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Flavor changing models with strictly massless neutrinos

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Theoretical models which present flavor changing neutrino interactions and simultaneously prevent this particle from acquiring any mass exist. We discuss some of them and their predictions for neutrino oscillations in matter which can account for the solar neutrino anomaly as well as the zenith-angle dependence of the deficit of atmospheric neutrinos observed by the SuperKamiokande experiment without invoking, therefore, neutrino masses nor mixing.

I. INTRODUCTION

Flavor changing neutrino interactions (FCNI) can induce neutrino oscillations in matter. This phenomenon was first investigated by Wolfenstein [1] who pointed out that interactions in a medium modify the dispersion relations of particles traveling through. Wolfenstein effect generates quantum phases in time evolution of phenomenological neutrinos eigenstates which consequently can oscillate. These oscillations can be resonantly enhanced even if neutrinos are massless and no mixing in the vacuum exists [2,3]. Similar phenomenon happens when neutrino masses are introduced and mixing angles in matter are induced by flavor changing interactions [2,4]. Wolfenstein effect has been invoked to obtain a good fit of the solar neutrino observations [5] when FCNI is assumed in the massless neutrino context [2,6] or when the neutrinos are assumed massive [2,4,7]. Furthermore it was recently pointed out that the SuperKamiokande results [8] showing a strong zenith-angle dependence of the μ flux induced by atmospheric μ-neutrinos, which have been interpreted as an indication of neutrino oscillations [9,10], can also be understood assuming non-resonant FCNI with ordinary matter in the Earth [11,12].

The presence of flavor changing neutrino-matter interactions implies a non-trivial structure for the neutrino evolution Hamiltonian in matter even if massless neutrinos and no mixing in the vacuum is assumed. The evolution equations describing the $\nu_\alpha \Rightarrow \nu_\beta$ transitions ($\alpha, \beta = e, \mu, \tau$ are flavor indices) are given by [2,3]:

$$i\frac{d\Psi_{\alpha\beta}}{dr} = \sqrt{2}\,G_F \begin{pmatrix} 0 & \epsilon^f_{\alpha\beta} n_f(r) \\ \epsilon^f_{\alpha\beta} n_f(r) & \epsilon'^f_{\alpha\beta} n_f(r) - \delta_{e\alpha} n_e(r) \end{pmatrix} \Psi_{\alpha\beta}, \qquad (1)$$

where $\Psi_{\alpha\beta} = (\nu_\alpha \;\; \nu_\beta)^T$; $\nu_\alpha \equiv \nu_\alpha(r)$ and $\nu_\beta \equiv \nu_\beta(r)$, are the probability amplitudes to find these neutrinos at a distance r from their creation position, $\sqrt{2}\,G_F n_f(r)\epsilon^f_{\alpha\beta}$ is the flavor-changing $\nu_\alpha + f \to \nu_\beta + f$ forward scattering amplitude with the interacting fermion f (charged lepton, d-like quark or u-like quark) and $\sqrt{2}\,G_F n_f(r)\epsilon'^f_{\alpha\beta}$ is the difference between the flavor diagonal $\nu_\alpha - f$ and $\nu_\beta - f$ elastic forward scattering amplitudes, with $n_f(r)$ being the number density of the fermions which induce such processes. In all cases of practical interest, electron-neutrinos will coherently scatter off the electrons present in matter through standard electroweak charged currents which introduce non trivial contributions to the neutrino evolution equations. These contributions are taken into account by the term $\sqrt{2}G_F\delta_{e\alpha}n_e(r)$ in Eq. (1). Therefore, when the electron neutrino evolves in matter ($\alpha = e$) and the fermion which the neutrino scatter off is not electron ($f \neq e$), a resonance can occur if the condition $\epsilon'^f_{e\beta} n_f(r) = n_e(r)$ is fulfilled at least in one layer of the matter crossed by the neutrinos [2]. In the case where no electron-neutrino participates in the oscillation, no resonance can happen.

In any case, flavor oscillations induced by FCNI can be relevant for understanding a considerable range of experimental data involving neutrinos which cannot be fitted by using only standard electroweak inputs. The possibility that these oscillations occur even in the absence of neutrino masses and mixing open a new perspective for interpretation of current neutrino data. Our purpose in this paper is to investigate how feasible this picture can be in several models which allow FCNI.

II. FCNI: A GENERAL REVIEW

Although FCNI do not appear in the standard model of the electroweak interactions they are quite naturally present in many of its extensions. Most of them, however, do not naturally admit strictly massless neutrinos. An example of these models are the R-parity broken supersymmetric models [13] which were invoked in the relevant analyses for the solar and atmospheric neutrinos previously cited [2,4,11]. In these models, the same FCNI interactions that induce the $\epsilon^f_{\alpha\beta}$ and $\epsilon'^f_{\alpha\beta}$ contributions to the Hamiltonian in Eq. (1) induce also contributions to the neutrino mass at the 1-loop level. A fine tuning in the model parameters is needed in order to keep neutrino masses negligible while endowing $\epsilon^f_{\alpha\beta}$ and $\epsilon'^f_{\alpha\beta}$ with appropriate values to fit the solar and atmospheric neutrino anomalies.

