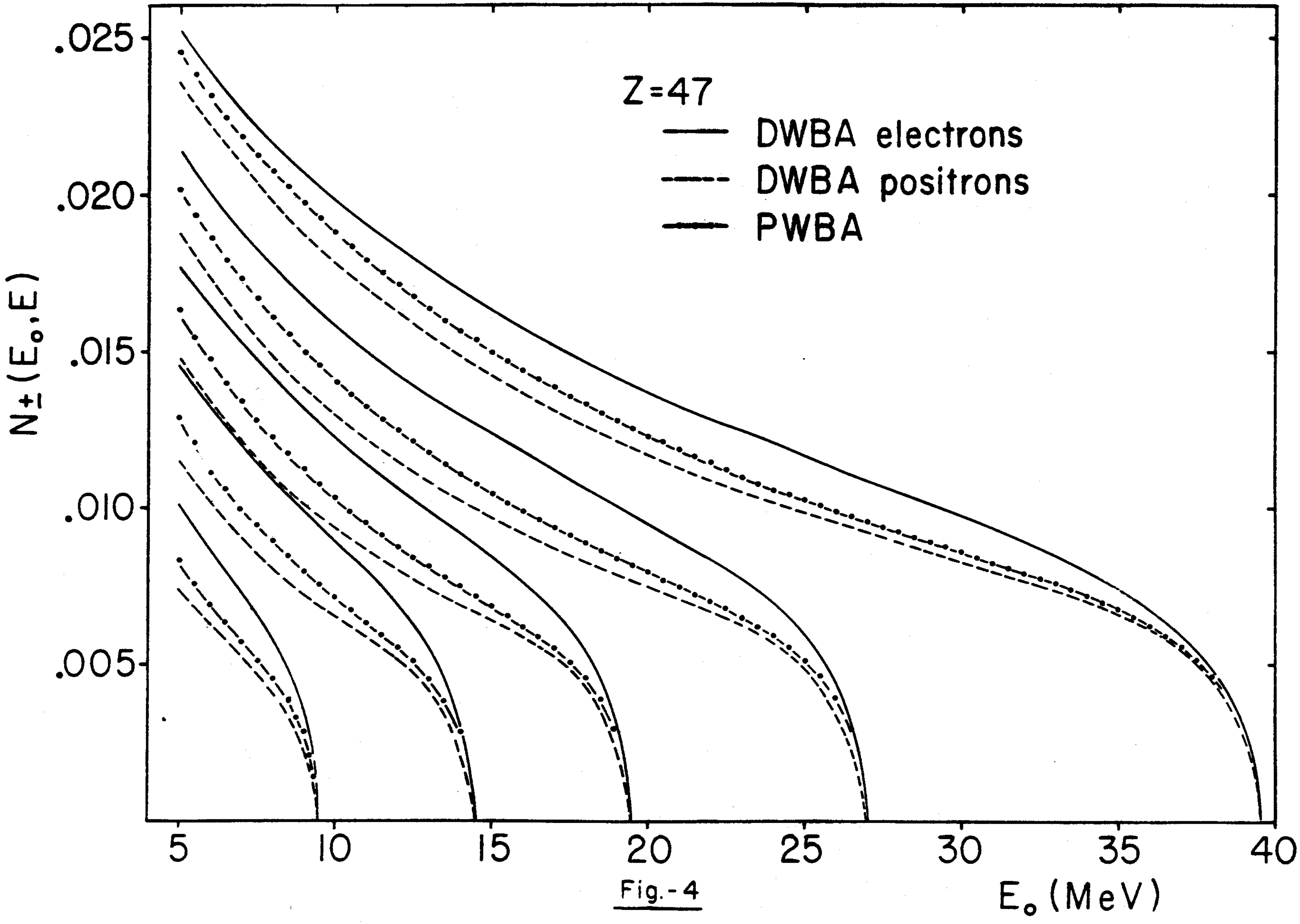