There are extensions of the standard $SU(2)\otimes U(1)$ model or of its $SU(5)$ grand unification extension in which neutrinos get naturally a small Dirac mass [14]. In order to obtain such effect extra symmetry and fields are added to the minimal models. For instance, in the $SU(2)\otimes U(1)$ model besides the right-handed neutrino ν_R, a pair of extra singlets s_L and s_R and one scalar singlet are added. The extra symmetry imply that for each family there are one light and one heavy neutral leptons. In this case the FCNI needed to induce non-zero $\epsilon^f_{\alpha\beta}$ and $\epsilon'^f_{\alpha\beta}$ are mediated by the gauge vector boson W and for this reason the only free parameters are the mixing angles. This has to be confronted with constraints imposed by the experimental limits on lepton number violating decays such as $\mu\to e\gamma$ in a similar way we will do in Sec. V

Also in this context it is worth to mention other models like the left-right symmetric one and their SO(10) grand unified extensions [15]. In this case three of the neutrinos can be massless at arbitrary order in perturbation theory. However, the size of flavor changing neutral currents in the neutrino sector depends on the value of the mass scale related to the pattern of symmetry breaking that reduces $SO(10)$ to $SU(3)_c\otimes SU(2)_L\otimes U(1)$. Only if $SO(10)$ breaks first to $SU(2)_L\otimes SU(2)_R\otimes U(1)$ it is possible to have intermediated mass scale related to the energy at which $SU(2)_R$ is broken. In general, grand-unified theories present both massless neutrinos and flavor changing interactions [16]. Nevertheless the effective flavor changing neutral currents are negligibly small since they are inversely proportional to the squared mass of the exotic vector boson which is fixed by the condition of unification. This is the case of all non-supersymmetric grand-unified models since in these models there exist the hierarchy problem.

A version of the Zee's model [17] in which only one extra singly charged scalar singlet (besides the usual doublet) and right-handed neutrinos are added was considered in Ref. [18]. In the simplest case the constraint coming from the muon decay implies that the atmospheric neutrino flux is reduced at most 20%. A way to overcome this difficulty is to add another singly charged scalar singlet. In this case the neutrino masses are also arbitrary and the constraints of the muon and other leptonic decays can be evaded. Notice that if we do not add right-handed neutrinos FCNI take place only for anti-neutrinos.

Nevertheless there are models where the mechanism pointed out in Refs. [2] and [11] can be exactly realized, keeping neutrinos strictly massless, while FCNI can exist with the required intensity. Basically, vanishing neutrino masses are guaranteed in these models due to the conservation of the total leptonic number L. Then, by imposing that there are no right-handed neutrinos to avoid Dirac masses, no vacuum expectation values associated with neutral scalars to prevent spontaneous violation of L and, consequently, no majoron-like Goldstone boson is generated, neutrino masses can be kept vanishing. Obviously, no L violation in the scalar sector is also allowed, like as in the R-parity violating supersymmetric models, since in this case neutrino masses can be generated radiatively [19].

In this paper we show that these conditions are naturally fulfilled in some theoretical scenarios and therefore FCNI can coexist with strictly massless neutrinos. In Sec. III we consider the gauge model based on $SU(3)_c\otimes SU(3)_L\otimes U(1)_N$ symmetry (331 model) [20] and in Sec. IV we consider the multi-Higgs doublet extension of the standard model [21].

III. FCNI IN 331 MODELS

Although the standard model of electroweak interactions accommodates all the present experimental results, it is not able to give answer for some questions in particle physics. One of these questions is the family replication problem, which can have an elegant solution in the simplest chiral extension of standard model, the gauge model based on $SU(3)_c\otimes SU(3)_L\otimes U(1)_N$ symmetry which broken to $SU(3)_c\otimes SU(2)_L\otimes U(1)_N$ in some energy scale higher than the Fermi one. In the 331 model of Ref. [20] neutrinos are kept massless as long as no extra right-handed neutrinos are added to the particle content of the model and the total lepton number L is conserved in the full Lagrangian [19]. We will be assuming here that this is the case. The 331 phenomenology was studied in many different contexts [23] but for the first time it is applied for neutrino phenomenology. The Yukawa interactions in the lepton sector are [20,22]

$$\begin{aligned} -\mathcal{L}_Y &= \frac{1}{\sqrt{2}}\,\overline{\nu}_L\,\mathcal{K}_{LR} l_R H_1^+ + \frac{1}{\sqrt{2}}\,\bar{l}_L\,\mathcal{K}_{LL}\nu_R^c H_2^- \\ &+ \frac{1}{2}\,\bar{l}_L\,\mathcal{K}_{LL}(l_L)^c H_1^{--} + \frac{1}{2}\,\overline{l^c}_L\,\mathcal{K}_{RR} l_R H_2^{++} \\ &+ 2\,\overline{\nu}_L\,\mathcal{K}'_{LR} l_R \eta_1^+ - 2\,\bar{l}_L\,\mathcal{K}'_{LL}\nu_R^c\eta_2^- + H.c., \end{aligned} \qquad (2)$$

where $\mathcal{K}_{LL} = E_L^\dagger G E_L^*$, $\mathcal{K}_{RR} = E_R^T G E_R$, $\mathcal{K}_{LR} = E_L^\dagger G E_R$; $\mathcal{K}'_{LR} = E_L^\dagger G' E_R$, $\mathcal{K}'_{LL} = E_L^\dagger G' E_L^*$; G and G' are symmetric and antisymmetric (they can be complex) Yukawa matrices. The symmetric (antisymmetric) property of the matrices above are because the correspondent Higgs field is in triplet (sextet) representation [20,22]. $E_{R,L}$ are the right- and left-handed mixing unitary matrices in the lepton sector relating symmetry eigenstates (primed fields) with mass eigenstates (unprimed fields) [24]:

$$l'_L = E_L l_L, \quad l'_R = E_R l_R, \quad \nu'_L = E_L \nu_L, \qquad (3)$$

where we have redefined the neutrino fields so that there is no mixing in the current coupled to the gauge vector boson W. Note, therefore, that while $\mathcal{K}_{LL}$ and $\mathcal{K}_{RR}$ are symmetric matrices and K'_{LL} is an antisymmetric matrix, no symmetry relation appears in $\mathcal{K}_{LR}$ and $\mathcal{K}'_{LR}$. None of the couplings in Eq. (2) depends directly on the charged lepton masses and all matrices in Eq. (2) are not unitary. When G and G' are real matrices the matrices $\mathcal{K}_{LL}$ and $\mathcal{K}_{RR}$ (and the respective primed matrices) are hermitian. All scalars in Eq. (2) are still symmetry eigenstates.

The interactions of the leptons with the gauge vector boson V [20] induce also FCNI and are given by

$$-\mathcal{L}_V = \bar{l}_R \gamma^\mu \mathcal{K} \nu_R^c V_\mu^- + H.c., \qquad (4)$$

with $\mathcal{K} = (g_3/\sqrt{2}) E_R^\dagger E_L^*$ being a unitary matrix and g_3 is the coupling of the gauge vector boson V with the leptons. This is not the g coupling of standard model gauge vector boson W.

We can now write the expressions for the parameter $\epsilon^f_{\alpha\beta}$ entering in Eq. (1) and for the analogous parameter $\bar{\epsilon}^f_{\alpha\beta}$, when anti-neutrinos are evolving instead of neutrinos, in the light of this model. We present, in Figs. 1, 2 and 3 the diagrams which illustrate scalar contributions for neutrinos, scalar contributions for anti-neutrinos and vector contributions for anti-neutrinos, respectively, in the 331 model. From Figs. 1, 2 and 3 and Eqs. (2) and (4), we write the nonstandard contributions to $\epsilon^e_{\alpha\beta}$ and $\bar{\epsilon}^e_{\alpha\beta}$ (the superscript e denotes that the relevant interactions are those with electrons):

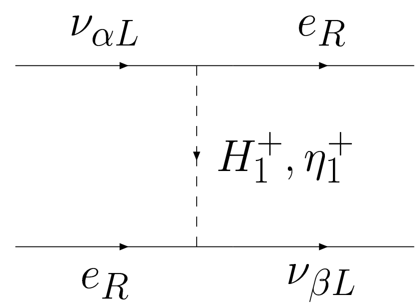

FIG. 1. Scalar contributions to FCNI in the 331 model.

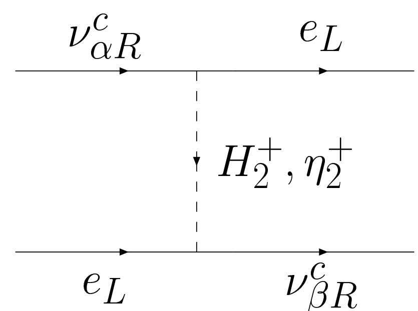

FIG. 2. Scalar contributions to anti-neutrinos FCNI in 331 model.

$$\epsilon^e_{\alpha\beta} = \frac{1}{4\sqrt{2}G_F M_W^2}\left[\left(\mathcal{K}^\dagger_{LR}\right)_{1\alpha}(\mathcal{K}_{LR})_{\beta 1}\,x + \left(\mathcal{K}'^\dagger_{LR}\right)_{1\alpha}(\mathcal{K}'_{LR})_{\beta 1}\,y\right], \qquad (5a)$$

where $x = M_W^2/M_{H_1}^2$, $y = M_W^2/M_{\eta_1}^2$, and

$$\bar{\epsilon}^e_{\alpha\beta} = \frac{1}{4\sqrt{2}G_F M_W^2}\left[(\mathcal{K})_{1\alpha}(\mathcal{K}^\dagger)_{\beta 1}\,z' + (\mathcal{K}_{LL})_{1\alpha}\left(\mathcal{K}^\dagger_{LL}\right)_{\beta 1}x' + (\mathcal{K}'_{LL})_{1\alpha}\left(\mathcal{K}'^\dagger_{LL}\right)_{\beta 1}y'\right], \qquad (5b)$$

where $z' = M_W^2/M_V^2$, $x' = M_W^2/M_{H_2}^2$, $y' = M_W^2/m_{\eta_2}^2$. Note also that we are assuming that all the matrices entering above are real. In case they are complex, only their real part will contribute to $\epsilon^e_{\alpha\beta}$ and $\bar{\epsilon}^e_{\alpha\beta}$. In this paper we always assume the convention that Greek subscript stands for neutrinos and Arabic subscript for charged leptons.

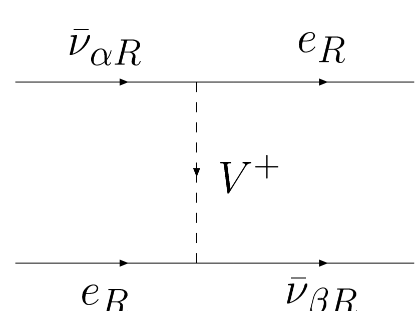

FIG. 3. Vector contributions to anti-neutrinos FCNI in 331 model.

From the above Eq. (5a) we can explicitly calculate $\epsilon^e_{\mu\tau}$ and $\epsilon'^e_{\mu\tau} = \epsilon^e_{\tau\tau} - \epsilon^e_{\mu\mu}$ that enter Eq. (1) for the case of transitions $\nu_\mu \to \nu_\tau$, which can be relevant to atmospheric neutrinos:

$$\epsilon^e_{\mu\tau} = \frac{1}{4\sqrt{2}G_F M_W^2} \left[\left(\mathcal{K}^\dagger_{LR}\right)_{1\mu} (\mathcal{K}_{LR})_{\tau 1}\, x + \left(\mathcal{K}'^\dagger_{LR}\right)_{1\mu} (\mathcal{K}'_{LR})_{\tau 1}\, y\right], \tag{6a}$$

and

$$\epsilon'^e_{\mu\tau} = \frac{1}{4\sqrt{2}G_F M_W^2} \left\{\left[\left(\mathcal{K}^\dagger_{LR}\right)_{1\tau} (\mathcal{K}_{LR})_{\tau 1} - \left(\mathcal{K}^\dagger_{LR}\right)_{1\mu} (\mathcal{K}_{LR})_{\mu 1}\right] x + \left[\left(\mathcal{K}'^\dagger_{LR}\right)_{1\tau} (\mathcal{K}'_{LR})_{\tau 1} - \left(\mathcal{K}'^\dagger_{LR}\right)_{\mu 1} (\mathcal{K}'_{LR})_{1\mu}\right]\right\}. \tag{6b}$$