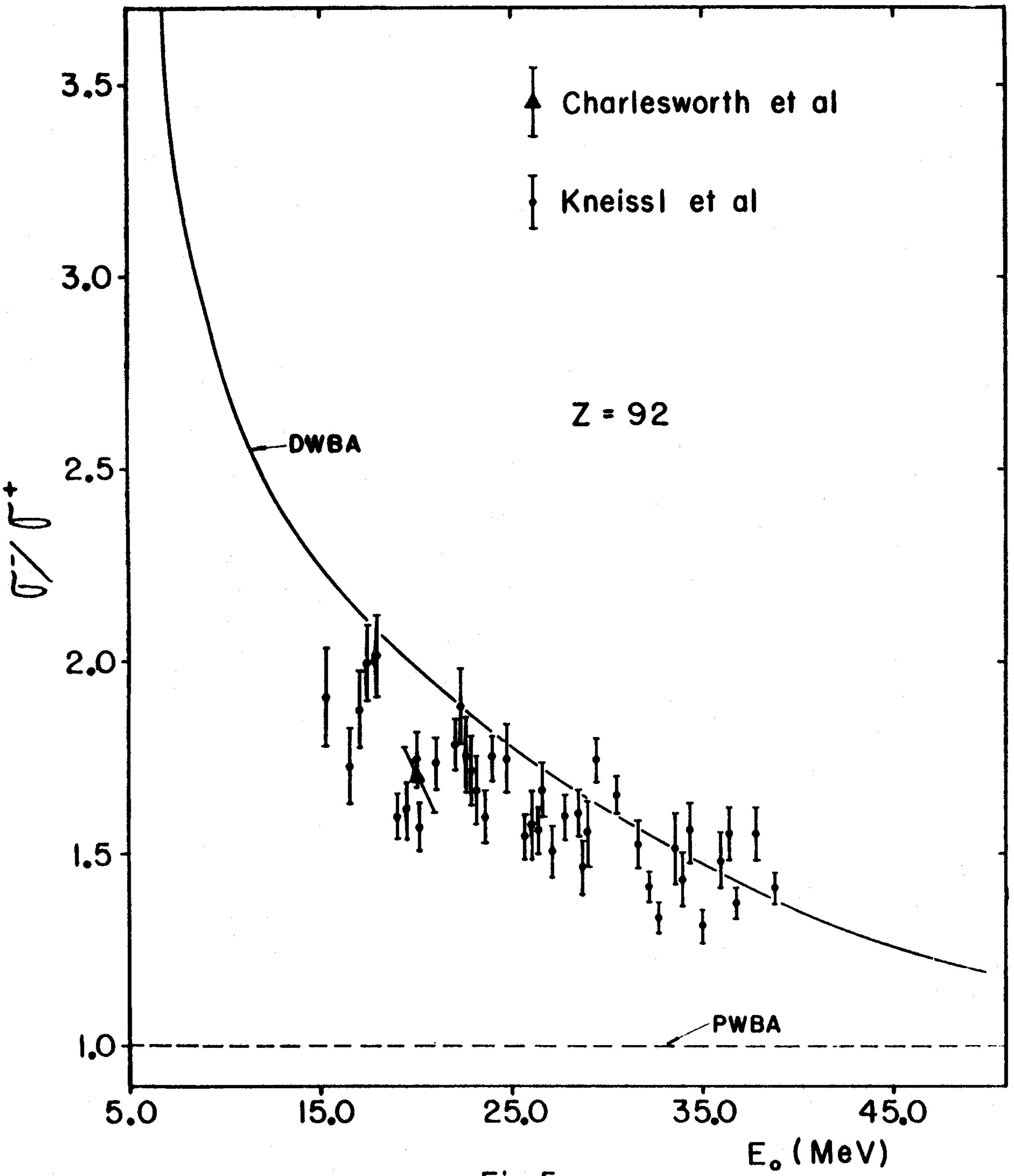