Similar for anti-neutrinos, where $\bar\nu_\mu \to \bar\nu_\tau$ transitions are at play, we obtain from Eq. (5b):

$$\bar\epsilon^e_{\mu\tau} = \frac{1}{4\sqrt{2}G_F M_W^2} \left\{(\mathcal{K})_{1\mu}(\mathcal{K}^\dagger)_{\tau 1} z' + (\mathcal{K}_{LL})_{1\mu}\left(\mathcal{K}^\dagger_{LL}\right)_{\tau 1} x' + (\mathcal{K}'_{LL})_{1\mu}\left(\mathcal{K}'^\dagger_{LL}\right)_{\tau 1} y'\right\}, \tag{7a}$$

and

$$\bar\epsilon'^e_{\mu\tau} = \frac{1}{4\sqrt{2}G_F M_W^2} \left\{\left[(\mathcal{K})_{1\tau}(\mathcal{K}^\dagger)_{\tau 1} - (\mathcal{K})_{1\mu}(\mathcal{K}^\dagger)_{\mu 1}\right] z' + [(\mathcal{K}_{LL})_{1\tau}\left(\mathcal{K}^\dagger_{LL}\right)_{\tau 1} - (\mathcal{K}_{LL})_{1\mu}\left(\mathcal{K}^\dagger_{LL}\right)_{\mu 1}] x' + \left[(\mathcal{K}'_{LL})_{\tau 1}\left(\mathcal{K}'^\dagger_{LL}\right)_{\tau 1} - (\mathcal{K}'_{LL})_{1\mu}\left(\mathcal{K}'^\dagger_{LL}\right)_{\mu 1}\right] y'\right\}. \tag{7b}$$

Notice that neutrinos and anti-neutrinos will not, in general, interact in the same way through the boson fields. This is not a CP violation effect but rather only reflects the fact that these particles have different interactions with matter in this theoretical scheme. In fact, if all couplings and the vacuum expectation values are real, CP is conserved in all the vertices of the model. The pure gauge boson interactions also conserve CP if the $SU(3)_L$ gauge and $U(1)_N$ vector bosons W^a with $a = 1, ...8$, and B, respectively transform as [25]

$$(W^1_\mu, W^2_\mu, W^3_\mu, W^4_\mu, W^5_\mu, W^6_\mu, W^7_\mu, W^8_\mu, B_\mu) \stackrel{CP}{\to} \tag{8}$$
$$-(W^{1\mu}, -W^{2\mu}, W^{3\mu}, -W^{4\mu}, W^{5\mu}, -W^{6\mu}, W^{7\mu}, W^{8\mu}, B^\mu). \tag{9}$$

It means that the physical fields transform as

$$(W^+_\mu, V^+_\mu, U^{++}_\mu, A_\mu, Z_\mu, Z'_\mu) \stackrel{CP}{\to} \tag{10}$$
$$-(W^{\mu -}, -V^{\mu -}, -U^{\mu --}, A^\mu, Z^\mu, Z'^\mu). \tag{11}$$

where W is the standard model gauge vector boson, V is a gauge vector boson , U^{++} is a doubly charged gauge boson and Z^μ and Z'^μ are neutral gauge vector bosons.

In order to obtain the relevant FCNI parameters for solar neutrino analysis, *i.e.*, transitions involving ν_e and ν_τ, it is sufficient to change appropriately the indexes $\mu \to e$ in the Eqs. (6a) and (6b). Notice also that in general $|\epsilon^f_{\alpha\beta}| \neq |\bar\epsilon^f_{\alpha\beta}|$, and similarly $|\epsilon'^f_{\alpha\beta}| \neq |\bar\epsilon'^f_{\alpha\beta}|$. In order to have $|\epsilon^f_{\alpha\beta}| \approx |\bar\epsilon^f_{\alpha\beta}|$ or $|\epsilon'^f_{\alpha\beta}| \approx |\bar\epsilon'^f_{\alpha\beta}|$, we need both the contributions of the gauge vector boson V to be small and a fine tuning among the Yukawa couplings and the mixing angles.

Another important remark concerning the model is the following. In order to get to all charged leptons an appropriate mass only the sextet of scalars is necessary [22]. However, unless we introduce an extra symmetry the triplet $\eta \sim (\mathbf{1}, \mathbf{3}, 0)$ [20,22] couples also to the leptons. In this case we have FCNI at the tree level mediated by scalars. The mass matrix is diagonalized by the biunitary transformation using the matrices $E_{L,R}$: $\hat{M}^l = E^\dagger_L(Gv_s + G'v_\eta)E_R$ ($\hat{M}^l = \mathrm{diag}(m_e, m_\mu, m_\tau)$ and v_s is the vacuum expectation value (VEV) of one of neutral Higgs (denoted σ^0_2 in the Eq. (23) of Ref [22]) contained in the sextet [22] and v_η is the VEV of one of triplets [20,22]) but this transformation does not diagonalize Gv_s nor $G'v_\eta$ separately. Hence, the interactions in Eq. (2) are not suppressed by the lepton mass. Since v_s is the VEV needed only in order to give mass to the charged leptons it is not necessarily of the order of the hundred GeV but may have a value of a few GeV. Hence at this stage the only constraints in the Yukawa parameters G and G' come from perturbation theory: $|G|^2/4\pi < 1$, $|G'|^2/4\pi < 1$. Since G and G' can be arbitrary complex matrices, the matrices defined in Eq. (2) are non-unitary matrices.

We observe that in order to obtain $\epsilon^f_{\alpha\beta}$ and $\epsilon'^f_{\alpha\beta}$ appropriate to solve solar or atmospheric neutrinos anomalies it is necessary that x, y, x', y' be not too small (that is not the case of grand-unified theories as discussed above). The charged scalars can not be so heavy. In fact, even the constraints coming from the neutral kaon system do not impose severe restriction to the mass of these scalar: $m_{h^+} \sim m_{h^0} \sim 150$ GeV (h^+ and h^0 denote any of the singly charged and neutral scalars) if at the same time we take into account the mixing angles in the quark sector [26]. It means $x \sim y \sim 0.8$ already give a value of $\epsilon^f_{\alpha\beta} \sim 0.1$ which is needed for solving the atmospheric neutrino anomaly [11]. A fine tuning of the mixing angles keep $\epsilon'^f_{\alpha\beta}$ almost zero. However, it is still possible that x, y, x', y' be larger than one because h^+ or h^0 contributing to the neutral kaons parameters are not necessarily the same component than the scalars in Eqs. (6). The same analysis is valid for the anti-neutrino parameters in Eqs. (7).

All scalars in Eq. (2) are still symmetry eigenstates. Hence, they are linear combinations of the mass eigenstates. It means that some of their components couple to quarks and induce contribution to the neutral kaon parameters. However, those contributions depend on mixing angles in the u- and d- quarks mixing matrices so they do not impose constraints on the couplings in Eq. (2). They will be constrained by some phenomenological processes as discussed later in Sec. V.

IV. MULTI-HIGGS DOUBLETS EXTENSION OF THE STANDARD MODEL

Let us consider the standard model plus an arbitrary number of scalar doublets $\Phi_i = (\phi_i^+, \phi_i^0)^T, \; i = 1, 2, ...N \geq 3$ In the lepton sector the Yukawa interactions are (with massless neutrinos)

$$-\mathcal{L}_Y^l = \sum_i \left(\bar{\nu}_L \, E_L^\dagger \Gamma_i^l E_R \, l_R \phi_i^+ + \bar{l}_L \, E_L^\dagger \Gamma_i^l E_R \, l_R \, \phi_i^0 \right) + H.c., \tag{12}$$

as in the previous model we have redefined the fields as in Eq. (3). The mass matrix for the charged leptons $M^l = \sum_i (v_i/\sqrt{2})\Gamma_i^l$ is diagonalized as follows:

$$E_L^\dagger M^l E_R = \hat{M}^l, \tag{13}$$

with $\hat{M}^l = \text{diag}(m_e, m_\mu, m_\tau)$. Hence, the unitary matrices $E_{L,R}$ diagonalize M^l but not each of the Γ_i^l separately. Although we have redefined the neutrino fields in the charged currents coupled to the vector bosons W, the same is not possible in the interactions with $\phi_i^\pm$. Hence, even with massless neutrinos we cannot avoid, in general, to have mixing in the charged current mediated by scalars in the lepton sector as well. Moreover, even if neutrinos are massless, there are flavor changing interactions mediated by neutral scalars in the charged lepton sector.

The charged currents in Eq. (12) can be written in terms of the physical charged scalar $\mathcal{H}_i^+$ (defined as $\phi_i^+ = \sum_i K_{ij} \mathcal{H}_j^+$) :

$$\mathcal{L}^{CC} = \sum_j \bar{\nu}_L \, \mathcal{V}_j \, l_R \, \mathcal{H}_j^+ + H.c., \tag{14}$$

where

$$(\mathcal{V}_j)_{k,\alpha} = \sum_i \left(E_L^\dagger \Gamma_i^l E_R K_{ij} \right)_{k\alpha}, \tag{15}$$

where $\alpha = e, \mu, \tau$ and $k = 1, 2, 3$. The matrices $\mathcal{V}_j$ are not unitary matrices since Γ^l are in general arbitrary complex matrices, with $|\Gamma|^2/4\pi < 1$.

In this case we have

$$\epsilon^e_{\alpha\beta} = \frac{1}{4\sqrt{2} G_F M_W^2} \sum_j \left(\mathcal{V}_j^\dagger \right)_{\alpha 1} (\mathcal{V}_j)_{\beta 1} \, \xi_j. \tag{16}$$

Explicitly

$$\epsilon^e_{\mu\tau} = \frac{1}{4\sqrt{2} G_F M_W^2} \sum_j \left(\mathcal{V}_j^\dagger \right)_{\mu 1} (\mathcal{V}_j)_{\tau 1} \, \xi_j, \tag{17}$$

$$\epsilon'^e_{\mu\tau} = \frac{1}{4\sqrt{2} G_F M_W^2} \left[\sum_j \left(\mathcal{V}_j^\dagger \right)_{\tau 1} (\mathcal{V}_j)_{\tau 1} - \left(\mathcal{V}_j^\dagger \right)_{\mu 1} (\mathcal{V}_j)_{\mu 1} \right] \xi_j, \tag{18}$$

with $\xi_j = M_W^2/M_{\mathcal{H}_j}^2$. Note again that in order to obtain the relevant FCNI parameters for solar neutrino analysis it is sufficient to change appropriately the indexes $\mu \to e$ in the Eqs. (17) and (18). In this case $\epsilon_{\alpha\beta} = -\bar{\epsilon}_{\alpha\beta}$ (similarly for the ϵ's) and the constraints coming from $\mu \to e\gamma$ and other processes are similar as for the previous model as it will be shown in the next section.

V. CONSTRAINTS ON FCNI PARAMETERS

In order to be sure that a realization of the oscillations induced by FCNI can happen in a quantitative way and a compatibility of data and related predictions be achieved we have finally to investigate the constraints to these FCNI that arise from the charged lepton decay measurements and from the absence of violation of the lepton number conservation laws [27].

The μ-decay is measured with large precision and it is consistent with the standard electroweak model predictions. This imposes severe constraints on exotic interactions involving neutrinos. Considering the 331 model, H_2^- and η_2^- mediate processes which should be summed coherently to the standard model contribution. The interference contribution of H_2^- and η_2^- with the standard model (SM) prediction, for the the left-handed μ-decay measurement, leads to [28]:

$$\left| Re \left[(\mathcal{K}'^\dagger_{LL})_{e2} (\mathcal{K}'_{LL})_{1\mu} \, y' + (\mathcal{K}^\dagger_{LL})_{e2} (\mathcal{K}_{LL})_{1\mu} \, x' \right] \right| < (G_F/\sqrt{2}) M_W^2 \Delta_\mu, \tag{19}$$

where Δ_μ is the combined experimental and theoretical error of μ-decay. Following the recipe of Ref. [29] we compute the G_F^{SM} value

$$G_F^{SM} = \frac{\pi \alpha(0)}{\sqrt{2} M_W^2 \, (1 - M_W^2/M_Z^2) \, (1 - \Delta r)} \, . \tag{20}$$

In Eq. (20) the fine structure constant $\alpha(0)$ is very accurately known to be $1/137.036$ and Δr is the radiative SM correction and is found to be $\Delta r = .0349 \mp .0019 \pm .0007$ [27]. However, in computing G_f^{SM} the largest error comes from M_W measurement with current value given as $M_W = 80.39 \pm .06$ GeV [30] and a much smaller error comes from $M_Z = 91.1867 \pm .0020$ GeV [27].