Z=47
DWBA electrons
DWBA positrons
PWBA
.025
.020
.015
.010
.005
$N_{\pm}(E_o,E)$
5
10
15
20
25
30
35
40
E_o (MeV)

Fig.-4



3.5
3.0
2.5
2.0
1.5
1.0
σ^-/σ^+
Charlesworth et al
Kneissl et al
Z = 92
DWBA
PWBA
5.0
15.0
25.0
35.0
45.0
E_o (MeV)

Fig.5

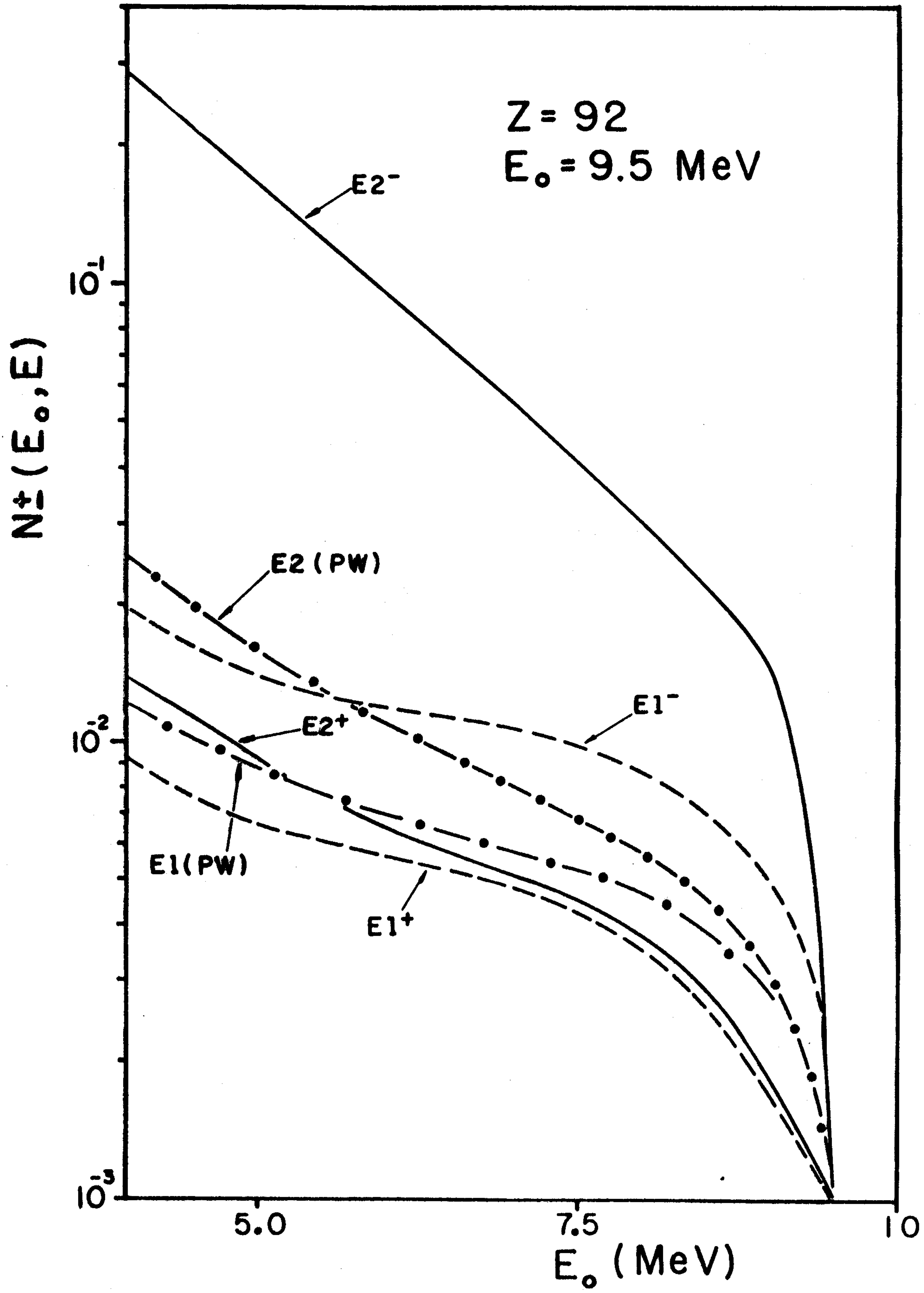

Z = 92
E₀ = 9.5 MeV
E2⁻
E2 (PW)
E1⁻
E2⁺
E1(PW)
E1⁺
N± (E₀, E)
10⁻¹
10⁻²
10⁻³
5.0
7.5
10
E₀ (MeV)