Using this and propagating the error of G_F^{SM} in the error of μ-decay width we get that at 1σ, $\Delta_\mu = 8.6 \times 10^{-3}$. The mainly error came from the G_F^{SM} uncertainty. Also in the case of τ-decay we need to take into account the τ-lifetime measurement error and τ-mass measurement error, then $\Delta_\tau = 9.5 \times 10^{-3}$ at 1σ.

We can consider also the τ-decay measurement

$$\left|Re\left[(\mathcal{K}'^{\dagger}_{LL})_{e3}(\mathcal{K}'_{LL})_{1\tau}\, y' + (\mathcal{K}^{\dagger}_{LL})_{e3}(\mathcal{K}_{LL})_{1\tau}\, x'\right]\right| < (G_F/\sqrt{2})M_W^2\Delta_\tau, \tag{21}$$

which constraint is obtained from Eq. (19) substituting appropriately the sub-indexes and imposing $\Delta_\tau = 9.5 \times 10^{-3}$ which leads to less stringent limits.

We can rewrite, using the symmetric (anti-symmetric) properties of $\mathcal{K}_{LL}$ ($\mathcal{K}'_{LL}$),

$$\left|Re\left[-(\mathcal{K}'^{\dagger}_{LL})_{2e}(\mathcal{K}'_{LL})_{1\mu}\, y' + (\mathcal{K}^{\dagger}_{LL})_{2e}(\mathcal{K}_{LL})_{1\mu}\, x'\right]\right| < (G_F/\sqrt{2})M_W^2\Delta_\mu, \tag{22}$$

and

$$\left|Re\left[-(\mathcal{K}'^{\dagger}_{LL})_{3e}(\mathcal{K}'_{LL})_{1\tau}\, y' + (\mathcal{K}^{\dagger}_{LL})_{3e}(\mathcal{K}_{LL})_{1\tau}\, x'\right]\right| < (G_F/\sqrt{2})M_W^2\Delta_\tau, \tag{23}$$

These constraints can give limits on the contribution of H_2^- and η_2^- to the $\bar{\epsilon}^e_{\alpha\alpha}$ defined in Eqs. (5b). Respectively the μ decay and the τ decay can give limits on $\bar{\epsilon}^e_{22}$ and $\bar{\epsilon}^e_{33}$. Nevertheless, Eqs. (22) and (23) can easily exhibits cancellations among the contributions which can eliminate the constraints.

There are also other contributions of the charged-lepton decay mediated by H_2^- and η_2^- which do not sum coherently to the standard electroweak process

$$\sum_{\alpha,\beta}\left|(\mathcal{K}'^{\dagger}_{LL})_{\alpha j}(\mathcal{K}'_{LL})_{i\beta}\, y' + (\mathcal{K}^{\dagger}_{LL})_{\alpha j}(\mathcal{K}_{LL})_{i\beta}\, x'\right|^2 < (G_F/\sqrt{2}M_W^2)^2\Delta_j, \tag{24}$$

where $j = \mu, \tau$ and $i \neq \alpha$ and $j \neq \beta$. These relevant constraint is to the $\bar{\epsilon}^e_{\alpha\beta}$ defined in Eq. (5b) is given for $i = 1$ and $\alpha = 1$ in Eq. (24). Then the H_2^- contribution to $\bar{\epsilon}^e_{1\beta}$ is

$$\left|\bar{\epsilon}^e_{1\beta}\right|_{H_2^-} < \sqrt{\Delta_\beta}/8 \tag{25}$$

where $\beta = 2, 3$. The contribution of η_2^- vanishes when $\alpha = 1$ because the anti-symmetric character of $\mathcal{K}_{LL}$.

Also we have the contributions of H_2^- and η_2^- that are constrained

$$|\bar{\epsilon}^e_{23}|_{H_2^-\eta_2^-} < \sqrt{\Delta_\tau}/8 \tag{26}$$

In the 331 model there are contributions to the right-handed charged-lepton decays mediated by V_μ^+, H_1^- and η_1^- do not sum coherently to the standard process. The relevant constraint is

$$\sum_{\alpha,\beta}\left|(\mathcal{K}^{\dagger}_{LR})_{i\beta}(\mathcal{K}_{LR})_{\alpha j}x + (\mathcal{K}'^{\dagger}_{LR})_{i\beta}(\mathcal{K}'_{LR})_{\alpha j}y\right|^2 + \sum_{\alpha,\beta}\left|(\mathcal{K}^{\dagger})_{\alpha j}(\mathcal{K})_{i\beta}\right|^2 z'^2 < (G_F^2/2)M_W^4\Delta_\mu \tag{27}$$

with α and β are related to the final neutrino states, j and i is related to the initial and final leptons respectively. If $j = 3$ ($j = 2$) implies τ (μ)-decay. Recall that although $\mathcal{K}$ is a unitary matrix, no symmetry relation exists in $\mathcal{K}_{LR}$ or $\mathcal{K}'_{LR}$ couplings. Each of elements of the sum defined in Eq. (27) also need to be constrained to be lower then the right side.

For example, the combination of product of matrices $\mathcal{K}$ given in Eq. (27 did not appear in the definition of $\bar{\epsilon}^e_{\alpha\beta}$ (Vide Eq. (5b)), but some of matrices element can be constrained to be small. The element $(\mathcal{K}^\dagger)_{\alpha j}$ with $j > 1$, or $(\mathcal{K})_{j\alpha}$ did not appear in the Eq. (5b), but can be constrained in the $\tau \to \mu\gamma$ decay (see below). Similar arguments apply for the $\mathcal{K}_{LR}$ or $\mathcal{K}'_{LR}$ couplings.

Concerning the contributions to the charged lepton decays arising in the multi-Higgs models of Section IV, there are no contributions to be coherently summed to the standard model contribution to the charged lepton decays. Therefore the relevant constraints coming from the charged lepton decay measurements can be written as

$$\sum_{\alpha,\beta}\left|\sum_j(\mathcal{V}_j)_{\alpha k}(\mathcal{V}_j^\dagger)_{l\beta}\xi_j\right|^2 < (M_W^2G_F/\sqrt{2})^2\Delta_j, \tag{28}$$

with α and β related to the final neutrino states, k and l is related to the initial and final leptons respectively. Also each term of sum of Eq. (28) is constrained to be smaller then the right side of Eq. (28).

Now, from Eq. (16) we can rewrite

$$\epsilon^e_{\alpha\beta} = \frac{1}{4\sqrt{2}G_FM_W^2}\sum_j\left(\mathcal{V}_j^*\right)_{1\alpha}\left(\mathcal{V}_j^{\dagger *}\right)_{1\beta}\xi_j. \tag{29}$$

Then comparing Eq. (29) and (28) we get

$$\left|\epsilon^e_{\alpha\beta}\right|^2 < \Delta_\alpha/64 \tag{30}$$

where $\alpha = 2, 3$ and any β.

These is a stronger constraint on the multi-Higgs model. Only the element ϵ^e_{11} is not constrained.

Lepton number conservation tests [27] can generate strong limits to FCNI parameters. In 331 model and in the multi-Higgs model there are constraints coming from $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$ and $\mu \to eee$. In the multi-Higgs doublets model there are contribution to the $\mu \to 3e$ decay mediated by the neutral Higgs flavor changing interactions given in Eq. (12) it constrains the same matrix elements that appear in the charged currents in Eq. (12) so, the mass of the neutral Higgs bosons and the respective mixing angles.

In 331, appear contributions to the $\mu \to e\gamma$ decay via the exchange of a gauge vector boson V ($\mu \to V\nu^c_\alpha \to e\gamma$); via exchange of charged Higgs H_1^- and η_1^- ($\mu \to H_1(\eta_1)\nu_\alpha \to e\gamma$), as well of H_2^- and η_2^- ; via exchange of doubly charged gauge boson U^{--} (the interactions are not showed here) [20] and via exchange of doubly charged Higgs H_1^{++} (H_2^{++}).

The amplitude of the contribution of gauge vector boson V is proportional to $\sum_\alpha \mathcal{K}_{2\alpha}\mathcal{K}^\dagger_{\alpha 1}$ and vanish because $\mathcal{K}$ is a unitary matrix.

The contribution of charged Higgs H_1^- and η_1^- can be written as

$$\left|\sum_\alpha (\mathcal{K}^\dagger_{LR})_{1\alpha}(\mathcal{K}_{LR})_{\alpha 2}x + \sum_\alpha (\mathcal{K}'^\dagger_{LR})_{1\alpha}(\mathcal{K}'_{LR})_{\alpha 2}y\right|^2 < (48\pi/\alpha(0))M_W^4 BR(\mu \to e\gamma) \quad (31)$$

where $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [31] is the branching ratio of the lepton violating process $\mu \to e\gamma$. Note, however, that the coupling $(\mathcal{K}_{LR})_{\alpha 2}$ and $(\mathcal{K}'_{LR})_{\alpha 2}$ do not appear in Eqs. (5a) and (5b) and therefore the limit above does not imply direct constraint on $\epsilon^e_{\alpha\beta}$ and $\bar\epsilon^e_{\alpha\beta}$.

The same happens with the contributions of H_2^- and η_2^-. Only the sum of some matrix elements is constrained:

$$\left|\sum_\alpha (\mathcal{K}^\dagger_{LL})_{\alpha 2}(\mathcal{K}_{LL})_{1\alpha}x' + \sum_\alpha (\mathcal{K}'^\dagger_{LL})_{\alpha 2}(\mathcal{K}'_{LL})_{1\alpha}y'\right|^2 < (48\pi/\alpha(0))M_W^4 BR(\mu \to e\gamma). \quad (32)$$

Even considering that the matrix $\mathcal{K}_{LL}$ ($\mathcal{K}'_{LL}$) is symmetric (antisymmetric), their combination appearing in Eq. (32) does not contribute to the definition of $\bar\epsilon^e_{\alpha\beta}$. Furthermore the constraint involves a coherent sum in which we can find cancellations making it weaker.

Finally there is a contribution of the doubly charged vector boson U^{--}, $\mu \to U^{--}l^+ \to e\gamma$; since the mass of the charged leptons are quite different, the amplitude only constrains $|\sum_\alpha \mathcal{K}_{\mu\alpha}\mathcal{K}^\dagger_{\alpha e}m_\alpha^2/m^2_{U^{--}}|^2$, with $m_\alpha = m_e, m_\mu, m_\tau$. Since the m_τ dominates, the decay constrains only $\mathcal{K}^\dagger_{\mu 3}\mathcal{K}_{3e}$. These contributions should be coherent summed to the H_2^{--} and to the H_1^{--}. The former have the couplings $\mathcal{K}_{RR}$ (See Eq. (2)) that did not appear in the Eqs (5a) and (5b) and the later $\mathcal{K}_{LL}$ that appear in those equations, but instead of H_2^- mass in the denominator of the Eqs (5a) and (5b) you have the H_2^{--} mass in the denominator. You can choose to put the masses of H_2^{--} field large enough to suppress the constraints on $\mathcal{K}_{LL}$.

The $\mu \to 3e$ decay arise by the interaction with the doubly charged Higgs H_1^{++} (H_2^{++}) and by the interaction with the doubly charged vector boson U^{++}. The Higgses H_1^{++} (H_2^{++}) constrain the matrix $\mathcal{K}_{LL}$ ($\mathcal{K}_{RR}$) and doubly charged vector boson U^{++} which constrains the $\mathcal{K}$ mixing matrix. The matrix elements involved are $\mathcal{K}^\dagger_{\mu 1}$ and $\mathcal{K}_{e1}$ and only the first one appears in Eq. (7a). Note, however, that the doubly charged vector and scalar bosons do not contribute to $\bar\epsilon^e_{\alpha\beta}$. Any constraint can be evaded by imposing that their masses are sufficiently large.