Fig. -6

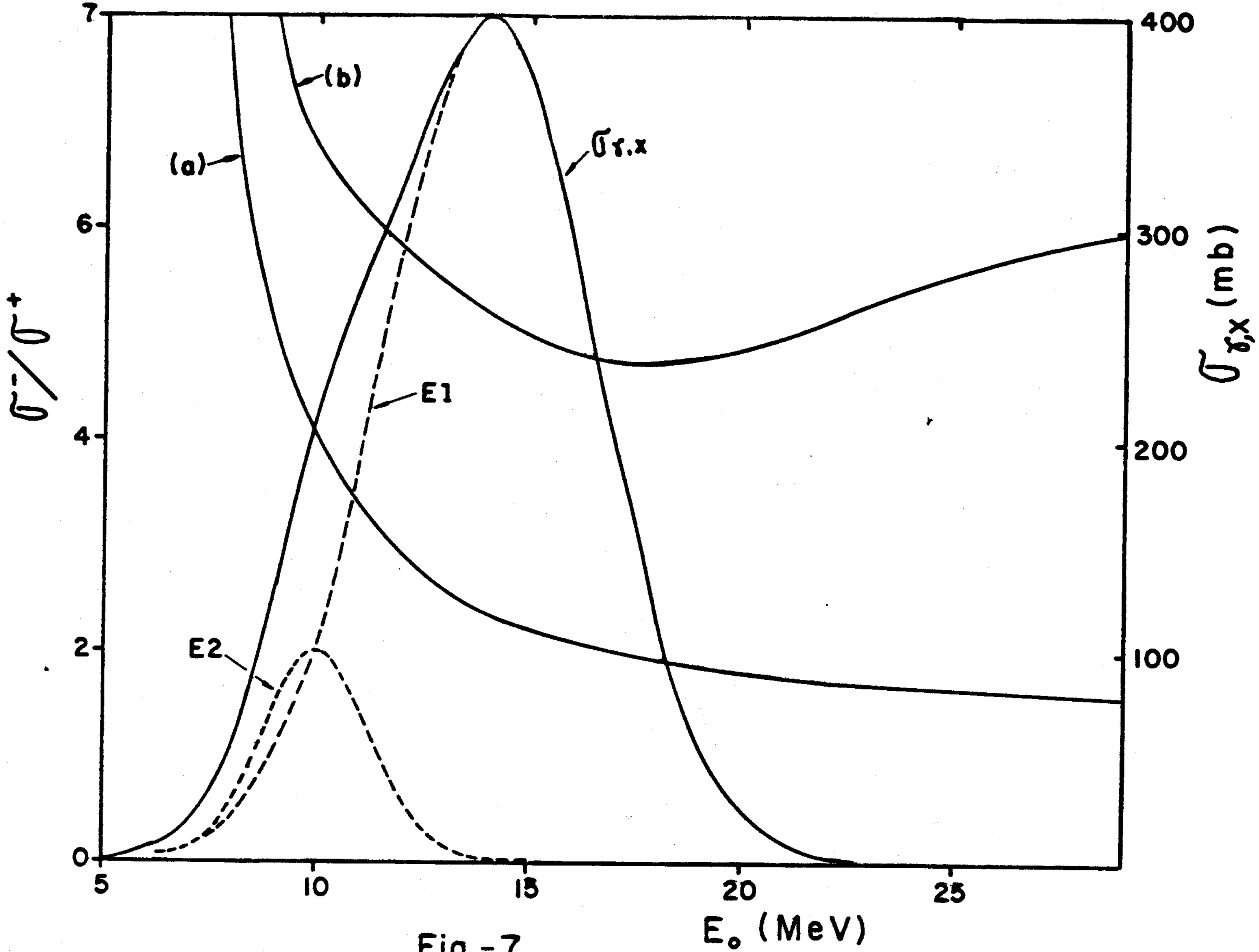

7
6
4
2
0
σ^{-}/σ^{+}
400
300
200
100
$\sigma_{\gamma,x}$ (mb)
5
10
15
20
25
E_o (MeV)
(a)
(b)
$\sigma_{\gamma,x}$
E1
E2

Fig.-7