At first sight the process $\mu\nu_e \to e\nu_\mu$ would impose constraints on $(\mathcal{K}_{LR})_{1\mu}$ and $(\mathcal{K}'_{LR})_{1\mu}$ and other parameters involving the μ. However, this process has a cross section which is 0.90 ± 0.20 times the prediction of the $V-A$ theory [32] (or 0.98 ± 0.18 [33]) which does not imply therefore strong constraints on the related couplings. The doubly charged scalars $H^{++}_{1,2}$ may contribute to the muonium to anti-muonium conversion with the same strength as the doubly charged vector bilepton. It depends on the values of the parameters such as vacuum expectation values of the model [34].

Finally we mention that the FCNI parameters can be constrained using only charged leptons decays and some violating lepton number processes. We get that the contributions of charged Higgs scalars H_2^- and η_2^- to $\bar\epsilon^e_{1\beta}$, with $\beta = 2, 3$ and $\bar\epsilon^e_{23}$ are severely limited. Also some combinations of couplings are constrained to be very small like Eqs (22), (23), (31) and (31) although have no direct comparison with the relevant couplings of $\epsilon^e_{\alpha\beta}$ and $\bar\epsilon^e_{\alpha\beta}$. For the other couplings like $\mathcal{K}_{LR}$, $\mathcal{K}'_{LR}$ and $\mathcal{K}$ we have a compromise between different processes but no direct limit can be obtained.

For the case of multi-Higgs, the situation is different. Only the charged lepton decays impose a stronger limits on the $\epsilon^e_{\alpha\beta}$ and $\bar\epsilon^e_{\alpha\beta}$. Unless you enlarge the model, *e. g.* adding a set of singly charged Higgs which can made less drastic the constraints, we have no hope to get FCNI parameters large enough to get a relevant oscillation.

Also is worth to mention that the $\epsilon^e_{\alpha\beta}$ and $\bar\epsilon^e_{\alpha\beta}$ parameters are essentially different, then even in the case of massless neutrinos, the neutrinos and anti-neutrinos have different evolution in the matter. This is not the case in the R-parity violating supersymmetric models.

VI. CONCLUSIONS

Both models discussed above can generate FCNI with strictly massless neutrinos with strength sufficient to induce appreciable effects both for solar as well as atmospheric neutrinos. These effects can either be confirmed or ruled out as the collected data sample increases and the solar and atmospheric results become more precise.

If confirmed, it is necessary to know what is the actual theoretical framework that can account for these type of FCNI. In fact, there are very few models in the literature that introduced FCNI while keeping neutrinos massless. If, on the other hand, future data points undoubtedly in the direction of conventional neutrino oscillation (in vacuum or resonantly enhanced by matter), FCNI present in these models will have to be suppressed and these models will either be ruled out or their FCNI couplings severely restricted by data.

It is worth to mention that a very interesting intermediate scenario occurs when oscillations induced by $\bar\epsilon^f_{\alpha\beta}$ and $\bar\epsilon'^f_{\alpha\beta}$ co-exist with conventional neutrino oscillations. It was pointed out that in this situation simultaneously solution to the solar, atmospheric and LSND observations [35] can be obtained in a context of only three neutrino families [36], dispensing therefore the introduction of a fourth light (electroweak singlet) neutrino.

Constraints from several processes previously analyzed can generate limits to the FCNI parameters $\epsilon^e_{\alpha\beta}$ and $\bar{\epsilon}^e_{\alpha\beta}$. We see above that the 331 model survives like a model able to generate FCNI parameters compatible with the constraints and a viable model of FCNI and still keeping the neutrinos massless. This is a different situation from what happens in R-parity broken supersymmetric models where the combination of matrices entering in the definition of the FCNI parameters appear also contributing to constrained lepton violating processes. It is also different from what happens in the Zee model. There the unique anti-symmetric coupling makes it impossible to avoid the constraints.

So far, neutrinos remain massless. In both models we can get massive neutrinos simply by adding the right-handed components and the neutrino masses are arbitrary. In the 331 model we have still the possibility that one of the neutral component of the scalar sextet [22] (denoted σ^0_1 in the Eq. (23) of Ref [22]) get a non-vanish VEV, here denoted by v_1, giving to the neutrinos a majorana mass term

$$\begin{pmatrix} Gv_1 & \frac{1}{2}G'v_\eta \\ \frac{1}{2}G'v_\eta & M^R \end{pmatrix} \tag{33}$$

where M^R is a possible Majorana mass term for the right-handed singlets. The neutrino masses are still arbitrary but there are FCNI in the scalar sector. It means that there are Yukawa couplings that are not proportional to the neutrino masses. In both models we can also add several neutral or doubly charged scalar singlets only to get calculable neutrino masses [17,37]. Or, it is possible to get calculable neutrino masses breaking explicitly the total lepton number in the scalar potential [19]. In all these possibilities the neutrino masses do not depend on the parameters entering in the ϵ and ϵ'. One example of new contribution to the FCNI effect, when the right-handed neutrino is introduced, is showed in Fig. 4.

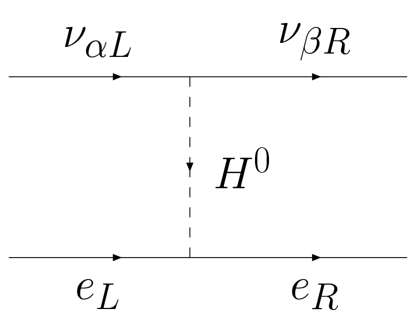

FIG. 4. Contribution if right-handed neutrinos are added.

Hence, we see that there are no strong constraints on all the parameters of the 331 model. And from this we hope that show a viable model for flavor changing neutrino interactions with strictly massless neutrinos that survive the constraints from charged lepton decays and some violating processes like $\mu \to e\gamma$, $\mu \to eee$.

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